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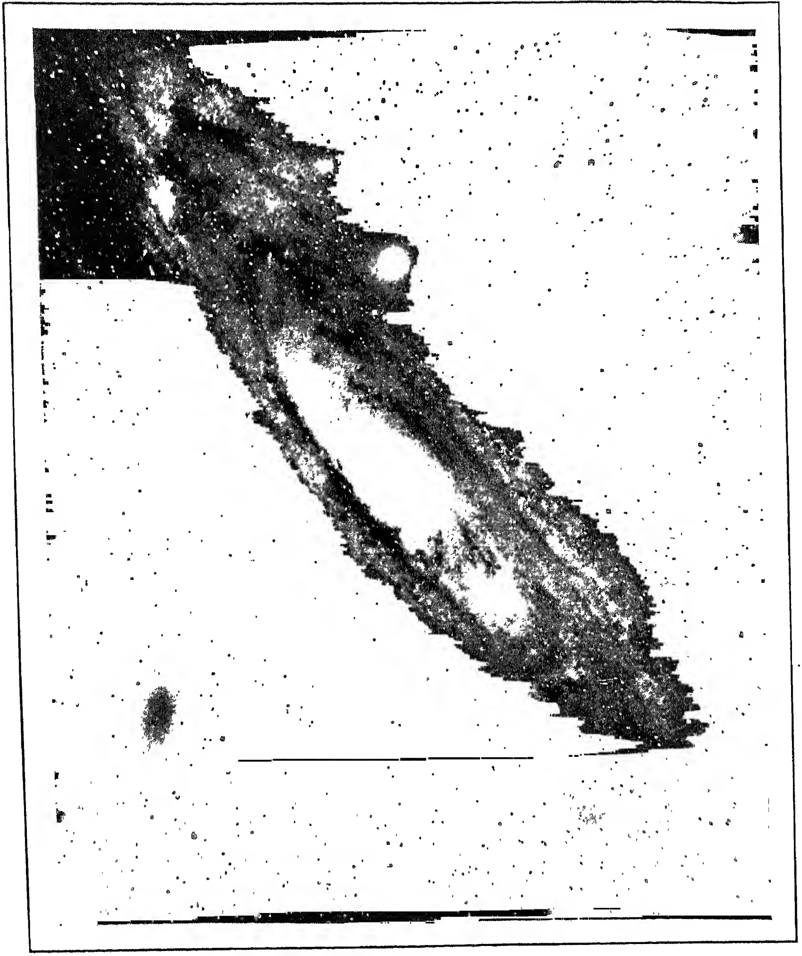

THE GREAT NEBULA IN ANDROMEDA (MESSIER 31 = N.G.C. 223)

The central portion of this nebula is conspicuous to the naked eye. The outer spiral arms are revealed only by photography. Recent investigations (p. 852) have shown that at least the outer parts of this nebula are composed of vast swarms of stars, and that its distance from the sun is of the order of a *million light-years*. The small nebula directly above the center of the large one and apparently almost on its edge (N.G.C. 221) has almost the same radial velocity, and is probably really a companion to it. This is also true of N.G.C. 205 (visible below and to the left). The numerous separate stars shown on the photograph are "foreground" objects, much nearer than the nebula. (Photographed by G. W. Ritchey, at Yerkes Observatory, with the 24-inch reflector)

ASTRONOMY

A REVISION OF
YOUNG'S MANUAL OF ASTRONOMY

II

ASTROPHYSICS AND STELLAR ASTRONOMY

BY

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Three branches of physics which possess highly important applications in astronomy are *mechanics*, the *theory of electromagnetic radiation*, and the *theory of atomic structure*. While it is by no means essential that the student of astronomy be also a master of these subdivisions of physics, he does benefit by a working knowledge of the fundamental facts and principles involved. Chapter X of this text gives some account of mechanics; Chapter XVII discusses atomic theory; and the present chapter treats of electromagnetic radiation and of the instruments for studying it. Radiation from hot bodies is dealt with in Chapter XVI. These discussions are primarily concerned with those parts of the various topics which have astronomical applications. For complete and detailed discussions of these subjects — which are among the most interesting in all modern science — the reader may refer to textbooks of physics, or to the works named at the ends of the chapters.

THE NATURE OF LIGHT

549. Electromagnetic Radiation. The passage of light and heat from the sun to the earth is an instance of the transfer through empty space of energy unassociated with the motion of material particles. Such transfer is accomplished by the process of electromagnetic radiation, described in the next section. The transfer of energy by means of material particles moving with high velocity is sometimes called corpuscular radiation; examples of this process are discussed in Chapter XVII.

Table VII exhibits the principal types of electromagnetic radiation recognized by the physicist: namely, Hertzian waves (radio), radiant heat, light, X-rays (or Röntgen rays), and gamma rays. Of these, light and heat are astronomically the most important; it is primarily through studies of their light and heat that knowledge of the heavenly bodies is acquired. Table VII also indicates the means by which radiation of the different types is detected and produced respectively.

Wave-length. Electromagnetic radiation of all types is believed to possess in empty space essentially the same properties. The only characteristic that distinguishes one type from another is the wave-length. This varies, as is indicated in Table VII, from many kilometers for the longest Hertzian waves to the billionth part of a centimeter for the shortest X-rays. The physical meaning of the term "wave-length" is explained in section 550.

The wave-length of Hertzian waves is generally expressed in meters. The wave-lengths of light and X-rays are expressed in angstroms (indicated throughout this book by the prefix λ) one angstrom is 10^{-8} centimeter, or one ten-millionth of a millimeter, or 0.3937×10^{-8} inch. (The name of this important unit was selected in compliment to A. J. Ångström, the Swedish physicist, who first made accurate measures of wave-lengths of light.) Thus, λ 6000 refers to a wave-length of 6000 angstroms or 6×10^{-5} centimeter, or 1/42,330 inch.

The velocity of light. There is good experimental evidence tha all types of radiant electromagnetic energy travel throug empty space with the same enormous speed (denoted by th symbol c), — a speed of very nearly 300,000 kilometers, c

TABLE VII. THE TYPES OF ELECTROMAGNETIC RADIATION

TYPE OF RADIATION	APPROXIMATE WAVE-LENGTH LIMITS (IN CENTIMETERS)	METHOD OF DETECTION	METHOD OF MEASUREMENT OF WAVE-LENGTH	METHOD OF PRODUCTION OF THE RADIATION
Hertzian (radio)	2,500,000.0 (arbitrary) 10.0	Resonant electric circuits	Electromagnetic constants of detecting circuit	Hertzian oscillators; oscillating electric circuits
Short electric waves	10.0 0.03	Nichols radiometer; thermal effect	Interference methods	Small electric oscillators
Infra-red radiation	0.03 0.00,008	Bolometer; thermal effect	Interference methods, with rock-salt prisms	Small changes in molecular systems and between nearly equal atomic energy states
Visible light	0.00,008 0.00,004	Eye; photographic plate	Interference methods	Relatively large changes in molecules; moderate changes in atomic energy states
Ultra-violet radiation . . .	0.00,004 0.00,000,14	Photographic plate	Interference methods	
Soft X-rays	0.00,000,14 0.00,000,022	Photo-electric effect	Indirect methods	Large changes in atomic energy states
X-rays	0.00,000,022 0.00,000,000,1	Photographic plate; ionization of gas	Crystal diffraction; ruled gratings at grazing incidence	
γ-rays	0.00,000,004 0.00,000,000,05	Ionization of gas	Energy of photo-electrons; absorption coefficients	Changes in energy states of atomic nuclei
Cosmic radiation	? 0.00,000,000,000,4	Ionization of gas	Absorption coefficients	Unknown cosmic origin; speculatively ascribed to transformation of matter into energy

This division into nine types is somewhat arbitrary, being based upon the methods of detection or production, and not at all upon differences in the quality of the radiation itself. Likewise, the wave-length limits assigned are subject to adjustment (for example, still smaller electric oscillators may hereafter be designed, pushing to smaller values the lower wave-length limit of the short electric waves); and, again, the limiting wave-lengths of visible light vary somewhat with different observers.

It will be noted that the range of wave-lengths tabulated is from 2.5×10^6 cm. (this upper limit, indeed, might be increased) to 4×10^{-12} cm., — a ratio of 6×10^{17}, or nearly 60 octaves.

The corresponding frequencies are obtained by dividing 3.00×10^{10} by the wave-lengths, and range, therefore, from 1.2×10^4 to 7.5×10^{21} cycles per second.

(The meaning of the term "energy states," used in the last column, is discussed in Chapter XVII.)

186,000 miles, a second. This quantity is one of the fundamental constants of physics and has been accurately determined by observations of light (§ 556).

550. The Electromagnetic Theory of Radiation; Wave-Motion Years of investigation culminated during the latter part of the nineteenth century in the theoretical discovery by Maxwell, and the observational confirmation by Hertz (1888), of the electromagnetic nature of light and analogous forms of radiation. Such radiation is characterized by a moving electric field (capable of exerting forces on electrically charged particles) and by a moving magnetic field.

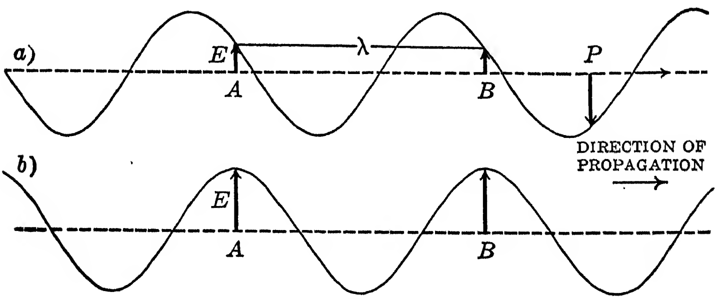

FIG. 184. Electromagnetic Wave-Motion

The ordinates of the sine-curve (*a*) (with respect to the central dotted line) represent the values of the electric vector *E* at points *A*, *B*, *P*, etc., along a line drawn in the direction of propagation of the radiation, at a certain instant of time. The magnitude of *E* is the same at *A* and *B*, which are spaced a wave-length, λ, apart. The sine-curve (*b*) is exactly similar except that it is displaced about one-eighth wave-length to the right. Its ordinates represent the magnitude of *E* at a slightly later instant. At the points *A* and *B*, *E* has increased to its maximum value and, at a slightly later instant still, would be found to have decreased, or even to have reversed its direction; but its value at *A* keeps equal to its value at *B*.

The *electric field* (or force) at a given point (*E* in Fig. 184) is, in free space, always at right angles to the direction in which the radiation is traveling (the direction of propagation); and at a given instant of time it possesses the same magnitude at points spaced a *wave-length*, λ, apart in the latter direction. Along a line drawn in the direction of propagation its magnitude at each point, at the same instant of time, is represented by the ordinates (heights) of a wavy curve (sine-curve), and the wave-length is merely the length of one wave in this curve, — the distance from crest to crest, or from trough to trough.

At any given point the magnitude of E changes with enormous pidity, oscillating back and forth from positive to negative lues. The manner of this change may be represented accu- tely by supposing the curve just described to move with the elocity of light in the direction of propagation of the radiation. hus, the points at which E has any given value move forward, hat is to say, the electric field moves with the radiation. At a xed point the field oscillates rapidly as the crests and troughs f the curve move past it; each wave in the curve corresponds o one such oscillation.

At right angles to the electric field and to the direction of ropagation there is a similar oscillating *magnetic field*, H. The wo fields, if measured in appropriate units, are, in free space, erywhere and always numerically equal.

requency. The number of oscillations, in unit time, of E and H at a given point is called the *frequency*, ν. In view of what has been said it is obvious that

$$\nu = c/\lambda,$$

when the radiation is propagated in free space. This equation is characteristic of a wave-motion.

The numerical value of ν is very great; for green light it is 6×10^{14}, that is, six hundred thousand billion vibrations per second.

In place of the wave-length or frequency the *wave-number*, n, or number of waves per centimeter, is sometimes employed; thus, $n = 1/\lambda$ (provided λ is measured in centimeters), or $n = 10^8/\lambda$ (if λ is measured in angstroms).

Refractive index. Electromagnetic waves travel through a transparent material medium with a velocity, V, usually less than c. The *index of refraction*, μ, is a number such that V is c/μ. The value of μ depends upon the properties of the medium and the wave-length of the radiation: for yellow light its value is 1.5 in flint glass; and for air at the normal density, 1.0003 (approaching unity as the density is decreased). The *frequency* is not changed when radiation passes from free space into a transparent medium, but the *wave-length* becomes λ', conforming to the relations

$$\nu = V/\lambda'; \quad \text{or} \quad \lambda' = \lambda/\mu.$$

Monochromatic radiation. Radiation specified by a single value of λ or ν is called monochromatic (singly-colored). The sine-curve of Fig. 184 refers to such radiation. When the radiation is not monochromatic, it may be regarded as a summation of a number of monochromatic wave-trains, for each of which a similar sine-curve of appropriate wave-length may be drawn. Such resolution of radiation into monochromatic components is actually effected by the spectroscope, considered below.

Wave-front and phase difference. At right angles to the direction in which radiant energy is being transferred, surfaces can be drawn such that at any given instant of time the value of E or H is the same at all points of a given surface, or *wave-front.* In the simplest case these are parallel planes (as when the source of the radiation is at a great distance). When the radiation is monochromatic, the distance in wave-lengths between two such surfaces is called their *phase difference.*

551. Polarization. The electromagnetic wave-motion, although always *transverse,* that is, across the direction in which the energy is transferred, may be of several different types, some of which are called polarized.

Plane-polarization. In the simplest case the direction of the electric field associated with the radiation remains constant, so that the moving curve whose ordinates represent the magnitude and direction of E lies in a single plane, as in Fig. 184. When this condition is satisfied, the radiation is said to be plane-polarized, and the plane through the direction of propagation and at right angles to E is conventionally known as the plane of polarization (at right angles to the paper in Fig. 184).

Circular polarization. In circularly polarized monochromatic radiation the magnitude of E remains constant, while its direction, at a given point in space, rotates uniformly (with the frequency of the radiation) about the direction of propagation. In this case the plane curve is to be replaced by a helix (like a screw-thread of pitch λ, winding about a circular cylinder) moving bodily forward with the speed of light. If, to an observer looking in the direction of propagation, the direction of rotation of E is clockwise, the circular polarization is said to be right-handed.

Elliptical polarization. Elliptically polarized radiation presents a slightly more complicated case; not only does the direction of E rotate, but its magnitude undergoes an oscillation in the same period. Here the variations of E can be described by means of a spiral drawn on an elliptical cylinder. Elliptically, as well as circularly, polarized radiation can be treated as the resultant of two component elements, differing in phase by the appropriate fraction of a wave and plane-polarized in mutually perpendicular planes.

Unpolarized radiation. These cases are simpler than the state of affairs which exists in ordinary light or in most types of radiation not produced by highly artificial means. Natural radiation, even when monochromatic, is the summation of contributions from innumerable atoms, each of which radiates but a very short time. The radiation from each atom is believed to be polarized, but usually there is no definite relation between the planes of polarization, so that the polarization of the resultant radiation changes, gradually but completely, in a random fashion, millions of times a second. Under these conditions the radiation exhibits identical properties in all directions transverse to the direction of propagation, and thus is unpolarized.

Polarized light is produced by natural means (1) when the light is reflected from a polished surface, (2) when the light is scattered by small particles, (3) when the source of light is in a magnetic field. All these phenomena have astronomical applications (§§ 228, 582, 596).

568. Radiant Flux, and Flux Density, or Intensity. Radiant energy traveling across a given surface in approximately a given direction is said to form a *beam*. The radiant flux is the amount of radiant energy that flows along the beam per unit time. The *flux density*, I, is defined as the amount of radiant energy that flows per unit time across unit area of a perfectly transparent plane surface placed at right angles to the beam, that is, as the flux per unit area. The flux density is often, although not quite accurately, called the *intensity* of the beam.

Units of intensity. Since the quantities, energy, time, and area enter into its definition, intensity must be measured in terms of a combination of units of each of these. Energy may be expressed

in several types of units; and since energy transferred per unit time is, by definition, equivalent to *power*, intensity may be measured in units of power. Thus a variety of units of intensity exist; the most useful are ergs per second per square centimeter; calories per second (or per minute) per square centimeter; watts per square centimeter, or horsepower per square foot. (A watt is 10^7 ergs per second, a horsepower is 746 watts, and a calorie is 4.186×10^7 ergs. Since a calorie is the standard unit of heat, and an erg of mechanical energy, the latter value is known as *the mechanical equivalent of heat.*)

The intensity of a beam of radiation is proportional to the (average) square of the magnitude of the associated electric field. When the field strength is given in volts per centimeter, the value of the intensity in ergs per second per square centimeter is, in free space, $c(E/300)^2/8\pi$.

The inverse-square law of intensity. When radiant energy is diverging from a point in space (or from a uniformly luminous sphere), the intensity is inversely proportional to the square of the distance from that point (or from the center of the sphere). Thus, the intensity of the light and heat from the sun (§ 604) is 1.35×10^6 ergs per second per square centimeter at the average distance of the earth's orbit; and at the sun's surface, which is 215 times nearer the sun's center, the intensity is 215^2 times as great, or 6.24×10^{10} ergs per second per square centimeter. (The value of the associated oscillating electric field is, in the latter case, 2170 volts per centimeter.)

553. Radiation Pressure. A beam of electromagnetic radiation carries momentum as well as energy, and thus exerts a pressure on a surface which absorbs or reflects it. This pressure was discovered theoretically by Maxwell, and its existence experimentally demonstrated (in 1900) by Lebedew, in Russia, and by Nichols and Hull in America.

The force per unit area at right angles to a beam of intensity I (that is, the radiation pressure) is I/c, provided the radiation is traveling in empty or nearly empty space. Thus, when sunlight falls at right angles on a black surface, it exerts a pressure of $(1.35 \times 10^6)/(3 \times 10^{10})$, or 4.5×10^{-5} dynes per square centimeter. This pressure, per square centimeter, is equivalent to only 1/28,000 of the weight of a cubic centimeter of air.

So tiny a pressure can be observed only with delicate instruments. One of the most interesting exhibits on public view in the building of the National Academy of Sciences in Washington, D. C., is an apparatus in which the radiation pressure of a beam of light is caused to rotate a small suspended mirror.

Although radiation pressure is usually too small to produce appreciable effects, it controls the form of comets' tails (§ 516); and, as Eddington has shown, in the deep interior of stars, where the radiation is extremely intense, it rises to high values and is of great importance (§ 952).

554. Direction. Often only the direction in which radiant energy is traveling is of interest, irrespective of wave-length and polarization. A line drawn in this direction is called a *ray* and represents a path of energy transfer. In empty space, or in any isotropic medium (such as water, glass, and crystals of the simple types), the rays are straight lines perpendicular to the wave-front.

The rectilinear propagation of electromagnetic radiation in free space is of much practical importance. Thus, the direction of a ray of starlight when it reaches the earth's orbit is in general the same as the direction of the star from the earth, and this fact simplifies the astronomer's survey of the universe.

Light rays are changed in direction by three processes: reflection, refraction, and diffraction. The last is evident when radiation passes through small openings in a screen, and is utilized in the diffraction grating (§ 560). When radiation, traveling in a given medium (as air) impinges on the boundary of a second medium (say, glass), in which the radiation travels with a different velocity, part is reflected back and part transmitted, but not in the original direction (refracted). In the case of many crystals (Iceland spar, mica, etc.) there are two refracted beams polarized at right angles to each other.

It is shown in textbooks of physics that these three processes are explained by the wave theory of light. The action of mirrors, lenses, and prisms — the essential features of most optical instruments — involves especially reflection and refraction. These matters are well treated in many elementary texts, particularly in works on geometrical optics, and need not be further discussed here.

ASTROPHYSICAL INSTRUMENTS

555. To summarize: light and the other forms of electromagnetic radiation are characterized by direction, speed, momentum and radiation pressure, wave-length, polarization, and intensity. Phenomena associated with each characteristic are of astronomical significance, and special instruments are employed for the investigation of each.

The instruments (described in Chapter II) employed in the old astronomy, or the astronomy of position, make use chiefly of the directional property of light-rays, and do not subject to analysis the light from celestial objects. Beginning about 1860 with the application of the spectroscope, and continuing with the development of accurate astronomical photometry (intensity measurement) and the utilization of polarization phenomena, the field of astrophysics owes its rapid expansion to investigations relating to the other properties of light and heat. In the remainder of this chapter the important types of instruments employed in these investigations are described.

556. Determination of the Velocity of Light. Before proceeding to a description of these instruments we may consider how the speed of light (commonly denoted by c) is determined.

The first two determinations of c were made by astronomical observations, namely, of the eclipses of Jupiter's satellites (§ 442) and of the aberration of light (§ 162). Measurements with terrestrial sources, however, can be made with greater accuracy; the best and most recent are those of Michelson (Mt. Wilson, 1926), yielding the value 299,796 kilometers per second, or 186,285 miles per second, as the speed in vacuum.

In this investigation the time was determined which was required for light to travel the 35,426.3 meters (about 22 miles) from a point on Mt. Wilson to a point on Mt. San Antonio, and back again after reflection by a fixed mirror. The source of light was a slit illuminated by a powerful arc. A rapidly rotating octagonal mirror (driven by an air-blast) reflected the light toward the distant station, not as a steady beam but as a rapid succession of nearly instantaneous flashes. The returning flashes were reflected, from another face of the same rotating mirror, to a micrometer eyepiece. The measurement consisted in adjusting the speed of rotation to such a value that during the time that each single flash occupied in traveling

the 70.8526 kilometers the octagonal mirror rotated exactly one eighth of a turn, as indicated by the direction of the returning ray after such reflection. The critical speed of rotation was determined as 528.76 turns per second; one eighth of a turn at that speed corresponded to 0.00023640 seconds, so that the indicated speed of light in air was 70.8526/0.00023640, or 299,711 kilometers per second. Multiplication by the index of refraction of air gave the value in vacuum stated above. Observations with mirrors with different numbers of sides and rates of rotation gave results agreeing within a few kilometers.

557. Study of Wave-Length; the Spectroscope. The angle through which a ray is deviated as the result of refraction or diffraction depends upon the wave-length. Consequently a beam of light which is not monochromatic can, by either process, be *dispersed* into separate monochromatic beams traveling in different directions.

A study of the monochromatic components of the original radiation yields important information concerning the luminous object, however distant, which is the source of the radiation. An instrument designed for this purpose is called a spectroscope. It is the most important astrophysical instrument. By its use the chemical composition and physical condition of stars can be deduced; the speeds with which they are moving toward or from us can be measured, and certain objects otherwise invisible, such as the solar prominences, can be observed.

The essential part of the spectroscope is either a prism or a train of prisms, or else a diffraction grating (§ 560). Either the prism or the grating performs the office of dispersing the rays of different wave-length and color. As is indicated in Table VII, radiation is perceptible as visible light only in a small range of wave-length. Within this range each wave-length corresponds to a *definite color*. To the physicist the wave-length is the important thing rather than the color, which is a sensation not directly expressible in precise terms; nevertheless convenience is often served by describing light by its color. (Light of wave-length λ 7000 is red; λ 6000, yellow; λ 5000, blue-green; λ 4000, violet.)

558. Formation of the Spectrum. The flame of a bunsen burner is normally only faintly luminous. If a compound of lithium is inserted (on the end of a wire, for example), the flame becomes much brighter and strongly red. Through a prism a single sharp

red image of the flame is seen, which indicates that luminous lithium vapor emits strongly, in the visible range, only a single monochromatic radiation. If a sodium compound (as common salt) is employed instead of the lithium, the flame is yellow, and only a yellow image appears with the prism. If both lithium and sodium are inserted, the prism gives two images, one red, the other yellow. The yellow rays are deviated more than the red.

The slit. This arrangement cannot clearly separate two beams of nearly the same wave-length, because the flame is so large that the two images overlap. Improvement is effected by placing, between the flame and the prism, an opaque screen, pierced with a narrow *slit* parallel to the refracting edge of the prism, so that images of the flame are replaced by images of the slit. Lithium

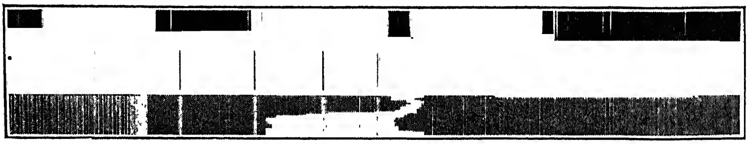

FIG. 185. Line and Band Spectra

The upper strip shows a small portion of the arc spectrum of iron, in the green; the middle, the same region in the spectrum of the titanium arc; and the lowest strip, that of an electric furnace in the same wave-lengths, showing lines due to titanium, and a conspicuous band due to some carbon compound

gives (conspicuously) a single red slit-image, or "bright line"; but sodium, with a powerful prism, gives two yellow images very close together. The combined spectrum, then, consists of two yellow lines of light, and one red, against a dark background. Thus, glowing sodium vapor strongly emits monochromatic radiation in two neighboring wave-lengths; lithium, in only one.

Spectrum analysis. If any luminous rarefied gas is thus observed, its spectrum is found to consist of bright lines, often numerous and sometimes crowded so closely together as to form bands (Fig. 185). Each spectrum is characteristic of the atom or molecule that is radiating; the wave-length of each line is a definite thing, independent of the method of exciting the gas to luminescence, whether by heat in the flame or by the electric arc or spark (see, however, § 564). The intensity, or conspicuousness, of each line depends, often greatly, on the conditions of excitation (§ 592).

The spectra of all the known chemical elements (except those just discovered) and of many compounds have been investigated and accurately mapped. When several kinds of radiating atoms or molecules are present in a luminous rarefied gas, the various characteristic spectra appear simultaneously and unchanged,— a fact of primary importance which permits the determination of chemical composition by spectrum analysis.

559. Details of Construction. *Collimator.* If non-parallel rays of light are allowed to pass through a prism, they cannot afterward

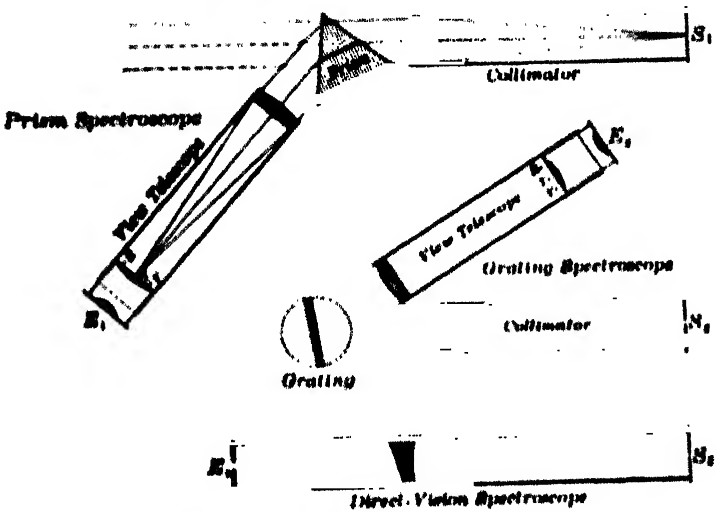

FIG. 186. Forms of Spectroscope

In each of the three forms of spectroscope here illustrated the letter *S* indicates the slit, and *E*, the eyepiece; *R*, *Y*, *V* mark the positions where the red, yellow, and violet images, respectively, of the slit are formed

be brought to a sharp focus. Consequently the rays from the slit, in the usual spectroscope, are made parallel, before passing through the prism, by an achromatic object-glass having the slit in its principal focus. The object-glass and slit are placed at the opposite ends of a tube, which excludes stray light. This arrangement constitutes the collimator.

View telescope or camera. The spectrum is usually magnified by a small view telescope. When photographs are to be made, the view telescope is replaced by a camera. The spectroscope,

therefore, as usually constructed, and as shown in the diagram (Fig. 186), consists of three parts; collimator, prism (or grating), and view telescope or camera.

Condensing lens. Often it is desirable that light from the source should be concentrated on the slit; this is accomplished by forming on the slit, with the aid of an auxiliary lens placed in front, an image of the source. The object-glass of an astronomical telescope is so employed when the spectra of celestial objects are studied.

560. The Diffraction Grating. This is a piece of speculum metal (reflection grating) either accurately plane or concave, ruled with many thousand straight, equidistant lines (grooves), from 10,000 to 20,000 in each inch. If a beam of monochromatic light falls on such a grating, a part is reflected in the ordinary manner; other portions are *diffracted* as beams proceeding in several different directions which depend upon the wave-length of the radiation.[1]

Thus several images of a single monochromatic source are formed by the diffracted rays. These lie both to the right and to the left of the ordinary image formed by the reflected rays, and are spoken of as first order, second order, and so on. By modern methods gratings can be ruled which will throw a large portion of the light into a single one of the diffracted beams, — a desirable economy of light.

A plane grating, like a prism, functions well only with parallel light and is employed with the same accessories: slit, collimator, and view telescope or camera.

With a concave grating neither collimator nor view telescope need be employed, as the grating itself focuses the light from the slit. An eyepiece may be used to view the spectrum; in photographic work this is replaced by a plate-holder, and thus glass is eliminated from the optical system, — a great advantage in studies of ultra-violet light, to which glass is opaque.

The ruling engines by which gratings are made are exceedingly precise mechanisms, and literally years of labor are required to put one into working order. Original gratings are therefore

[1] For a full discussion see a textbook of optics; for example, O. M. Stewart, *Physics*, p. 690 (Ginn and Company, 1924).

expensive, but replicas, or casts in celluloid from a metal original, may be purchased for a few dollars. These are transparent and may be used either with reflected light or, better, with transmitted light (*transmission grating*). Such a replica, together with an ordinary field-glass and an improvised slit, forms a spectroscope of nominal cost and very considerable power, with which the student may make many observations of much interest.

561. Relative Advantages of Prism and Grating. Three desiderata in a spectroscope are *high dispersion, large resolving power, and a bright spectrum.* By high dispersion is meant large angular separation of the beams corresponding to different wave-lengths. Large resolving power implies the capacity of forming separate slit-images corresponding to wave-lengths which are nearly equal.

In the matter of intensity the prism is superior to the grating, since in the latter a large proportion of the incident light is wasted in the unnecessary duplication of images in different orders. Consequently, prism spectroscopes are always used with faint sources, such as the stars.

A grating gives much higher dispersion than a single prism. With a train of prisms increased dispersion can be obtained; but owing to absorption in the glass and partial reflection at each surface, intensity is correspondingly sacrificed. Therefore gratings are used in all high dispersion work, whether with the sun or with brilliant laboratory sources. For ultra-violet light, glass is opaque, and gratings, or quartz or fluorite prisms, must be used. To still shorter waves air also is opaque, and the whole apparatus must be placed in a vacuum.

As regards dispersion the grating possesses another advantage in giving an approximately *normal* spectrum, that is, one in which equal distances correspond to equal differences in wave-length. A prism spreads out the blue end of the spectrum in comparison with the red.

The *resolving power* of a grating varies as the product of the order of the spectrum by the number of ruled lines in the grating which are actually utilized in dispersing the initial beam. This product is arithmetically equal to the ratio $\lambda/\delta\lambda$, where $\delta\lambda$ is the difference in wave-length, in the neighborhood of wave-length λ, which corresponds to the closest spectral lines that can clearly be separated.

The same ratio, when resolution is effected by a train of prisms, is expressed by the product of a quantity (that depends upon the material of the prism and the wave-length of the light) by the total effective thickness of all the prisms in the train. The employment of large prisms therefore secures greater resolving power. As optically perfect glass is required, large

prisms are expensive, and gratings are used for most work requiring high resolving power.

It will be recalled that the term "resolving power" is applied in essentially the same sense to the object-glass of a telescope (§ 53).

562. Important Types of Spectroscope. The two main types, prism and grating, are designed in various forms for special purposes. Prisms may be made of glass, or quartz, or other material transparent to the radiation studied, and gratings are ruled with many or relatively few grooves to the inch, and with grooves of special shape.

Hertzian waves can be analyzed with prisms made of pitch or with coarse gratings constructed of parallel copper wires. X-rays are analyzed by reflection from suitable crystals, in which the ordered rows of atoms afford a sort of natural grating, with tens of millions of lines to the inch; ordinary gratings also have been successfully employed.

The *concave grating* is of much importance for laboratory investigations; it gives a spectrum that is exactly normal and requires no auxiliary lenses. Rowland employed this type of spectroscope for his classical studies of the solar spectrum (at Johns Hopkins University, about 1890).

With a plane grating the *Littrow mounting* is convenient, in which the diffracted beam is sent directly back through the object-glass of the collimator, which acts also as a view telescope, with the slit a short distance above its axis and the eyepiece a little below.

The *direct-vision spectroscope* (Fig. 186) is a handy instrument for low-dispersion work. It employs a train of prisms so arranged that collimator, prism, and view telescope lie along the axis of a single tube containing the whole system.

A spectroscope arranged for photography is called a *spectrograph*. A spectroscope especially adapted to the measurement of wave-lengths is called a *spectrometer*.

563. Measurement of Wave-Length. A grating spectrometer is capable of yielding fairly good fundamental measurements of wave-length, deduced from accurate observations of the angles of diffraction and from a precise knowledge of the ruling interval. Still more accurate fundamental measurements can be obtained with a suitable type of interferometer. The accepted international primary standard of wave-length was determined, with the interferometer, by very careful comparisons with the standard meter; it is the wave-length, namely, 6438.4696 angstroms, of a certain red line of cad-

mium. By interferometer comparisons with this primary standard the wave lengths of a considerable number of secondary and tertiary standard lines, mainly in the iron spectrum, have been determined to within one part in a million. With the aid of these the wave-lengths of other lines may be found by differential methods with an ordinary grating; if the lines are sharp and the dispersion high, the results should be good to 0.01 angstrom or better.

Corrections. In precise work wave-lengths are referred to vacuum; wave lengths in air, therefore, must be multiplied by the appropriate index of refraction (see the table on page 494). Modern measures are expressed in "international angstroms"; Rowland's values were determined on an older and less accurate scale and need to be diminished by about 1/30,000, but as the factor varies somewhat a conversion table must be employed.

Comparison spectra. The wave-lengths of the radiation from any source may be obtained by comparing its spectrum with a second spectrum which includes many sharp, strong lines of accurately measured wave-length. The two spectra are secured simultaneously, with the same instrument, as follows: light from one of the two is focused on the upper half of the slit, and light from the other is focused on the lower half. For greater accuracy a comparison spectrum may be put on each side of the unknown spectrum (Fig. 187).

564. Changes in Wave-Length. When a luminous gas is emitting monochromatic radiation, an increase in its density is often accompanied by changes in wave-length (of the order of a fraction of an angstrom, and usually an increase). This so-called *pressure shift* comes about because of the mutual influences of neighboring atoms (§ 590). Somewhat similar changes are observed when the emitting gas is near one pole of an electric arc (*pole effect*). These types of change are complicated and vary with the emission line concerned. The normal wave-length of a given line refers to radiation from the given substance when in the form of a gas at very low density, not subjected to an electric field or to a magnetic field (§§ 595, 636).

Doppler effect. When the distance between the source of radiation and the device which receives it is changing at a uniform speed, v, all wave-lengths concerned are changed according to the following simple relation,

$$\lambda' = \lambda(1 + v/c),$$

where λ is the normal wave-length, λ' the changed wave-length, and c the velocity of light, and where v is counted positive if the distance is increasing, negative if it is decreasing.

This type of change of wave-length is known as the Doppler effect, and the wave theory of light, as well as the special theory of relativity, affords a simple explanation of it. It is closely related to the aberration of light (§ 162), and both phenomena are best observed astronomically. The Doppler effect is of much astrophysical importance, since it makes possible the direct measurement of the velocity of stars and other heavenly bodies in the line of sight (§§ 579, 729). A given chemical element, in all probability, emits radiation of very nearly the same wave-lengths, whether it is shining in a star or in a laboratory. Consequently a discrepancy in the measured wave-length in the two cases may usually be ascribed to Doppler effect, and the radial velocity, v, may be calculated accordingly (Fig. 187). For a given source the Doppler shift, for all lines, is proportional simply to the wave-length, and so may be distinguished from the more complicated pressure shift and pole effect, which are very different for different lines even of the same element. (Doppler announced the principle, in 1843, as affecting color; Fizeau, in 1848, was the first to point out its effect in shifting the lines of the spectrum.) The minute "relativity shift" (§ 364) behaves exactly like a Doppler shift.

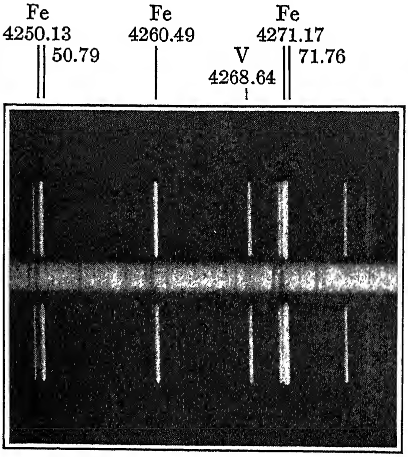

FIG. 187. Spectrum of the Star *Procyon*, showing the "Doppler Shift"

The narrow central strip is the spectrum of the star. The bright lines on each side come from an electric arc near the spectroscope, and are due to iron and vanadium. The iron lines are conspicuous as dark lines in the spectrum of the star, and are shifted toward the violet by an amount corresponding to a velocity of approach of 30 km./sec. Most of this is due to the earth's orbital motion. (From photograph by V. M. Slipher, Lowell Observatory)

In the case of sunlight reflected from a planet the Doppler effect corresponds to the rate of change of the *sum* of the distances from the sun to the planet and from the planet to the earth. The shifts due to the rotation of Saturn and its rings, for example, thus correspond to twice the rotational velocity (Fig. 188).

565. Polarizing devices are of many types, of which only those of astrophysical importance are mentioned here. One of the most useful is the Nicol prism; this is a long, narrow block of Iceland spar, consisting of two parts cemented together, through which light is sent lengthwise. It transmits only that component of the

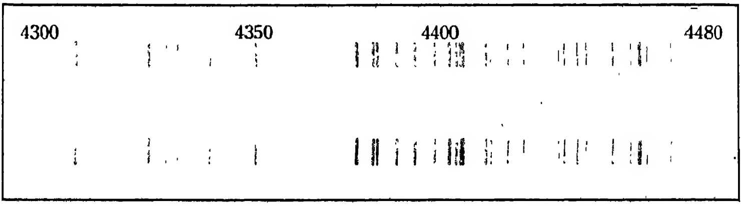

FIG. 188. Spectrum of the Ball and Rings of Saturn

A small region in the violet photographed with high dispersion. The bright-line spectrum on each side is that of iron, photographed as a standard for comparison. The upper part of the spectrum of the planet and rings comes from the eastern side, which is approaching the earth, — owing to the planet's rotation, — so that the lines are shifted toward the violet. At the time of observation the earth was approaching Saturn, and the shift due to this cause is added to that arising from the rotation, — about doubling it at the eastern limb, and nearly neutralizing it at the western (the bottom). This is evident on comparing the strong bright iron lines in the comparison spectrum with the corresponding dark lines in the planet's spectrum. Close inspection shows that the displacement of the lines in the spectrum of the ring is greatest at the inner edge, which indicates that the particles at the outer edge are moving more slowly than those at the inner edge (§ 460). (From photograph by V. M. Slipher, Lowell Observatory)

original radiation which is polarized in the *principal plane* of the Nicol (perpendicular to the short diagonal of its cross-section), and cuts off the remainder. When it is rotated about the direction of the incident beam as an axis, the variation in intensity of the transmitted light serves as an index to the degree of polarization of the original beam. If this is unpolarized, there is no such variation; if it is plane-polarized, the intensity falls from a maximum to zero as the Nicol is rotated 90° from the appropriate initial position.

Since an unpolarized beam (it will be recalled that ordinary light is unpolarized) becomes completely plane-polarized by

passage through such a prism, if a second Nicol is inserted in the optical path, with its principal plane at right angles to the first, no light at all is transmitted through it; at intermediate angles a calculable fraction of the light is transmitted. Thus, *crossed Nicols* afford an excellent quantitative means of varying the intensity of a beam (§ 569).

When unpolarized light is reflected from a smooth surface, it becomes partially polarized; the plane of polarization is that of the incident ray and the perpendicular to the surface. If the angle between the latter two directions is about 57°, in the case of a glass surface, the reflected beam is almost completely polarized, and is thus incapable of reflection from a second glass surface set also at that angle and turned so that the two respective planes of incidence are mutually perpendicular. If these planes are not quite perpendicular, only a small fraction of the light incident on the first mirror is reflected from the second. This is the principle of the common polarizing eyepiece used in astronomical telescopes for cutting down the brightness of the solar image (p. 195, Vol. I).

The *quarter-wave plate* is a very thin plate of mica or other doubly refracting crystal; when plane-polarized light is incident upon such a plate it is decomposed into two beams, mutually polarized at right angles and transmitted with different velocities. The emergent components join to form a single beam which is circularly polarized, since the thickness of the plate is such as to permit the phase of one component to gain a quarter-wave upon the other. On the other hand, and for the same reason, if the incident light is circularly polarized, the emergent beam is plane-polarized, and the direction of its plane of polarization differs by 90° according as the circular polarization of the incident beam is right-handed or left-handed.

Circularly polarized light observed with a Nicol prism is indistinguishable from unpolarized light; but if a quarter-wave plate is inserted ahead of the Nicol, the light becomes plane-polarized and thus can be extinguished if the Nicol is rotated.

566. Types of Instrument for the Measurement of Intensity. There are two main types: (1) energy-measuring devices, affected by radiation of any wave-length; (2) photometers, affected only by radiation of a specific band of wave-lengths (for example, in the visible region, which lies, roughly, between 4000 and 8000 angstroms), and measuring light rather than radiant flux.

567. Energy-Measuring Devices. Among these are the bolometer, spectrobolometer, thermocouple, and pyrheliometer. The last is a special device for measuring the heat from the sun (Chapter XVI).

Bolometer. In this instrument radiant energy is concentrated (by a lens or other means) on a tiny metal strip (for example, of platinum) which forms one branch of an electrical circuit known as a Wheatstone's bridge. The opposite branch is formed by an exactly similar, neighboring platinum strip, which is, however, shielded from radiation. The energy of the radiation is absorbed into the first metal strip, increasing its temperature. This results in a corresponding increase in its electrical resistance, which is accurately measured by a galvanometer in the bridge circuit. The value of the intensity of the radiation which impinges on the bolometer can be deduced from the galvanometer readings.

Spectrobolometer. This is the combination of bolometer and spectroscope; such an instrument measures the intensity of radiation included in a very narrow band of wave-lengths, cut out of any portion of the spectrum and impinging on the platinum strip of the bolometer. Measurements of this sort are especially important in giving the relation between the intensity and the wave-length of radiation emitted from hot bodies (§ 609).

Thermocouple. This is another electrical device. The radiation is made to strike the junction of two wires of dissimilar metals (as bismuth and antimony respectively); as a result of the consequent heating of the junction an electric current is set up in the circuit. This is measured by a sensitive galvanometer and is proportional to the intensity of the initial radiation. The thermocouple has been particularly useful in measuring the heat radiation from the stars and planets.

568. Units of Light. *Light* is not primarily an objective entity in nature, such as energy is supposed to be, but a sensation produced when radiation of a certain limited band of wave-lengths (§ 549) strikes the eye. Nevertheless, the illuminating engineer and the astrophysicist alike need to deal with light in an objective manner. For this purpose the following terminology has been devised (applications are given in section 600).

Luminous flux; visibility. The total quantity of energy transferred per second in any beam of electromagnetic radiation (the *radiant flux*) can be evaluated in watts, but the *luminous flux* (which refers to the capacity of the radiation to produce the sensation of light) is measured in a special unit called the *lumen*. For light of a given color the luminous flux is proportional to the radiant flux, but the factor of proportionality (lumens per watt, known as the *visibility* of radiation of the given wave-length) changes with the wave-length.

It is a maximum in the green and falls off rapidly in the red and violet. Great complications are introduced by the fact that the relative visibility of different wave-lengths differs for strong and weak illumination; the eye is relatively more sensitive to red light when the illumination is strong. The standard visibility curve (relation of visibility to wave-length) of illuminating engineers refers to much more intense light than is usually met with in observations of the stars.

For the ordinary photographic plate the visibility, or sensitivity, is a maximum in the violet, drops to zero in the green, and is high in the ultra-violet (where the eye is altogether insensitive); and the sensitivity curve differs very considerably for different plates.

Candlepower. In practice, measurements of light start with the adoption of a certain definite laboratory source — a candle, a lamp, or a hot piece of metal at known temperature — as the primary standard. Years ago such a source was always a candle of carefully specified size and material, and the term "candlepower" has been retained in reference to the quantity of light radiated from any source.

The lumen and the meter-candle. The value of the lumen is, by definition, such that the total luminous flux from a source of unit candle-power which radiates equally in all directions is 4π lumens. The *illumination* of a surface is the rate per unit area at which it receives light, and may be expressed in units of one lumen per square centimeter or per square meter. The latter unit is called the *meter-candle*, since it is the illumination produced by a standard candle at a distance of one meter.

Luminous efficiency. The *luminous efficiency* of any source is equal to the ratio of the luminous flux to the radiant flux from the source, and is expressed in lumens per watt, or in candles per watt. For monochromatic light it is identical with the visibility, and is a maximum in the green (664 lumens, or 53 candles per watt at λ 5560). Usually the luminous efficiency is very much less, since much of the radiation from such sources as hot bodies is invisible.

Surface brightness; the lambert. The lambert is a unit of brightness of an extended surface; it is the brightness of a luminous surface which is emitting one lumen per square centimeter. The brightness of an extended surface, as judged by the eye, is independent of its distance.

Albedo. The albedo of a reflecting surface is the ratio of the total reflected luminous flux to the incident luminous flux, and is of course never greater than unity, its value for a perfectly reflecting surface (§ 204).

569. Photometers. *Visual photometers.* The relative brightness of two light sources may be determined by varying the brightness of one in a known ratio until the two appear equally bright. This may be accomplished (1) by varying the distance of one source, (2) by the use of a "wedge" to weaken it by absorption, (3) by polarizing the light from one or both sources and equalizing by means of a Nicol prism. The wedge and the polarizing devices are extensively used in visual astronomical photometry.

Photographic photometry. The blackness, or density, of a developed photographic image, and also the size of the image (if of a star), depend upon the intensity of the light which falls upon the plate, and also upon the length of exposure, the kind of plate, the method of development, etc., in a very complicated manner. The only principle upon which accurate photographic photometry has yet been carried out is the simple one that two beams of light which produce equal effects for equal exposure times on the same plate under similar conditions are of equal intensity. By diminishing the intensity of the light from one of two sources in a known ratio this result can always be obtained. For lights of different color the relative sensitiveness of the ordinary photographic plate is entirely different from that of the eye (Fig. 217). Different types of plate differ greatly in this respect; some are sensitive even to the red. Advantage may be taken of these differences to obtain valuable information concerning the colors of the stars or other sources of light.

Selenium and photo-electric photometers. The electrical resistance of selenium and the rate of emission of electrons from an alkali metal vary with the intensity of illumination. These properties are taken advantage of in the construction of physical photometers which measure the light-intensity by means of such electrical effects. For the brighter stars these instruments give more accurate results than any others, but they would fail even to detect fainter objects which are readily seen or photographed.

EXERCISES

1. Express the wave-lengths of the various radiations listed in Table VII in the notation by powers of 10 (§ 125) and find the corresponding frequencies (§ 550) expressed in the same notation.

(Illustration: For soft X-rays the upper limit is 1.4×10^{-6} cm., and the corresponding frequency is $(3.00 \times 10^{10})/(1.4 \times 10^{-6})$, or 2.1×10^{16} cycles per second.)

2. How many light-waves per centimeter are there in the case of the red line of lithium, for which the wave-length is 6707.85 angstroms? (Both these numbers refer to the waves in *air* under standard conditions.)

3. Find the wave-numbers in air for the calcium lines at wave-lengths (referred to air) λ 4226.73; 3968.47; 3933.66.

4. Find the wave-lengths and wave-numbers for these lines in a vacuum (§ 550), given the following table for the *index of refraction of air under standard conditions.*

Wave-Length	Index of Refraction	Wave-Length	Index of Refraction
7000	1.000275	4000	1.000282
6000	1.000276	3500	1.000285
5000	1.000278	3000	1.000291

Ans.	6709.70	14903.8
	4227.91	23652.4
	3969.59	25191.6
	3934.77	25414.4

5. Measure the positions of the lines in Fig. 187 with a millimeter scale. Compare with the wave-lengths noted for the comparison lines, and so verify that the radial velocity of the star is − 30 km./sec.

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CHAPTER XV

THE SOLAR SPECTRUM

FRAUNHOFER LINES · KIRCHHOFF'S LAWS · PHOTOSPHERE, REVERSING LAYER, AND FLASH SPECTRUM · SPECTRUM ANALYSIS OF REVERSING LAYER AND THE CHEMICAL COMPOSITION OF THE SUN · ELEMENTS AND COMPOUNDS PRESENT · QUANTITATIVE ANALYSIS · CHROMOSPHERE, PROMINENCES, AND CORONA · THE SPECTROHELIOGRAPH AND ITS APPLICATIONS · DETAILED STUDY OF FRAUNHOFER LINES · FLAME, ARC, AND SPARK SPECTRA · STUDY OF SUN-SPOT SPECTRUM · MAGNETIC FIELD IN SPOTS · THEORY OF SUN-SPOTS

570. The Solar Spectrum. When the slit of a spectroscope is illuminated by sunlight (whether direct or reflected from the sky), the solar spectrum can readily be observed or photographed. It consists (Fig. 189) of a continuous bright background, of colors ranging from red for the longest visible wave-lengths to violet for the shortest; and this bright background is crossed by many transverse narrow dark lines called Fraunhofer lines, in honor of the observer who first studied them (1814). It has been photographed from just below 3000 angstroms in the ultra-violet to about 10,000 angstroms in the infra-red. When a bolometer is substituted for the photographic plate, observations extending still farther into the infra-red can be made. In the ultra-violet, however, the earth's atmosphere is opaque to radiation of wave-lengths shorter than about 2.9×10^{-5} centimeters, or 2900 angstroms, so that there is little hope of making observations of the spectra of celestial bodies in wave-lengths less than that value.

To a few of the more conspicuous dark lines, or groups of lines, Fraunhofer assigned letters of the alphabet which are still retained as designations. Thus, (A) is a group of strong lines at the extreme red end of the spectrum; (D) is a close pair in the yellow; and (H) and (K) are a pair at its violet extremity. All the lines are now designated by their wave-lengths in angstroms; thus, the (D_1) and (D_2) lines are λ 5890 and λ 5896, — more precisely, (D_1) is λ 5889.963 (§ 634). Present maps of the solar

spectrum contain more than 10,000 lines, some strong and heavy, others so fine as to be hardly visible; but each (except for certain slight changes in width and blackness) as permanent a feature of the spectrum as are rivers and cities on a geographical map. The spectra of the stars resemble, in essentials, that of the sun.

571. Kinds of Spectra; Kirchhoff's Laws. The interpretation of the Fraunhofer lines remained a mystery for many years after their discovery, until the laboratory investigations of Kirchhoff

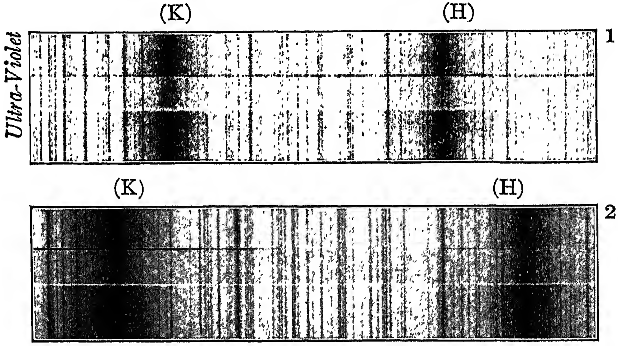


FIG. 189. (H) and (K) Regions of the Solar Spectrum

Two photographs, with high dispersion, of a small portion of the violet region of the solar spectrum, including the great (H) and (K) lines. The central strip in each is made by light from the very edge of the sun; the strips above and below, by light from the center of the disk. There are notable differences in the appearance of some of the lines in the two cases (§ 664). (From photograph by Jewell, Johns Hopkins University)

and Bunsen (1859–1862). These explained at once their general nature, but after sixty years the mine of information obtainable from these lines is far from being exhausted.

Line spectra; Kirchhoff's second law. Radiation from a glowing rarefied gas, composed of atoms of elementary chemical substances, is found in the laboratory to possess a spectrum of bright lines crossing a dark background. This generalization is known as Kirchhoff's second law. Each bright line is a slit-image produced by light of a certain narrow range of wave-lengths (§ 558). Such gases, it may be concluded, do not emit appreciable radiation except in certain definite wave-lengths. The line spectrum of each chemical element so produced is distinctive, and is characteristic of the given element. The spectra of some elements contain few lines; those of others, thousands.

Band spectra. When the glowing rarefied gas is composed, not of simple atoms of elementary substances, but of molecules, composed of two or more atoms in combination, the spectrum again consists of a dark background crossed by bright lines; but in this case the lines are extremely numerous, and are crowded so close together in certain regions as to form bright bands of considerable width and usually sharp at one edge (Fig. 185). Such band spectra are characteristic of chemical compounds which exist undecomposed in the gas, while line spectra are characteristic of the free elements.

If several kinds of atoms or molecules are present in the gas, the lines or bands of all appear in the spectrum without mutual interference.

Continuous spectra; Kirchhoff's first law. The first law expresses the observed fact that when the source of radiation is an incandescent solid or liquid, or a dense gas, the spectrum consists of a continuous background of light showing the colors of the spectrum, from red to violet. This indicates that all wave-lengths are represented in the radiation, and the bright background results from the overlapping of the slit-images formed by light of the various wave-lengths. Experiment shows that a spectrum of this sort is characteristic of the physical conditions (in particular, the temperature) of the source, and not of its chemical composition.

Reversals, or dark-line spectra; Kirchhoff's third law. When light from such a source is passed through a comparatively cool gas (which by itself would emit a spectrum of bright lines), and then into a spectroscope, the continuous bright background of the initial beam is found crossed by narrow dark lines. Kirchhoff's third law, based on his laboratory investigations, and since generally confirmed, states that the dark lines correspond exactly in wave-length to bright lines which the gas is capable of emitting. That is to say, a gas is transparent to radiation of most wave-lengths, but selectively opaque to radiation of wave-lengths spectroscopically characteristic of it; and when it transmits radiation from another, hotter source, the bright lines found in the spectrum of the gas when it is glowing appear as dark lines (that is, *reversed*) in the spectrum of the transmitted beam. Band

spectra, as well as line spectra, are observed thus reversed (according as the absorbing gas consists of molecules or of simple atoms).

The reversed lines are not quite black; if the column of gas is not thick, a fraction of the original radiation in the affected wave-lengths still makes its way through. In addition, if the gas itself is hot, it contributes some light in the reversed lines.

Bright lines on continuous bright background. Thus, reversals only appear when the brightness of the lines in the spectrum of the intermediate gas (if the gas is hot enough to be luminous) is less than that of the neighboring regions of the continuous spectrum of the solid or liquid source. Experiment shows that this condition requires that the temperature of the absorbing gas be less than that of the source giving the continuous spectrum. If the temperature of the gas is higher, the spectrum of the transmitted beam consists of bright lines, due to the gas, on a less bright background, due to the source.

572. Application to the Solar Spectrum. *Photosphere.* With the aid of these principles, certain conclusions about the sun's constitution are readily deduced from a study of the solar spectrum. The continuous background is presumably due to radiation emergent from the hot, comparatively dense, photosphere. As compared with the spectrum of such a terrestrial source as the glowing carbon in the electric arc, the continuous background of the solar spectrum is very much brighter toward the violet. This is a consequence of the higher temperature of the photosphere (§ 610).

Reversing layer. Kirchhoff's third law suggests that the dark Fraunhofer lines are produced by a rarefied and cooler gaseous atmosphere, or outer solar layer, called the reversing layer. The photosphere is what is seen in ordinary visual or telescopic observation of the sun; the atmosphere (except during total solar eclipses) is only observable spectroscopically, since it is transparent to radiation of most wave-lengths. Formerly the photosphere was supposed to consist of solid or liquid material, but various lines of evidence now prove that it, and indeed the whole sun, is gaseous (§ 613).

Flash spectrum. If the interpretation of the solar spectrum suggested by Kirchhoff's laws is correct, light from the sun's

atmosphere, if seen alone, should show a spectrum of bright lines agreeing in wave-length with the Fraunhofer lines. Fortunately this crucial test can be made at the time of a total eclipse of the sun, and the theory is triumphantly verified. Just after the photosphere is hidden by the moon, the outer solar atmosphere appears as a very narrow bright crescent along the edge of the dark moon. Its spectrum is actually found to consist of bright lines on a dark background (Fig. 190), which flash out as the photosphere is hidden, and disappear in

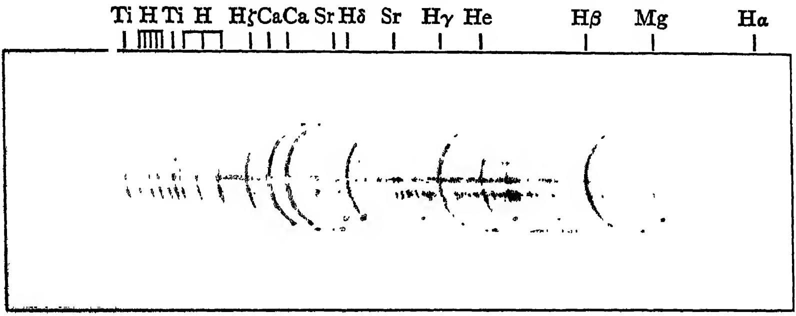


FIG. 190. The Flash Spectrum

From a photograph obtained by Dr. Baade (of the Hamburg Observatory) at the eclipse in Norway, June 29, 1927. The origins of some of the stronger lines are indicated. Those of Ca, H, and He are the most conspicuous. The isolated bright patches are prominences, rising above other parts of the moon's limb, and emitting these lines

a few seconds as the moon moves in front of the sun's atmosphere. This is called the flash spectrum. It was first observed by Professor Young, in Spain, during the eclipse of 1870.

Photography has here an immense advantage, on account of the transient character of the flash. Photographs at the more recent eclipses (taken by the method described in § 580) show many hundreds of lines, agreeing excellently in position (although in many cases not in intensity) with all the stronger Fraunhofer lines. The differences in intensity are not surprising, since corresponding lines in the two spectra are not necessarily produced at exactly the same depth in the solar atmosphere.

573. Spectrum Analysis of Reversing Layer. Comparison of the Fraunhofer lines with the spectra of glowing gases in the laboratory (the method of spectrum analysis) serves to show that

many of the chemical elements are present in the reversing layer in gaseous form. In order to effect the comparison, half the slit of a powerful spectroscope is covered with a little reflector, or a so-called comparison prism, which reflects the sunlight into it, while the other half of the slit receives directly the light from the luminous vapor. Upon looking into the view telescope the observer will have the two spectra, of the sunlight and of the metal, side by side, and can at once see what bright lines of the metallic spectrum do or do not exactly coincide with the dark lines of the solar spectrum. If he finds that every one of the conspicuous bright lines matches a conspicuous dark line, he can be certain that the substance exists as a vapor in the sun's atmosphere.

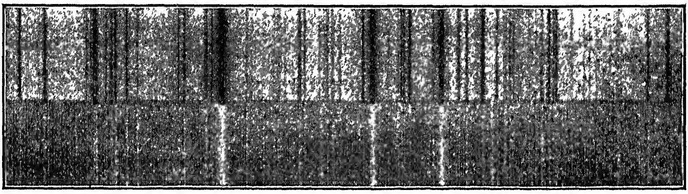


FIG. 191. The Spectrum of Iron in the Sun

In such comparisons photography may be effectively used. Fig. 191 is a reproduction of a negative which indicates the presence of iron in the sun. The lower half is part of the violet portion of the sun's spectrum (the dark lines appearing in the negative as bright lines); the upper half is part of an electric arc charged with the vapor of iron. In the original every line of the iron spectrum agrees perfectly with a correlative in the solar spectrum, although some of the coincidences do not appear in the engraving. There are, on the other hand, many lines in the solar spectrum, due to other elements, which do not find any correlative in that of iron.

574. Qualitative Determination of the Chemical Composition of the Sun. As the result of such comparisons between the wave-lengths of lines produced in the laboratory and in the sun (first made by Kirchhoff, but repeated and greatly extended by later investigators, and especially by Rowland, about 1890), forty-nine, or more than half, of the ninety chemical elements known terrestrially have been found to exist in the solar atmosphere,

— in gaseous form, owing to the high temperature. The very important results of these sixty years of study are presented in Table VIII.

A large majority of the strong Fraunhofer lines have been identified, and thousands of the weaker ones. Conspicuous among the former are the lines which Fraunhofer designated as (C), λ 6563, now known to be due to hydrogen; (D), λλ 5890, 5896, to sodium; (H) and (K), λλ 3968, 3933, to calcium. The hydrogen lines are now usually known as H α (Fraunhofer's (C) in the red); H β, λ 4861 ((F) in the blue); H γ, λ 4340; H δ, λ 4101; etc. The weak lines have not yet been sufficiently studied; it is probable that in the future all these also will be referred to known elements, with possible additions to the list of substances identified in the sun.

575. Quantitative Chemical Analysis of the Solar Atmosphere. One of the outstanding recent developments in astrophysics has been the rough determination of the relative abundance in the solar atmosphere of the elements (listed in Table VIII) which give rise to the Fraunhofer lines. Studies of the width and blackness of the Fraunhofer lines, as distinct from their wave-lengths merely, have been the chief factor in the quantitative chemical analysis of a body ninety-three millions of miles distant. It is found in the laboratory that when only a small quantity of a given substance is present in a gas, only a few of the very strongest lines in its spectrum appear. These are called *raies ultimes* (§ 639). When these are weak, the quantity present is very minute; as this quantity is increased the *raies ultimes* become stronger and wider, and other *subordinate* lines make their appearance.

It is therefore concluded that those elements, such as silver and palladium, which are represented in the Fraunhofer spectrum only by their *raies ultimes*, are present in the solar atmosphere only in small quantities, especially when those lines are weak. On the other hand, elements such as iron, calcium, and magnesium, for which weak subordinate lines appear strongly in the solar spectrum (and for which the strong lines are much widened) must be relatively abundant. A complete study from this point of view remains to be made; only rough results are at present

available, but it is not too much to hope that within a few years the analysis can be made approximately quantitative (§ 934). The conclusion is strongly indicated that the order of abundance of the elements in the solar atmosphere is much the same as in the earth's crust. (Kirchhoff and Rowland were of this opinion.) It is interesting to note, in Table VIII, that most of the more abundant elements are of even atomic number, agreeing with data on the chemical constitution of the earth's crust.

Some elements are at a serious disadvantage in the spectroscopic analysis. For example, silicon has but one strong line (λ 3905) in the observable region, and this a subordinate one; and the only available oxygen lines are not merely of a pronounced subordinate type but are so far in the red (beyond λ 7700) that Rowland, who had no plates sensitive to this region, did not record them in the sun at all. It is therefore not surprising that these elements, although very abundant in the earth's crust, are only moderately or feebly represented in the Fraunhofer spectrum.

576. Chemical Compounds in the Sun. Nitrogen is represented in the solar spectrum, not by the line spectrum of the element, but by bands in the blue and ultra-violet, resolved into finely packed lines, which are believed to be due to a compound of carbon and nitrogen, — probably cyanogen.

Other bands occur in the solar spectrum. Fraunhofer's (G) group is a band due to some compound of carbon and hydrogen, with many metallic lines superposed on it. Bands due to ammonia or some other compound of nitrogen and hydrogen, and to hydroxyl (OH), have recently been detected in the ultra-violet. These chemical compounds, then, exist in the sun's atmosphere without being completely decomposed by the high temperatures.

577. Explanation of Absences. Forty-one of the known chemical elements, as listed in Table VIII, possess no identified lines in the solar spectrum (Fraunhofer or flash). The conclusion does not follow, however, that these elements are absent from the sun. In the first place, only the material above the photosphere can produce line absorption or emission; consequently only a tiny fraction of the whole solar mass is subject to spectrum analysis.

TABLE VIII. CHEMICAL ELEMENTS REPRESENTED IN THE FRAUNHOFER SPECTRUM

REPRESENTED	ABSENT	REPRESENTED	ABSENT	REPRESENTED	ABSENT	REPRESENTED	ABSENT	REPRESENTED	ABSENT
H		K		Rb[2]			Cs[2]		Ta
He[1, 4]		**Ca**		**Sr**		Ba		W	
Li[2]		Sc		Y		La			Re[5]
Be		Ti		Zr		Ce			Os
	B[4]	V		Cb		Pr			Ir
C		Cr		Mo		Nd			Pt
N		Mn			Ma[5]		Il[5]		Au[4]
O		**Fe**		Ru		Sm[3]			Hg[4]
	F[4]	Co		Rh		Eu		Tl	
	Ne[4]	**Ni**		Pd			Gd[5]	Pb	
Na		Cu		Ag			Tb[5]		Bi
Mg		Zn		Cd			Dy[5]		Po[5]
Al		Ga		In[2]			Ho[5]		—[6]
Si		Ge		Sn			Er[5]		Rn[5]
	P[4]		As[4]	Sb[3]			Tm[5]		—[6]
	S[4]		Se[4]		Te[5]		Yb[5]		Ra
	Cl[4]		Br[4]		I[4]		Lu[5]		Ac[5]
	A[4]		Kr[4, 5]		Xe[4, 5]		Hf[5]		Th
									Pa
									U

The symbols refer to the ninety-two chemical elements, a list of which is given in Table XXXVI, in the Appendix. They are tabulated in the order of increasing atomic number; those which possess identified lines in the Fraunhofer spectrum are listed as "represented," and all others as "absent." Those with the strongest lines are listed in heavy type. The numbers refer to the following notes:

NOTE 1. Helium lines are absent from the Fraunhofer spectrum (except occasionally in the spectra of regions above sun-spots), but appear in the flash spectrum.

NOTE 2. Li, Rb, In, possess no lines in the normal Fraunhofer spectrum, but show faint lines in the spectra of sun-spots. The ultimate lines of Cs are in the infra-red, and have not yet been examined in spots.

NOTE 3. Identifications of Sb and Sm are somewhat doubtful.

NOTE 4. The strongest lines of these elements are in the astronomically unobservable region short of λ 2900.

NOTE 5. The spectra of these elements have been incompletely investigated in the laboratory; further study may show that some of them are represented in the Fraunhofer spectrum.

NOTE 6. These two gaps represent chemical elements predicted by theory but not yet isolated by chemists.

Secondly, there are many spectra (Note 5, Table VIII) which still need laboratory investigation, — especially those of recently discovered elements, of several radioactive elements, of other elements which show different spectra under different conditions, and of still other elements for which the ultimate lines are not yet known. Thirdly, the *raies ultimes* (§ 639) of several elements (Note 4) lie in wave-lengths too short to be transmitted through the earth's atmosphere; the spectroscopic test is not capable of detecting the presence of small quantities of these. Consequently a number of the elements not yet identified in the solar spectrum may nevertheless be present in the reversing layer.

There remain tungsten, bismuth, radium, tantalum, osmium, iridium, platinum, thorium, and uranium. If present in appreciable quantities not too low in the solar atmosphere, there is little doubt that these would possess recognizable lines in the Fraunhofer spectrum, — especially the first three. Since all these elements are of great atomic weight and occur in but very small proportions in the earth's crust, the supposition appears plausible that such heavy atoms would, for the most part, sink so low as to leave almost none in the reversing layer.

There is no sufficient reason, then, to conclude that any element is really absent from the sun.

Indeed, there is one instance of the discovery, on the sun, of an element (namely, helium) which was not isolated in the chemical laboratory until years later. The history of this element has been anything but dull. First recognized, at the total solar eclipse of 1868, as the source of a yellow line in the flash spectrum, it was named from the Greek word for *sun*. It was discovered anew in 1895, in a classical investigation by Ramsay, as a chemical constituent of the earth's atmosphere with surprising properties. About 1917 it was found (as a constituent of the natural gas from wells in certain localities) in quantities sufficient to make practicable its use in dirigibles.

578. The Telluric Lines. Not all the dark lines in the solar spectrum are due to substances in the sun's atmosphere. Before it reaches the spectroscope, sunlight must also pass through the earth's atmosphere; and certain of the constituents of the latter (principally the oxygen and water-vapor) imprint their characteristic bands on the Fraunhofer spectrum. The telluric oxygen bands (Fraunhofer's (A) and (B)) are due to molecular oxygen in

the earth's atmosphere; there are solar oxygen lines, on the other hand, produced by atomic oxygen. In the infra-red the telluric lines are numerous, while the solar lines are few. Beyond λ 10,000 there are many wide and heavy absorption bands due to water-vapor and carbon dioxide in the earth's atmosphere. A great band in the ultra-violet, due to ozone in the upper atmosphere, cuts off entirely all radiation from the sun and stars of wave-lengths shorter than about λ 2900.

Telluric lines in general are distinguished from those of solar origin by their variability with the sun's altitude, being stronger when its rays pass more nearly horizontally through a greater length of air; they appear the same at the limb as at the center of the sun's disk (§ 664), are not affected in spots (§ 593), and do not show the Doppler shift due to the sun's rotation.

579. Solar Rotation from Doppler Effect. If the spectra of the eastern and western limbs of the sun are photographed, simultaneously or in immediate succession on the same plate, the lines in the spectrum of the eastern limb (that is, the edge of the sun's disk seen toward the east in the sky) are found displaced toward the violet with respect to those in the spectrum of the western limb. This arises from the Doppler effect due to the sun's rotation. Points on the eastern limb are approaching the earth at a velocity of about 2 kilometers per second, and the lines in its spectrum are therefore slightly shifted toward the violet. Telluric lines, of course, are not displaced at all, and are thus immediately distinguishable from the others.

Using the equation

$$\Delta\lambda/\lambda = v/c,$$

the shift $\Delta\lambda$ comes out as 1/150,000 of the wave-length λ in each direction, or 1/75,000 when corresponding lines from the two limbs are compared. This amounts to only 0.08*A* at $\lambda = 6000$ (in the orange); but the solar spectrum can be observed with very high dispersion, so that the shifts can be not only detected but measured with considerable precision.

In this way the velocity of rotation of the sun's surface, and consequently the period of rotation, can be determined spectroscopically (Fig. 192). Thus, even in high solar latitudes,

where no sun-spots occur, Adams has found that the rotation period continues to increase toward the pole (§ 226).

Individual plates sometimes give discordant results, showing temporary currents in the solar atmosphere, or solar winds, with velocities of several hundred miles an hour.

580. The Chromosphere. The upper part of the sun's atmosphere extends several thousand miles above the layer which produces the flash spectrum, and is conspicuous during a total solar eclipse. It is called the *chromosphere*, or color-sphere,

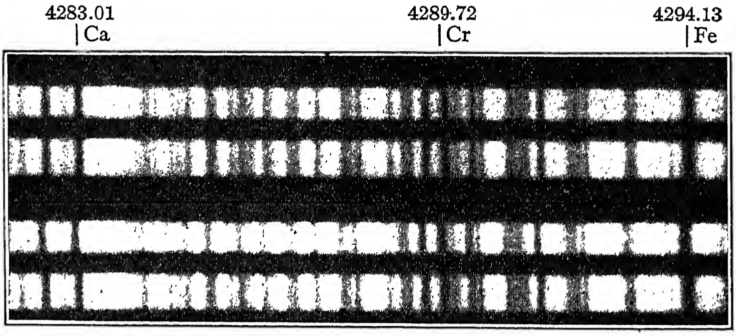


FIG. 192. Doppler Effect due to the Sun's Rotation

A small region of the solar spectrum in the violet, photographed with very high dispersion. A few prominent lines are identified by their wave-lengths in international angstroms. The upper pair of spectra are formed with the light from points at the northern and southern limbs of the sun, and the lower pair with light from the eastern and western limbs. The lines in the spectrum of the eastern limb, which is approaching, are shifted toward the violet; those in that of the western (receding) limb, toward the red. At the poles no shift appears. It may be verified by measurements with a fine scale on this print that the shift between the eastern and western limbs is 0.06 A, which corresponds to a relative velocity of 4.2 km./sec.—or an equatorial rotational velocity of 2.1 km./sec. (From photograph by Mt. Wilson Observatory)

because it is then seen as brilliantly scarlet, owing its strong color to hydrogen, the most conspicuous of its constituents.

The changing crescent of chromosphere visible at any instant as the moon passes over the sun is so narrow that no slit is needed to obtain its spectrum; a camera of long focus, with a prism or transmission grating placed outside its object-glass, or a concave grating, gives on a photographic plate a crescent image for every wave-length represented in the light emitted from the chromosphere (Fig. 190). The length of an arc is a rough indica-

tion of the height to which the corresponding kind of atom extends in the chromosphere, but it is influenced also by the brightness of the line.

Such photographs have been obtained at every observed eclipse since 1896, and indicate that the principal chromospheric constituents, and those which extend the highest, are hydrogen, helium, and calcium vapor, which ascend to about 12,000 kilometers. Ultimate lines of several other elements (strontium, titanium, scandium, magnesium) are found extending as high as 5000 kilometers. Strong lines of sodium, iron, and a few other elements run up to 1000 or 1500 kilometers, while most of the lines in the flash spectrum stop at a height of 400 or 500 kilometers.

The distinction between the chromosphere and the reversing layer, although convenient, is somewhat arbitrary; no sharp boundary separates these two divisions of the solar atmosphere. Most of the Fraunhofer lines are probably produced low in the reversing layer; the chromosphere accounts only for the central narrow dark cores of some of the strongest lines.

581. The Prominences and the Corona. *Prominences*. Prominences are projections from the chromosphere, or isolated clouds of chromospheric material. During a total solar eclipse, after the sun is fairly hidden by the moon, they are seen as scarlet, starlike objects, blazing like rubies around the edge of the dark moon. They were first proved to be appendages of the sun (rather than of the moon) during the eclipse of 1860, by means of photographs which showed that the moon's disk moved over them as it passed across the sun. (Compare Figs. 107 A, 107 B.)

The spectrum of a prominence resembles that of the upper chromosphere, containing lines due to hydrogen, helium, and calcium, and sometimes other substances (§§ 585, 586).

Corona. The spectrum of the corona is composite. By far the greater part of the light possesses a continuous spectrum, on which the Fraunhofer lines are faintly visible. Probably much of this radiation is ordinary sunlight which has been reflected or scattered in passing outward through the corona. The remainder is probably emitted by the coronal material itself. Superposed on the continuous spectrum, but relatively faint, is a characteristic bright-line spectrum which must be emitted by some gaseous

constituent of the corona. The most conspicuous of these lines has usually been one in the green at λ 5303 (first observed by Young in 1869).

Not one of the thirty coronal lines which have been observed has yet been duplicated in the laboratory. On this account they have been ascribed by some investigators to a hypothetical, very light gas, "coronium." The modern theory of atomic structure leaves no place for such an unknown element, and it appears more probable that the coronal lines are due to some known substance which shines in this peculiar way under the special physical conditions prevalent in the corona. (For example, the extreme rarefaction of coronal material combined with the great thickness is impossible to duplicate in the laboratory.)

582. Nature of the Corona. The total amount of time during which astronomers have been able to photograph the corona and its spectrum is much less than one hour, — the sum of the minutes of total obscuration of the sun by the moon since the development of photography. Drawings of it, made of necessity during the limited time of totality, are not very satisfactory. Consequently knowledge of the corona is not extensive.

Its form and size vary during the sun-spot cycle in a fairly regular manner (Fig. 96). According to measurements of several observers in 1918, 1922, and 1925 its brightness varies little from one eclipse to another, and is one millionth that of the sun. Thus it produces about half the illumination of the full moon (§ 203). Its heat radiation is also one millionth of the sun's, indicating that the quality of its light is approximately that of sunlight; the outer parts are bluer than the inner. Its light is distinctly polarized. Its brightness falls off very rapidly with moderate distances from the limb of the sun, giving rise to a rather sharp distinction between the *inner* and the *outer corona.*

The former, or the portion near the sun, appears as a bright, homogeneous ring, of a pale yellowish tinge contrasting finely with the scarlet prominences; the bright coronal spectral lines probably originate in this part, within 100,000 miles of the sun's surface. The outer corona, on the other hand, is pearly white in color and consists of fainter filaments or rays which diverge more or less radially but are strangely curved and intertwined

(taking, perhaps, the shape of magnetic lines of force). Long, pointed streamers, composed of individual filaments, may run out for several millions of miles from the sun's surface.

Photographs made at widely separated stations in the track of an eclipse, over intervals of an hour or so, show, in the main, identical details — proving that the coronal material is not in very rapid motion unless the motion is along the curved streamers. Spectroscopic observations during the eclipse of 1923 showed displacements of the Fraunhofer lines in the coronal spectrum which indicated (through Doppler effect) that the coronal material was moving outward from both limbs of the sun at the rate of about 20 kilometers per second.

The partial polarization of the light of the corona, and the color of its outer portions, are indications that much of its light has been scattered from very fine particles, or molecules; and doubtless it is in large measure composed of gas. Its density must be exceedingly small, for several comets (themselves bodies of very low density) have passed right through it, at velocities of over 500 kilometers per second, without experiencing the least observable resistance to their motions. (The comet of 1843 went within 100,000 miles of the sun's surface.)

The corona (even the inner corona) is so faint that no attempts to photograph it, except at times of total eclipse (and a few seconds before and after), have yet succeeded. When the sun is shining with its full brightness, this most beautiful attendant is overwhelmed in the glare of the sky close to the sun's disk. If the earth had no atmosphere, there would be no such glare, and the corona would be visible at all times; but there would be no astronomers to see it.

583. The Prominences and Chromosphere Observable at Any Time with the Spectroscope. During the eclipse of 1868 Janssen was so struck with the brightness of the hydrogen lines in the spectrum of the prominences that he believed it possible to observe them in full daylight, and the next day he found this to be so. He also found that by a proper management of his instrument he could make out the forms and structure of the prominences which he had seen the day before during the eclipse. Lockyer, in England, a few days later, but quite independently,

made the same discovery, and ascertained that the prominences were mere extensions from a hydrogen envelope completely surrounding the sun; it was he who gave to this envelope the now familiar designation of "chromosphere." His name is always, and justly, associated with that of Janssen as a co-discoverer. A little later Huggins showed that by simply opening the slit of the spectroscope the form and structure of the prominence, if not too large, could be observed as a whole, and not merely by piecemeal as before.

Simply screening off the sun's disk fails to render the prominences and chromosphere visible, because the brilliant illumination of the air near the sun drowns them out, as daylight does the stars. But when the sun's image is permitted to fall (Fig. 193) with the limb just tangent to the slit of a spectroscope, then, if there be a prominence at that point, two overlying spectra are produced: one, the spectrum of the illuminated air; the other, superposed upon this background, that of the prominence itself. Now the latter is a spectrum consisting of bright lines or, if the slit be opened a little, of bright images of whatever part of the prominence may fall between the jaws of the slit, and the brightness of these lines or images is independent (even with the most powerful instruments) of the dispersive power of the spectroscope; increase of dispersion merely sets the images farther apart, without making them fainter. The spectrum of the aërial illumination, on the other hand, is that of sunlight, — a continuous spectrum showing the usual Fraunhofer lines; and this spectrum is made faint by great dispersion. Moreover, it presents dark lines or spaces just at the very places in the spectrum where the bright images of the prominences fall, and therefore they become easily visible.

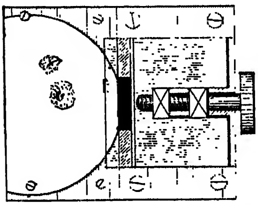


FIG. 193. Spectroscope Slit adjusted for Observation of Prominences

584. The Spectroheliograph. A great step forward in the study of the prominences, and indeed of the entire outer region of the sun, was made in 1890 with the invention (independently by Hale and Deslandres) of the spectroheliograph, — an instru-

ment by which photographs of the sun may be taken exclusively by light of any desired wave-length. Its essential feature is the introduction of a second slit in a spectrograph, just in front of the photographic plate, which thus permits only a nearly monochromatic beam, chosen at will, to affect the plate. If an image of the sun is formed (by the object-glass of a large telescope) on the first, or ordinary, slit of the spectroscope, this slit (which must be longer than the diameter of the image) cuts a narrow segment out of the solar image. An image of this segment, produced solely by light of the single wave-length which is isolated

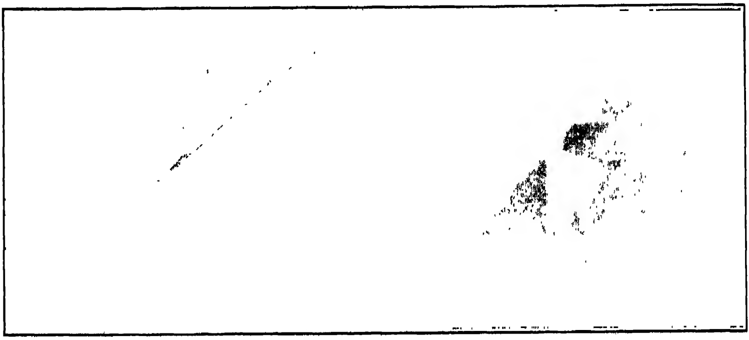


FIG. 194. Prominence Projected on the Sun's Disk

The left-hand picture, in which the photosphere is hidden by an occulting disk, shows the prominence bright against the sky; the right-hand, with a shorter exposure, shows it faintly against the sky, and dark against the photosphere, since it absorbs the sort of light which it emits. (Photographed with the H α line, May 22, 1916, at Mt. Wilson Observatory)

by the second slit, is formed on the plate. The sun's image is permitted to drift slowly over the first slit while the plate is moved with the same velocity behind the second slit; thus the image of segment after segment of the disk is recorded on the plate, until a complete photograph of the sun has been built up, in the selected wave-length.

In photographing the prominences the disk of the sun is concealed by an opaque screen of proper size, and the second slit is adjusted to pass radiation corresponding to one of the strong bright lines in the prominence spectrum (Fig. 195). Regular records of the whole circumference of the sun are thus obtained daily at a number of observatories.

585. Quiescent Prominences. Prominences may be broadly divided into two classes, the *quiescent* and the *eruptive* (Fig. 196). The quiescent prominences are immense clouds, often from 50,000 to 100,000 kilometers in height and of corresponding horizontal dimensions, either resting directly upon the chromosphere as a base or connected with it by stems and columns, although in some cases they appear to be entirely detached (Fig. 195). They are not very brilliant, and ordinarily show only the lines of hydrogen, helium, and the (H) and (K) lines of calcium in their

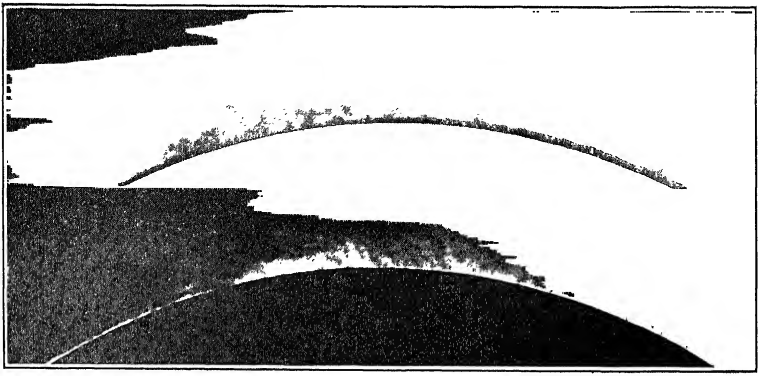


FIG. 195. Quiescent Prominences

Showing the characteristic appearance, like a "grove of banyan trees," March 17 (above) and 18 (below), 1910. On the second day the prominence was being carried in front of the sun's disk by the sun's rotation, so that only the upper parts of it projected. This group of prominences was observed at five successive presentations on the eastern and western limbs of the sun, and at its best was over 500,000 kilometers long and 77,000 kilometers high (From photograph by Frederick Slocum, Yerkes Observatory)

spectra. Their changes are not rapid; quiescent prominences often continue sensibly unaltered until the sun's rotation carries them out of sight, and can sometimes be recognized when they appear at the opposite limb. Their forms and behavior indicate that they are supported in some way against solar gravity; but recent investigation makes it probable that they are not floating in an atmosphere, like terrestrial clouds, but are held up by radiation pressure. They are found on all portions of the sun's limb and are not confined to the sun-spot zones, although they show two zones of maximum abundance. One lies about 25° each side of the sun's equator; and the other, starting in about

40° of latitude at the time of sun-spot maximum, moves gradually toward the pole until the next maximum. Their number rises and falls with that of the spots, but to a less marked degree.

586. The eruptive prominences, on the other hand, appear only in the spot-zones, and as a rule in connection with active spots. Their spectra usually contain the bright lines of hydrogen, helium, calcium, and various other substances, most of them metals), including iron, magnesium, titanium, barium, and sodium. They originate not in the spots themselves but in the disturbed faculous region just outside. Ordinarily they are not very large, though very brilliant; but at times they become enormous, reaching elevations as great as 700,000 kilometers. They are most fascinating objects to watch, because of the rapidity of their changes. Sometimes the motion of their filaments can be perceived directly, like that of the minute-hand of a clock, and this implies a velocity of at least 250 miles a second. In such cases the lines of the prominence spectrum are greatly displaced and distorted, and often indicate motions in the line of sight of a similar order of magnitude.

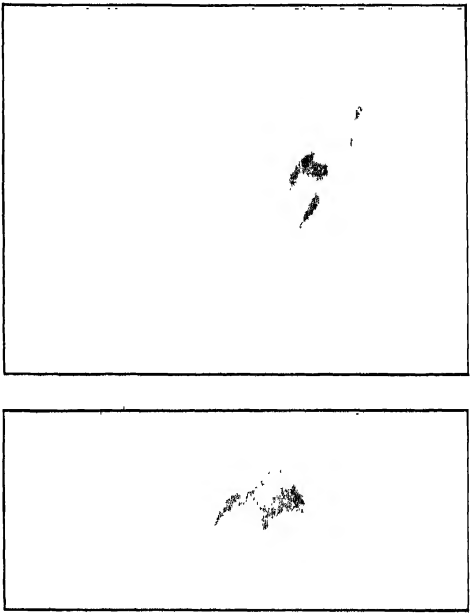


FIG. 196. Eruptive Prominence

The two photographs, taken 33 minutes apart, show an increase of the height from 120,000 kilometers to 252,000 kilometers, and great changes in shape. (Photographed with the (H) line of calcium (λ 3968), March 25, 1910, by Frederick Slocum, Yerkes Observatory)

The number of prominences visible at one time is variable; it is not at all unusual to find as many as 20 at once on the sun's limb.

587. Spectroheliograms of the Sun's Disk. The second slit of the spectroheliograph can be adjusted to pass radiation of any narrow band of wave-lengths, whether the band falls within a wide Fraunhofer line or in the continuous background of the solar spectrum. There is a radical difference between spectroheliograms of these two types. The latter depict merely the photosphere, of nearly uniform appearance (except for sun-spots and faculæ) over the entire sun. Spectroheliograms corresponding to

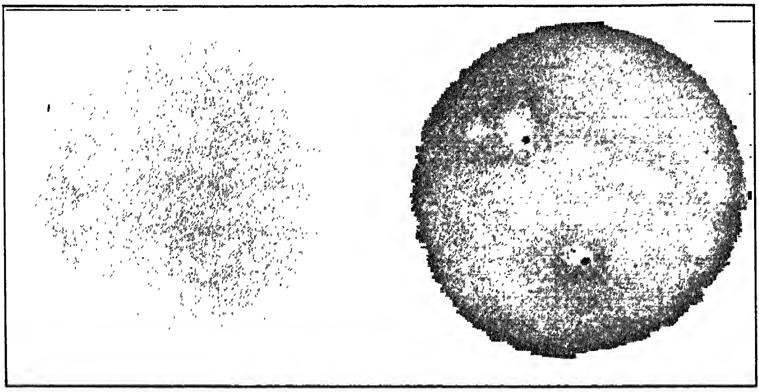


FIG. 197. Comparison of Direct Photograph and Spectroheliogram of the Sun

The direct photograph, on the left, shows only the prominent sun-spot groups. The spectroheliogram, with the (H) line of calcium, shows the characteristic flocculi all over the surface, and a long series of bright flocculi in the two sun-spot zones. (From photograph by Mt. Wilson Observatory)

Fraunhofer lines are more interesting. Such photographs of the sun yield information concerning the distribution in the solar atmosphere of those atoms which emit, and absorb, radiation of the particular wave-lengths concerned (Fig. 197). For example, spectroheliograms taken with a narrow strip at the center of the wide (K) line (due to calcium) tell a great deal about the distribution (both horizontally, or over the surface of the sun, and vertically, in the solar atmosphere) of the calcium atoms concerned in producing it.

Wherever the calcium vapor high in the solar atmosphere is hotter than usual, the light given off from it is unusually strong, and a bright marking appears on the photograph. Where it is cool, and present in relatively large quantities, it gives off less

light, while absorbing almost completely the light from the layers below, and dark markings appear. Both the light and the dark markings were named *flocculi* by Hale.

A spectroheliogram taken near the edge, instead of near the center, of a wide Fraunhofer line is believed to represent the distribution of the atoms concerned at a somewhat lower level in the solar atmosphere. Thus, a series of spectroheliograms running from the center to the edge of the (K) line, for example,

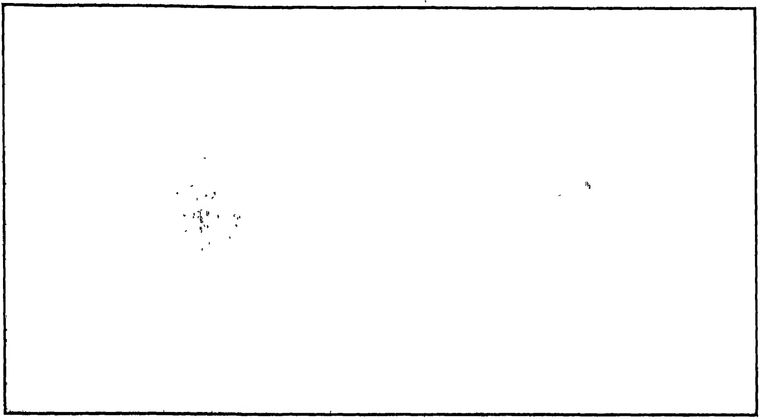


FIG. 198. Direct Photograph of the Sun, and Spectroheliogram

The direct photograph shows numerous spots, and faculæ near the limb. The spectroheliogram (with the red hydrogen line H α) shows flocculi all over the disk (very different in form from the calcium flocculi), bright flocculi in the spot-zones, and dark markings in the higher latitudes, which are prominences projected upon the disk

affords valuable knowledge of the distribution in depth of the calcium vapor involved in producing this particular line.

588. Details of Spectroheliograms. The flocculi of calcium and of hydrogen have been best observed. Both are thought to represent clouds of material in the chromosphere, but there are reasons for believing that the observed hydrogen flocculi exist several thousand kilometers higher in the solar atmosphere; and this difference in elevation is associated with important differences in the characteristics of the flocculi.

The calcium flocculi in general show bright against the sun's disk. Those at the lowest levels agree closely in form with the faculæ (§ 230) which appear on direct photographs. Spots are

always attended by these flocculi, which appear sooner and last longer than the spots themselves; and they often appear where no spots occur. Spectroheliograms taken with light in the strong iron line at λ 4045 show flocculi of similar appearance, agreeing with the flash-spectrum observations, which show that this line ascends to a relatively small height in the chromosphere.

Calcium flocculi at a higher level are much larger and brighter, and often completely cover the attendant spots. The whole surface of the sun on such photographs has a very characteristic appearance, resembling the skin of a rough orange. The positions of these flocculi can be measured with tolerable accuracy, and they yield values of the solar rotation and its equatorial acceleration very nearly the same as those derived from the spots.

Calcium flocculi at still higher levels (photographed with light in the very center of the calcium lines) are even larger and more conspicuous, and include prominent dark markings. These are closely associated with quiescent prominences, and have sometimes been identified as the actual projections of prominences on the disk which were not seen in silhouette until the solar rotation had carried them to the limb of the sun.

The prominences, being relatively cool, absorb powerfully and hence appear as dark markings, and, being of low density, give narrow absorption lines.

The hydrogen flocculi are quite different in shape and general aspect, although they also are usually associated with sun-spots; they are in general dark, and they change rapidly in form and position. The remarkable feature of the hydrogen flocculi is the striking curved structure which they present around sun-spots (Fig. 204); this at once suggests that the spots are associated with vortices (great cyclonic whirls of gas in the solar atmosphere) which make their effects known in this manner only at high levels. The apparent direction of a whirl may be either right-handed or left-handed, but it is almost always opposite for the eastern and western components of the characteristic spot-pairs.

Photographs made with the center of Hα refer to the highest level of all, and show the prominences very black against the disk.

589. Doppler Shifts of Fraunhofer Lines. Precise measurements show minute differences in wave-length between the Fraunhofer

lines and the corresponding lines produced in the laboratory, — not large enough to cast any doubt upon the identification of elements present in the solar atmosphere, but giving very valuable information.

The Doppler shift due to the sun's rotation has already been described. Similar shifts, found by Evershed, and known as the *Evershed effect* (Fig. 199), in the spectrum of the photosphere, just outside of sun-spots, can be interpreted as due to a flow of material outward in all directions from the sun-spot, along the sun's surface. The faintest lines are shifted most, and indicate an outflow at a speed of about 1 kilometer per second. Stronger lines, which rise higher in the flash spectrum, show smaller velocities,

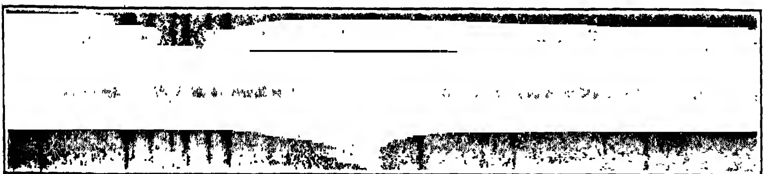


FIG. 199. The "Evershed Effect" in Sun-Spots

Spectra of two sun-spots near the limb, with the slit set across them in the direction of the sun's radius. The lines in the spectrum of the penumbra are widened, and show a shift toward the red at the upper edge (nearer the sun's limb), and toward the violet at the edge near the center of the disk. Some lines are conspicuously strengthened in the spot spectrum. (From photograph by C. E. St. John, Mt. Wilson Observatory)

and the lines which rise high in the chromosphere indicate inward motions which increase to 2 kilometers per second at the top.

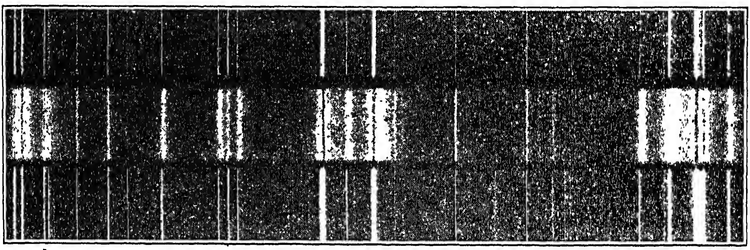
Occasional irregular and turbulent motions, often much more rapid, and usually near active sun-spots, are detected in the same way.

590. Effects of Pressure and Density. In the laboratory, when a luminous gas is subjected to high pressure, almost all the lines are widened, and most of them shifted, usually toward the red. Unlike the Doppler effect, these *pressure shifts* influence neighboring lines very differently, and so may be distinguished from the former (Fig. 200).

This effect is not negligible, even at atmospheric pressure, as is found by comparing the spectrum of a vacuum arc with that of an electric arc in air. Comparison with solar wave-lengths,

making due allowance for other shifts (§ 591), gives results indicating that the pressure in the reversing layer is at most a small fraction of one atmosphere. St. John (Mt. Wilson, 1923) thus found a pressure of 0.1 ± 0.1 atmosphere. There is little doubt that these effects depend really on the *density* rather than on the pressure, but the term "pressure shift" is conventional.

Many lines that are fuzzy in the arc in air become sharp in a vacuum; such lines are always conspicuously sharp in the sun, indicating that the density of the reversing layer is very low. Other considerations (§ 662) indicate that the density at the very bottom of the reversing layer is probably less than 1/10,000



λ 5200 5270

FIG. 200. Widening and Shifting of Spectrum Lines

The outer strips show the ordinary arc spectrum of iron; the central strip that of an iron arc carrying a current of 1000 amperes, in which the density of the vapor is great. The comparison shows that the lines in its spectrum are much widened, many are shifted in wave-length from their normal positions, and some are crossed by dark reversals due to absorption by the cooler gases in the outer parts of the arc. (From photograph by A. S. King, Mt. Wilson Observatory)

that of atmospheric air, while that in the chromosphere may be 1,000,000 times smaller, or 10^{-10} times that of air.

The density of the gas in the reversing layer is comparable to that of a moderate vacuum in the laboratory; the density of the gas in the chromosphere is that of a high vacuum. There must be less matter per cubic centimeter in the corona, and probably also in the prominences, than in any vacuum that has been artificially produced. It is only because of the great thickness of these objects that they can be seen at all.

591. Other Shifts. At the center of the sun's disk the Fraunhofer lines are slightly shifted toward the red in comparison with laboratory vacuum wave-lengths. The average amount of this shift, according to St. John, agrees very closely with that

predicted by Einstein, on the theory of general relativity, as the result of the sun's gravitation (§ 364).

The stronger lines are shifted more than this, and the weaker lines less, which St. John explains as due to ascending and descending currents in different regions of the atmosphere. These minute shifts, however, are not completely explained. There are also small differences in wave-length between the center and the limb of the sun (after correction for Doppler effect due to rotation), usually toward the red at the limb, and amounting to about a hundredth of an angstrom at most.

592. Flame, Arc, and Spark Spectra. The spectrum emitted or absorbed by a given element is not the same under all conditions of excitation. If, for example, a calcium compound is put into the flame of a Bunsen burner, the spectrum shows bands in the red, differing with the compound, and a single sharp line in the violet (λ 4227), due to the metal. When the same compound is put into an arc between carbon poles, a great number of other lines appear which cannot be obtained in the flame. On passing to a high-tension spark between terminals containing calcium, certain lines are found to be greatly strengthened, or *enhanced*, in intensity. For other elements, such as iron, new lines appear in the spark which are not in the arc spectrum at all. These also are classed as enhanced lines.

Grades of excitation intermediate between the flame and the arc can be obtained in the electric furnace, — a long carbon tube heated by a heavy, low-voltage current, water-cooled at both ends, and closed by glass plates. Such a furnace may be run at any temperature up to almost 3000° centigrade, and even the most refractory metals may be volatilized in it. The light from the vapor escapes at the end of the tube and may be observed without contamination from the continuous spectrum of the glowing walls.

Observations with the furnace show that the lines in most spectra can be divided into several *temperature classes*, denoted by A. S. King as I, II, III, etc. Lines of Class I appear at the lowest temperatures, and often in the Bunsen flame. As the temperature is raised, lines of classes II, III, and IV successively make their appearance, while the light grows brighter and

all the previously observed lines become stronger. Lines of Class V appear in the arc only. Additional classes would be required to include those lines which are produced only in the spark. (King, who has devoted years to this work in the Mt. Wilson laboratory, has thus classified many thousands of lines in all the more important spectra.)

In passing to the arc and the spark, the effects of powerful electrical disturbances are added to those of higher temperature, but the fact that the enhanced lines correspond to a still higher degree of excitation of the atoms is unquestionable. Some enhanced lines appear even in the high-temperature furnace, — some strongly and others faintly in the arc, and some only in the spark, — so that they too may be classified in a similar manner.

The term "enhanced lines" was introduced by Lockyer, who was the first to point out the great importance, both in physics and in astronomy, of the differences here discussed.

593. Application to the Sun-Spot Spectrum. If the image of a sun-spot is thrown on the slit, a spectrum is obtained which, though generally similar to that of the photosphere, differs in many important details (Fig. 201). The spectra of different spots are remarkably similar. Certain lines are strengthened and others weakened. Many lines appear which are not present in the normal solar spectrum.

The strengthened lines are always low-temperature lines, those of Class I showing the greatest strengthening, and most of the weakened lines are enhanced lines. The correlation between the changes in intensity in passing from the photosphere to the spots, and from the arc to the furnace, is so good as to leave no doubt whatever that these differences arise because the spots are cooler than the normal photosphere.

Further evidence of this is afforded by the lines peculiar to the spot spectrum, thousands of which have been identified as belonging to bands of titanium oxide and of hydrides of magnesium and calcium. These compounds exist at the lower temperature of the spot but are dissociated in the ordinary reversing layer.

These important conclusions are due to the work of Hale and his colleagues at Mt. Wilson. The spectra of faculæ show differences from the normal solar spectrum which are less conspicuous

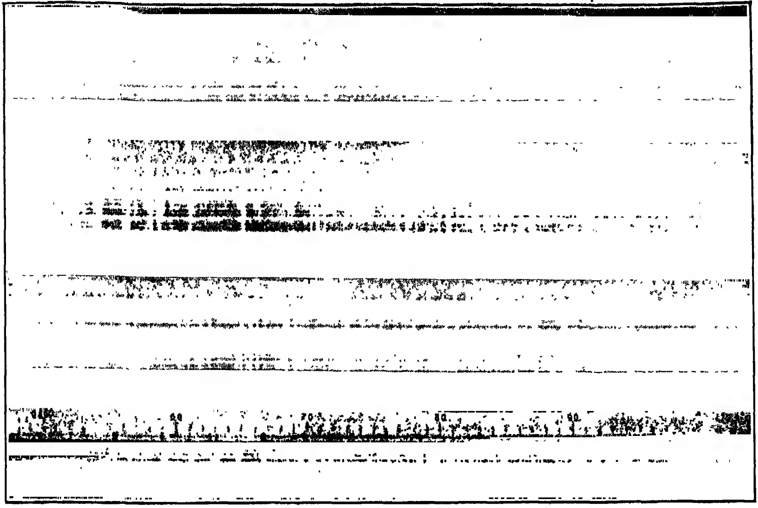


FIG. 201. The Sun-Spot Spectrum

This is a portion of the "map" photographed with the great tower-telescope and spectrograph at Mt. Wilson. An image of the spot was thrown on the middle of the slit, so that the spectrum of the umbra appears along the middle of the strip, and that of the photosphere at the edges. The exposure was graduated so as to make the spectrum of the spot equally strong.

A Nicol prism and compound quarter-wave plate were placed in front of the slit, so that thirteen or fourteen narrow strips were formed alternately by light circularly polarized in opposite directions. Consequently lines subject to Zeeman effect (§ 595) in the spot spectrum present a zigzag appearance — the component shifted toward the red being cut off on one strip, and that shifted toward the violet being eliminated on the two adjacent strips. The magnitude of the Zeeman effect differs greatly for different lines.

Certain lines are much strengthened in the spot spectrum, and others are conspicuously weakened, while a number of telluric lines (§ 578) are absolutely unaffected. The band lines, produced by compounds, show no Zeeman effect, and occur only in the spot.

Among the most notable individual lines are those given in the following table; they may readily be identified by the scale of wave-lengths (in angstroms) at the top of the spectra.

STRENGTHENED LINES	WEAKENED LINES	TELLURIC LINES	BAND LINES
6304.0 Ti	6322.3 Ni	6302.2	6371.1
05.9 Sc	61.0 Ni	03.0	74.4
06.2 Sc	62.5 Zn	06.0	78.2
12.4 Sc	66.7 Ni	06.8	79.0
25.4 Ti	69.7 Fe +	10.1	82.2
30.3 Cr	71.5 Si +	10.8	etc.
60.6 Ti	78.4 Ni	14.5	
63.0 Cr	81.0 Fe	15.2	

Those listed as strengthened are all characteristic low-temperature lines. Those weakened are either high-temperature lines or enhanced lines. The latter — denoted in the table by the suffix "+," as Fe +, Si +, etc. — are practically obliterated in the spot. The telluric lines are due to oxygen in the earth's atmosphere, the band lines to a compound of calcium and hydrogen in the sun-spot.

but in the opposite direction, in that the low-temperature lines are weakened and the enhanced lines strengthened. Consequently the faculæ must be hotter than the photosphere.

594. Magnetic Field in Sun-Spots. When the spectrum of a spot is observed with high dispersion, many of the lines, especially in the yellow and red, are found to be widened, and some divided into two components separated by a bright interval. This doubling of lines was first observed by Young, at Princeton, in 1892. Hale, in 1908, showed that for spots near the center of the sun the two components of each doublet were circularly polarized (§ 551) in opposite directions. The same disposition of the polarizing apparatus which extinguished the red (or violet) components of the doublets also cut off the red (or violet) edges of the numerous widened lines, showing that these were really very narrow doublets, incompletely resolved.

The only known way in which such double, circularly polarized lines can be produced is by the action of a powerful magnetic field upon the luminous gases.

595. The Zeeman Effect. When an electric spark is placed between the poles of an electromagnet, so that the atoms emit light while in a powerful magnetic field, each spectral line is broken up into a close group of lines, the light of which is polarized. When the light comes off at right angles to the magnetic field, each line, in the simplest case, is split into a triplet, in which the outer components are plane-polarized at right angles to the magnetic field, and the central component in a plane parallel to it. The separation of the lines is proportional to the magnetic field but is very different for different lines. A few lines in arc spectra and most of those in band spectra are not affected at all. Most lines, however, are resolved into three groups of closely packed lines (Fig. 202). (These phenomena were first observed by a Dutch physicist, Zeeman.)

When the light comes off parallel to the magnetic field (through a hole in one of the pole-pieces), the central component of a triplet (or central group for a complex line) disappears, while the outer components (or groups) form a doublet in which the two members are circularly polarized in opposite directions. Absorption lines are affected in the same manner.

596. Zeeman Effect in the Spot Spectrum. When a spot is near the center of the sun's disk, the lines in its spectrum are double and circularly polarized, showing that the magnetic field is directed in the line of sight, or at right angles to the sun's surface. When the same spot is near the sun's limb, the lines become triple and plane-polarized, indicating a field perpendicular to the line of sight

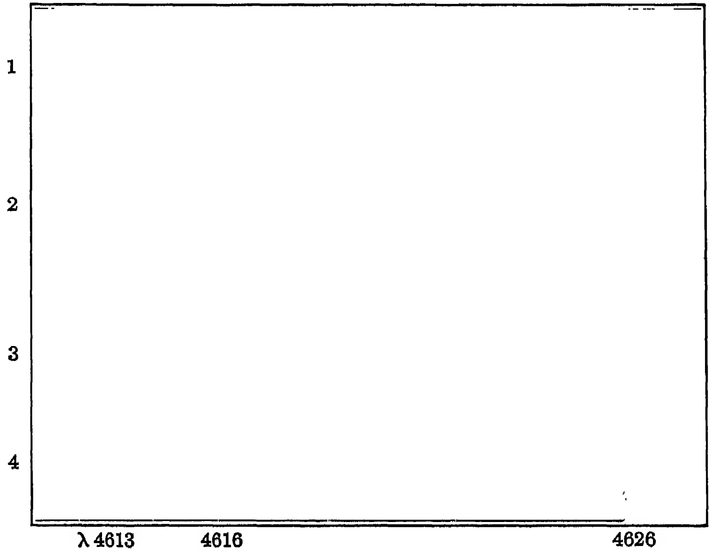


FIG. 202. Zeeman Effect for Chromium

Strips 1 and 4 show a small portion of the spark spectrum of chromium, in the blue; strips 2 and 3 show the same under the influence of a magnetic field at right angles to the line of sight. A Nicol prism was interposed and set so that the components which are polarized at right angles to the magnetic field are shown in strip 2; those polarized in the plane parallel to it in 3. The line λ 4613 is divided into three components, λ 4626 into 6, λ 4616 into 14. With lower dispersion the complex lines would appear as blurred triplets, as is the case with some other lines on the plates. (From photograph by H. D. Babcock, Mt. Wilson Observatory)

but still at right angles to the sun's surface. For spots part way to the limb the lines are double when the light comes from the edge of the spot near the sun's center, and triple when it comes from the edge near the limb, showing that the lines of magnetic force spread outward from the spot (Fig. 203).

The separation of the doublets or triplets for different lines differs just as it does in the laboratory (complex groups being unresolved), and the band lines are unaffected.

This detailed agreement proves conclusively the existence of a powerful magnetic field in sun-spots. The intensity varies in different spots and apparently diminishes in upper levels of the atmosphere. In large spots it may reach 4000 gausses, which is about one third of that between the pole-pieces of some dynamos. The field in the spots, however, is maintained over an area thousands of miles in diameter.

597. Polarity of Spots. Spots in which the directions of vortical rotation, as shown in the Hα spectroheliograms, are opposite have oppositely directed magnetic fields. The red components of the doublets in the spectrum of one spot are circularly polarized in the same manner as the violet components of the others. The leading and following spots of a typical pair (§ 233) are always opposite in polarity, but their field strengths may be very different. Furthermore, the leading spots of each pair in the southern hemisphere of the sun possess fields opposite to those of the leading spots in the northern hemisphere; and the polarity exhibited by the leading spots, in both hemispheres, is reversed when a new sun-spot cycle begins after a minimum. Thus, the magnetic period of the sun is twice the familiar (11-year) spot-period.

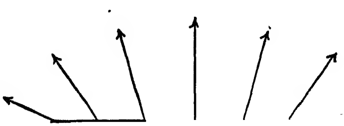


FIG. 203. The Magnetic Field in Sun-Spots

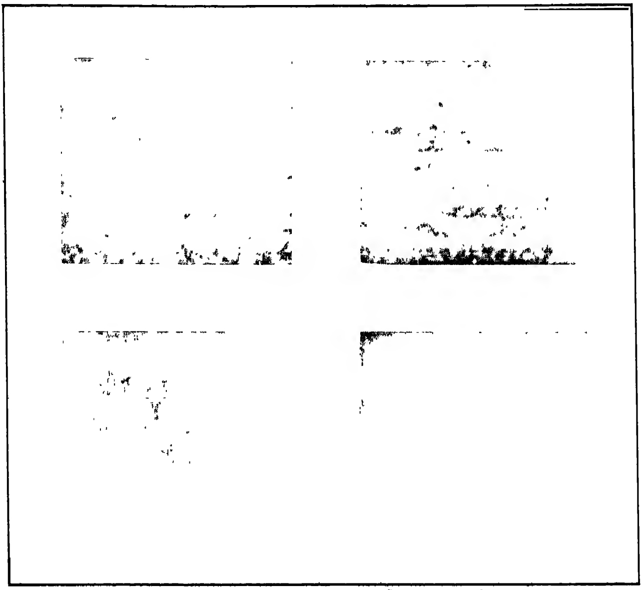
In complicated spot-groups adjacent umbræ often show opposite polarities in an apparently chaotic arrangement.

598. General Magnetic Field of the Sun. One of the finest triumphs of precise measurement has been the discovery by Hale and his colleagues that the sun possesses a general magnetic field much like that of the earth. The sun's north magnetic pole, like that of the earth, is near the north pole of rotation but not coincident with it. The Zeeman effect produced by this field is at best sufficient only to widen the lines very slightly, but careful measures taken with suitable polarizing apparatus reveal slight shifts from which the direction and magnitude of the magnetic field may be found.

Even the small changes which occur as the magnetic axis is tipped toward or from the earth by the sun's rotation have been

detected; and it is thus found that the magnetic pole is 4° from the pole of rotation, and revolves around it in a period of 31.5 days. The field intensity decreases rapidly with height, falling from 50 gausses to an amount too small to measure, between the base and the top of the reversing layer. The former value is about one hundred times as great as the maximum value of the magnetic field at the surface of the earth.

June 2. $16^h\ 10^m$ A.M. June 3. $5^h\ 14^m$ P.M.



June 3. $4^h\ 58^m$ P.M. June 3. $5^h\ 22^m$ P.M.

FIG. 204. Motion of a Dark Hydrogen Flocculus

The dark flocculus, after remaining almost quiescent near a spot, was suddenly sucked into it, from a distance of about 140,000 kilometers. This indicated the vortical nature of the spot. (Photographed at Mt. Wilson Observatory, June 3, 1908)

599. Hale's Theory of Sun-Spots. This mass of evidence bearing on the nature of sun-spots has not been obtained by accidental or random investigations but as the result of a deliberately planned campaign instituted by Hale, so that the results and the theory to which they give rise are justly associated mainly with his name. The facts outlined above have led to the formation of a theory of sun-spots which, although it accounts for almost

all the phenomena, is still regarded by its author (with true scientific caution) as tentative.

According to this theory, a sun-spot is a great funnel-shaped vortex in the outer layers of the sun, within which the gases are ascending spirally upward and outward (with a motion exactly opposite to that of water running out of a wash-basin).

The rapid expansion at the throat of the funnel cools the ascending gases and may maintain for weeks a temperature fully 2000° centigrade below that of the surrounding photosphere. Thus a sun-spot is the greatest refrigerating mechanism known to exist.

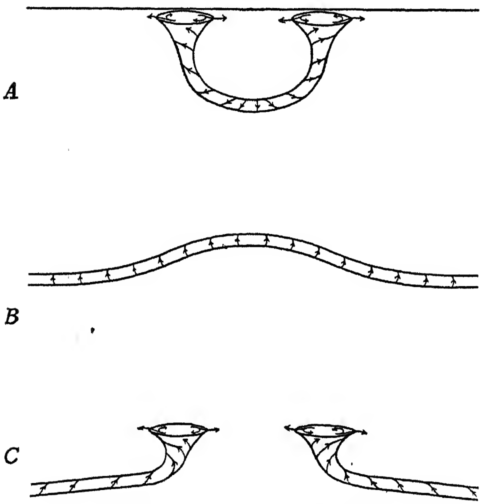


Fig. 205. Sun-Spots as Vortices

This cooling accounts for the darkness of the spot (§611), for the existence in it of compounds giving band spectra, and for the changes in the intensity of the lines. The gases finally flow out along the surface almost directly away from the spot, thus producing the Evershed effect. The most rapid flow may well be below the visible surface, for we can probably see down into a sun-spot for a distance that is only a very small fraction of its diameter. The inward motion in the chromosphere and the vortical structure shown by the flocculi probably represent upper eddies setting in the opposite direction.

The origin of the magnetic field is doubtless associated with the rapid rotation in the deep-lying vortex, opposite polarities presumably corresponding to right-handed and left-handed whirls but the exact nature of the electromagnetic action involved is still uncertain. It may well be related to the fact that the ascending gases carry great numbers of electrically charged particles.

It is fairly probable that the sun's general magnetic field is connected in a similar fashion with the axial rotation of the sun.

The components of a bi-polar group must be in some way connected, since the polarity is nearly always opposite in the two cases. They may be the free ends of a single U-shaped vortex (Fig. 205 *A*) like that produced by drawing an oar through water. Bjerknes has recently suggested that a primary vortex may extend horizontally east and west far below the surface, — perhaps entirely around the sun (Fig. 205 *B*), — and at times bend up to the surface (Fig. 205 *C*) and discharge much of its material and energy. This may explain how gas can flow out of both members of a bi-polar spot for weeks at a time, why spots may disappear and break out again in the same place, and perhaps why the spot-zones work toward the equator from each side during the spot-period. The reversal of polarity in the new cycle can be accounted for by a reversal of the direction of rotation in the primary submerged vortices, which is supposed to be always opposite in the two hemispheres.

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CHAPTER XVI

THE SUN'S LIGHT AND HEAT

LIGHT OF THE SUN • THE SOLAR CONSTANT • ATMOSPHERIC TRANSMISSION • ENERGY OF SOLAR RADIATION • SUN'S TEMPERATURE; STEFAN'S LAW, WIEN'S LAW, AND PLANCK'S LAW • TEMPERATURES OF CENTER AND LIMB, SUN-SPOTS, AND FACULÆ • TEMPERATURE OF THE SOLAR ATMOSPHERE • THEORY OF PLANETARY TEMPERATURES • RADIOMETRIC MEASUREMENTS OF PLANETARY TEMPERATURES • SPECTRA OF THE PLANETS

In the study of the sun's light and heat, astronomy comes into contact with matters of far-reaching practical significance. Not only is sunlight a daily necessity for the support of life on the earth; it is also the ultimate source of the mechanical energy employed in industry, transportation, and communication, which is an essential factor of civilization. The energy liberated when coal and oil are burned represents sunlight that was stored in trees and plants thousands of centuries ago; the energy extracted from winds and waterfalls is a product of the sunshine that warms oceans and deserts.

600. The Light of the Sun. The most casual observation, without instrumental aid, shows that the light of the sun is very great. The average illumination produced by the sun at the zenith has been measured (by Kimball, 1913) as 103,000 meter-candles (§ 568). Passage through the earth's atmosphere considerably reduces the intensity of sunlight; the illumination just outside the atmosphere is probably about 135,000 meter-candles. Compared with the light of other heavenly bodies (as received at the earth) the sun's light is about 465,000 times that of the mean full moon. It is 900 million times as great as that received from Venus, the brightest of the planets, at her best, and 11,400 million times as much as that from Sirius, the brightest of the stars. Enormous as these numbers are, the agreement of the results of several observers, working by quite different methods, indicates that they are correct within a few per cent.

The candlepower of the sun is found by multiplying 135,000 by the square of 1.495×10^{11} (the number of meters in an astronomical unit), which gives 3.02×10^{27}. Since the superficial area of the sun is 6.07×10^{22} square centimeters, *a single square centimeter of the sun's surface shines with a light of about 50,000 candlepower.* A sphere only 1 inch in diameter, of the same surface brightness, would have a candlepower of nearly a million. No artificial source of the same size would even approach this.

According to the best available data, the sun's surface brightness is about 600,000 times that of a candle flame, 10,000 times that of melting platinum, and more than 10 times that of the hottest part of the carbons in an electric arc. Even the brilliant arc looks like a dark spot when interposed between the eye and the sun.

Sunlight is also much whiter than the light from ordinary artificial sources. The "daylight lamps," which are designed to give light of about the same color as the sun, illustrate this.

From the above data the total luminous flux from the sun is easily computed to be 3.79×10^{28} lumens (§ 568), and its surface brightness 624,000 lamberts. A sphere 1 astronomical unit in radius has an area of 2.806×10^{27} square centimeters; hence the surface brightness of a perfectly white surface illuminated by full sunlight at the earth's distance would be 13.5 lamberts.

The luminous efficiency of sunlight is obtained by dividing the total luminous flux by the total radiant flux, 3.79×10^{26} watts (§ 604), and comes out 100 lumens, or about 8 candles per watt. This is to be compared with 1 or 2 candles per watt for the most efficient artificial sources. The difference is a consequence of the very high temperature of the sun (§ 605).

The sun's disk is brightest at the center, but the variation is slight until near the edge, where the brightness falls off rapidly, so that at the limb itself the brightness is not more than one third of that of the center. The color is modified also, verging toward chocolate, because the blue and violet rays are much more affected than the red and yellow; this is the reason why the darkening at the limb of the sun is so much more conspicuous in photographs.

601. Measurement of the Intensity of the Sun's Radiation. As explained in section 568, there is a distinction between the

light radiated from a luminous source and the energy radiated. The accurate determination of the intensity of the sun's energy radiation is a very important problem.

The solar constant. This intensity, at the earth's mean distance (that is, the quantity of energy that falls in unit time on unit area of a surface placed at right angles to a sunbeam just outside the earth's atmosphere) is known as the *solar constant.* It is usually measured in heat units, and Abbot, from the mean of hundreds of measures, makes it 1.938 calories per minute per square centimeter. It is subject to fluctuations of a few per cent (§ 603).

The pyrheliometer. Such measures are made with the pyrheliometer. This is an instrument in which the radiant energy of the sun, of all wave-lengths, is transformed into heat and then measured in calories. (A calorie is the quantity of heat required to raise 1 gram of water, or equivalent mass of other material, 1° centigrade. It is equivalent to 4.186×10^7 ergs.)

The measurement is made by permitting a beam of sunlight, of known cross-section, to be completely absorbed by a body of known heat capacity (for example, a blackened silver disk), and noting accurately the consequent rate of rise of temperature. It is necessary to determine and allow for all heat lost by the receiving body during the experiment, or received by it from other sources. When this is done, observations with instruments of different types are in good agreement; but only that portion of the heat which has penetrated through the earth's atmosphere to the surface is thus measured, and the observations therefore require correction.

Atmospheric opacity. When the sun is low in the sky, as seen from a given point on the surface of the earth, its rays are passing through scores of miles of air, and their intensity is much diminished, especially in the shorter wave-lengths. Thus, comparatively little heat and light comes from the setting sun, and what light there is exhibits a strongly reddish hue. Even when the sun is near the zenith and the air-path of the rays is much shorter, the energy of solar radiation is considerably reduced by atmospheric opacity. Furthermore, the normal opacity may be much increased by the presence of dust, smoke, and water-vapor.

Only about 70 per cent of the initial solar radiation gets through to sea-level when the sun is at the zenith and the air is free from dust and clouds.

Determination of atmospheric transmission with the spectrobolometer. For radiation of a definite wave-length the depletion in the atmosphere varies in a simple manner; the logarithm of the fraction of radiation which gets through in the direct beam is proportional to the total quantity of air traversed by the rays. Sunlight consists of radiation of many wave-lengths, which are affected very differently; and the only trustworthy way of dealing with it is to measure with the spectrobolometer (§ 567) the energy received, when the sun is at various altitudes, separately for each different wave-length; to determine the depletion for each; and to find the total solar radiation outside the atmosphere by summing up the values thus computed. The process is accurate but laborious.

A very important and long-continued series of measurements of the solar constant have been made by the Astrophysical Observatory of the Smithsonian Institution, under the direction of Dr. Abbot. Observing-stations have been located in deserts and on mountain peaks, in order to avoid, so far as possible, difficulties due to water-vapor and dust in the atmosphere; and pyrheliometers have even been attached to small balloons and sent into the upper air 16 miles above the ground. It is planned to continue these measurements indefinitely.

602. By-Products of the Measurement. *Atmospheric dust.* One interesting by-product relates to the presence of fine dust many miles high; this dust is blown up by volcanic eruptions and requires years to settle back to the ground. Meanwhile air-currents carry it in invisible clouds many times around the earth; its presence, however, is indicated by diminution in the solar radiation that reaches the ground.

The blue of the sky. The purest air is not altogether transparent to light; each molecule diverts energy from the transmitted beam, scattering it out in all directions. Just as a tiny chip on a water surface reflects most strongly the shorter waves, so the air molecules are more efficient in impeding the transmission of blue light than red light. The law that governs such *scattering* was discovered by the late Lord Rayleigh. The removal of blue light leaves the rays from the setting sun reddish in color; and the blue light thus removed is not lost but appears as the

blue of the sky. Thus light from the sky is merely sunlight scattered by air molecules, and it is blue because molecules scatter the blue light more strongly than the red. Dust particles which are small compared with the light-waves act in the same fashion, so far decreasing atmospheric transparency.

Number of air molecules. From observations of the transmission of sunlight through the atmosphere when most transparent, the Smithsonian investigators and others have calculated, using Rayleigh's formula, the number of air molecules that are active in scattering solar radiation, and from this the number per cubic centimeter under standard conditions. The latter result agrees closely with the value 2.70×10^{19}, obtained in other ways. This proves that in clear mountain weather the depletion is due almost entirely to molecular scattering, — that is, that the high atmosphere is very nearly as transparent as it can possibly be.

603. Fluctuations in the Solar Constant. The term "solar constant" is to some extent a misnomer, for the sun's radiation is slightly variable. The value of the solar constant given above (1.938 calories per minute per square centimeter) is the mean indicated by the Smithsonian observations; the actual value at a given time may differ from this by ± 0.05, or even somewhat more, as indicated by simultaneous measurements at stations located in widely separated parts of the earth.

The most conspicuous of these fluctuations was a drop from 1.95 calories per minute per square centimeter in January, 1922, to 1.90 at the end of May, followed by low values which continued for over a year, during the period of sun-spot minimum. The more usual fluctuations are irregular and occupy a few days or weeks.

The solar constant has been found to be greater (by about 2 per cent) in years when sun-spots are numerous; probably this is a general rule, but another decade or two of observation will be required to confirm it.

There is some evidence that the variation in the solar constant is accompanied by a proportionate slight variation in the light of the sun, as measured by the amount reflected from the planets.

Possible effects on climate. Attempts are being made to utilize the observed fluctuations of the solar constant for the pre-

diction of the general run of terrestrial temperatures; the effects are small. Long-continued changes in solar radiation would undoubtedly result in important alterations of climate. The palæontological and biological evidence showing the continued evolution of life on the earth since Cambrian times is strong evidence that the solar constant has not undergone any large permanent changes in several hundred millions of years. Temporary changes afford a plausible explanation of the recurrent ice ages, when glaciers covered much of Europe and North America. Since it requires 80 calories to melt 1 gram of ice of density 0.92, it follows that if there were no absorption in our atmosphere the quantity of solar heat received from a vertical sun in one hour would melt a layer of ice 15.8 mm. (or $\frac{5}{8}$ inch) thick. According to this the earth receives from the sun every year enough heat to melt a layer of ice 35 meters (114 feet) thick, covering its entire surface. For a plane surface always exposed to a vertical sun the thickness of the ice melted would be four times as great.

An unusual amount of volcanic activity, throwing up great quantities of dust into the atmosphere, is thought to produce cold seasons, since the quantity of heat reaching the lower air is then reduced by the resultant increase in atmospheric opacity.

604. Energy of Solar Radiation. A clearer realization of the enormous amount of energy which the earth receives from the sun can be obtained by expressing it in the more familiar mechanical units. According to the known value of the "mechanical equivalent of heat" 1 calorie is equivalent to 41,860,000 ergs. Ten million ergs per second constitute the familiar electrical unit of power, the *watt*, 746 of which equal 1 *horsepower*. Abbot's value of the solar constant therefore corresponds to 1.35×10^6 ergs per second per square centimeter, or 0.135 watts per square centimeter, or 1.81 horsepower per square meter. This is equivalent to 4,690,000 horsepower per square mile. If the solar radiation which strikes the earth in a single second could be converted into power, its value at the low rate of 1 cent per kilowatt hour would be $478,000,000. Yet the amount which strikes the relatively tiny earth is only one part in 2200 millions of the truly enormous energy which the sun radiates in all directions.

Solar engines. Numerous inventors have devised mechanisms for directly utilizing solar energy. There are many regions of the earth in which, during the daylight hours, solar heat equivalent to one horsepower could be absorbed on a surface of about two square yards. Unfortunately, owing to the comparatively small temperature difference which can conveniently be set up, the process of direct transformation of this heat into mechanical work is inefficient, and no solar engine has yet been devised which is commercially successful. The increased cultivation of vegetation for its fuel value appears to be a more hopeful project.

The sun's heat is directly utilized in southern California and elsewhere for heating water for domestic purposes.

Radiation from the sun's surface. If now we calculate the amount of radiation at the sun's surface itself, we come to results which are simply amazing. The total solar radiant energy which passes outward through a spherical surface surrounding the sun at the earth's mean distance is readily calculated from the measured value of the solar constant: it is 1.35×10^6 multiplied by 2.806×10^{27} (the area of the surface in square centimeters), or 3.79×10^{33} ergs per second. This, then, is the rate at which the sun radiates energy (since the space between the sun and the earth is almost perfectly transparent). Dividing by the surface area of the sun, or 6.07×10^{22} square centimeters, gives 6.25×10^{10} ergs per second per square centimeter as the rate of radiation from unit area of the sun's surface. This is equivalent to 89,500 calories per square centimeter per minute, and this amounts to nearly 84,000 horsepower, continually acting, for every square meter. If the sun were frozen over completely to a depth of 40 feet, the heat emitted would be sufficient to melt this whole shell in one minute of time; and if an ice bridge could be formed from the earth to the sun by a column of ice 2 miles in diameter and extending across the whole 93,000,000 of miles, and if by some means the whole solar radiation could be concentrated on it, it would be melted in one second; and in eight seconds more would be dissipated in vapor.

The entire heat radiation of the sun is 5.43×10^{27} calories per minute, which corresponds to 5.08×10^{23} horsepower (508 thousand billion billions). If the whole of this power could be set at work drawing the earth away from the sun, against the pull of solar gravitation (which amounts to 3,600,000 millions of mil-

lions of tons weight), it would raise it at the rate of more than 10 kilometers per second. Since the force grows smaller at greater distance, the energy emitted in six months would suffice to pull the earth entirely away from the sun into infinite space; and that radiated in thirty-three years would do the same for all the planets of the solar system.

605. The Sun's Temperature and the Laws of Radiation. Radiation of so large an amount of energy is proof that the sun's visible surface is very hot. It is only "by courtesy" that the sun can be said to have *a* temperature, since the temperature is now known to increase with increasing depth within its outer, visible regions. The laws of radiation from hot bodies are, however, well known, and it is possible to calculate the *effective temperature* of the sun, that is, the temperature which a perfect radiating body (§ 606) of the same size and at the same distance would have if it emitted radiation like the sun's. It is highly probable that this represents a sort of average of the temperatures of the visible layers.

The perfect radiator may be matched with the sun as regards the *whole amount of radiation from it*, or the *amount of radiation of a given wave-length*, or the *distribution of the radiation among the different wave-lengths*. These three comparisons give three slightly different results. The good agreement among them shows that, while the sun is not an exactly perfect radiator, it approximates this condition. All three agree in indicating an effective temperature of about 6000° K. (Absolute temperature, denoted by the letter "K," in honor of Lord Kelvin, is reckoned, in degrees centigrade, from the absolute zero, which is 273° below centigrade zero.)

606. Perfect Radiators, or Black Bodies. As a body is heated to incandescence it radiates an increasing amount of energy, and its light becomes continually brighter and whiter. Of different substances, those which are the blackest when cold shine brightest and radiate most heat when hot. That is to say, the emitting power of a body is large if its absorbing power is large. An ideally *perfect radiator* would appear absolutely black when cold. The properties of the radiation from such a perfect radiator, or *black body*, are simpler than those of the radiation from

ordinary bodies; these properties have been deduced theoretically and verified experimentally, and are the basis for the estimates of the sun's temperature. A surface well covered with lampblack is a tolerable imitation of a black body.

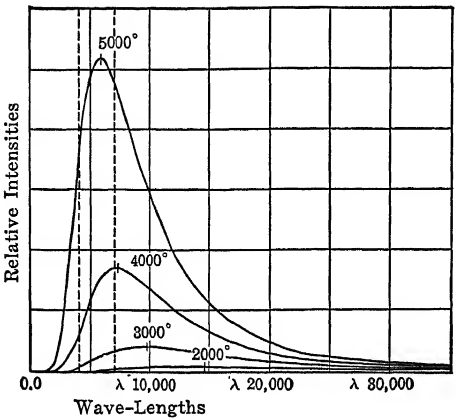

FIG. 206. Energy Curves of a Black Body

These curves give on a uniform scale the *intensity of radiation at different wave-lengths* emitted by black bodies at absolute temperatures 2000, 3000, 4000, and 5000 degrees, respectively. As the temperature rises, the intensity for each wave-length increases, but much more rapidly for the short wave-lengths. The wave-length of maximum intensity, indicated by a short vertical line crossing each curve, varies as $1/T$ (Wien's law). The area included under the curves, which represents the total radiation, is proportional to T^4 (Stefan's law). The two vertical dotted lines represent approximately the limits of the visible portion of the spectrum. It is evident that the fraction of the total radiation which is visible — and therefore the luminous efficiency (§ 568) — is very small at low temperatures, but increases rapidly

607. Total Radiation and Stefan's Law. The total radiation from a black body follows Stefan's law, which states that the total energy in ergs, in all wave-lengths, radiated per second from each square centimeter of a hot, black surface, is

$$E = \sigma T^4.$$

In this equation T represents the temperature in degrees K, and σ is a constant which has been determined by experiment as 5.72×10^{-5} (when E and T are measured in the units stated). The radiation from a surface which is not a perfect radiator is less than this.

In section 604 it was shown, from the average value of the solar constant, that 6.25×10^{10} ergs per second are radiated per square centimeter of the sun's surface; and when E is assigned this value in Stefan's equation, the value of T calculated is 5750° K.

608. The Distribution of Radiation and Wien's Law. Any hot body radiates energy in a wide range of wave-lengths (theoretically an infinite range). From a perfect radiator the intensity is a maximum for a certain wave-length, which is shorter

the higher the temperature. Wien's law states that the wave-length corresponding to maximum intensity, from a perfect radiator at absolute temperature T, is, in centimeters,

$$\lambda_m = \frac{0.289}{T}.$$

Spectrobolometric study of the sun's radiation shows that λ_m for the sun is 4.70×10^{-5} centimeters. Accordingly Wien's formula indicates a solar surface temperature of 6150° K.

609. Radiation in a Given Wave-Length and Planck's Formula. With the spectrobolometer (§ 567) the intensity of radiation from a black body (or, indeed, from any source whatsoever) in any narrow region of wave-length can be measured. The result can be illustrated graphically by plotting intensities as ordinates against wave-lengths (Fig. 206) as abscissæ.

The theoretical equation for these curves, derived by Planck, is in excellent agreement with observation.

This equation is

$$E_\lambda = \frac{c_1 \lambda^{-5}}{\epsilon^{\frac{c_2}{\lambda T}} - 1},$$

where $c_1 = 3.71 \times 10^{-5}$; $c_2 = 1.435$; λ is to be expressed in centimeters; ϵ is the base of natural logarithms (or 2.718); and $E_\lambda \Delta\lambda$ is the quantity of radiant energy, in ergs, emitted per second per square centimeter of a perfectly radiating surface at the absolute temperature T, and lying in the narrow band between the limiting wave-lengths λ and $\lambda + \Delta\lambda$, where $\Delta\lambda$ is a small increment in wave-length.

For example, when T in the formula is 6000°, the calculated value of E_λ in the neighborhood of wave-length 4.7×10^{-5} centimeters is 1.0×10^{15}. Accordingly, from a perfect radiator at 6000°, 1.0×10^{8} ergs per second per square centimeter of surface are emitted between wave-lengths 4700 and 4710 angstroms ($\Delta\lambda = 10^{-7}$ centimeters).

Stefan's law as well as Wien's law can be derived from Planck's formula by application of the calculus.

Gray bodies. Any source excited to radiation by temperature alone emits, at each wave-length, radiation of intensity which may be less than that computed from Planck's formula, but not greater. If it is the same fraction of the black-body intensity for all wave-lengths, the radiating body is called gray.

Transparent bodies, when of moderate thickness, emit very little light even if very hot, on account of their almost negligible absorbing power, as is illustrated by comparing the brightness of a Welsbach mantle with that of the gas flame which surrounds it and is the hotter of the two. No substance, however, is completely transparent, and a layer of hot gas thick enough to be opaque always shines brightly.

610. The Sun's Energy Curve. The curve which shows the variation of the intensity of solar radiation throughout the spectrum, as it would be observed outside the earth's atmosphere, is an important product of the Smithsonian spectrobolometric investigations. It is represented by the heavy line in Fig. 207. The maximum lies at wave-length 4.7×10^{-5} centimeters. The depressions corresponding to individual Fraunhofer lines are too narrow to show on so small-scale a diagram; the inflections in the curve in the violet are due to great groups of lines.

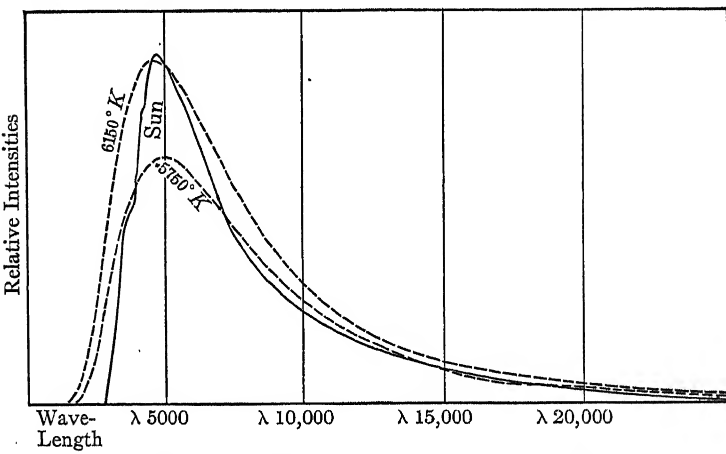

FIG. 207. Distribution of Energy in the Solar Spectrum

The two dotted curves in the figure represent the theoretical radiation, according to Planck's formula, of two black bodies, of the size and distance of the sun, at temperatures 5750° K and 6150° K, respectively. At wave-length 5×10^{-5} centimeters (in the green) the sun gives off a little more radiation than the black body at the higher of these temperatures; hence

the regions of its surface concerned in this radiation must actually average at least as hot as this, or hotter. In the extreme red the radiation corresponds to that from a black body at only 5800°, and the same is true in the violet. In the infra-red the radiation follows closely that of a black body at 5600°; but in the ultra-violet (where there are many heavy Fraunhofer lines) it falls off very rapidly. From a study of the regions between the ultra-violet Fraunhofer lines, however, Fabry has concluded that the unobstructed radiation in this region also corresponds to that from a black body at 5900°. Wave-lengths shorter than 2.9×10^{-5} centimeters are unobservable (§ 578).

The intensity for individual wave-lengths therefore indicates effective temperatures for the sun ranging from 5800° to 6300°. Wien's law gives 6150°; Stefan's law, 5750°. The round number 6000° K may therefore be taken as within 5 per cent of the sun's effective temperature.

It is not surprising that the temperatures computed by the three radiation laws are not exactly the same. There is good reason to believe that the sun is entirely gaseous and grows hotter toward the interior. Thus, the light in some wave-lengths may, on the average, come from deeper down, where the gas is hotter, while in other wave-lengths the light comes from higher up, where the gas is comparatively cool. Different methods of averaging these temperatures may well lead to different results. Consequently it is not to be expected that the solar radiation will be found identical with that of a black body.

611. Temperature of Different Parts of the Sun's Disk. *Radiation from center and limb.* Radiation from the center of the sun's disk comes from deeper and hotter layers than that from the limb. This is because the path through the solar atmosphere of rays from a considerable depth, which would emerge approximately tangential to the sun's surface, is so long that such rays are blocked off by the opacity of the gas; and the rays which do emerge at the limb start from higher levels (Fig. 212 *A*). Thus, the falling off in the brightness of the solar disk toward the limb is explained, as is its redder color.

A study of the radiation from separate regions of the disk indicates an effective temperature of 6300° K for the center and

5000° for the extreme limb, where the light comes from the shallowest layer, and hence from cooler regions.

The temperature of sun-spots and faculæ. The umbræ of sun-spots (§ 231), although they show by contrast as black against the solar disk, nevertheless emit measurable radiation. From their relative brightness, compared with that of the photosphere, in different parts of the visible spectrum, the temperature of the umbræ appears to average somewhat over 4000° K. Faculæ, on the other hand, are brighter and emit more radiation than do the surrounding areas of the photosphere. Thus their temperature is probably a few hundred degrees higher than the average of the photosphere.

612. Temperatures of the Sun's Surroundings. In order to obtain some idea of the temperatures prevailing in the upper solar atmosphere, and generally throughout the solar system, we may make use of the conception of a small black sphere supposed to be a perfect absorber, radiator, and conductor of heat, so that it takes up all the solar radiation that falls upon it, and radiates this energy as fast as received, equally from all parts of its surface and in all directions. Stefan's law permits ready calculation of the temperature of such a sphere, at any distance from the sun and warmed by the sun's radiation alone. For convenience the computed temperature of such a sphere (which is approximated by a meteorite) may be called the *black-sphere temperature* at a given distance from the sun.

The direct computation of the temperature of such a black sphere at the earth's distance from the sun is of interest. If of radius a centimeters, it would absorb $(1.35 \times 10^6)\pi a^2$ ergs per second, by § 604. (The cross-sectional area exposed to solar radiation is πa^2 square centimeters.) The surface area is $4\pi a^2$ square centimeters, and at a temperature of T, absolute, it would, by Stefan's law, radiate σT^4 ergs per second per square centimeter, where $\sigma = 5.72 \times 10^{-5}$. Equating energy radiated to energy absorbed and solving for T leads to $T = 277°$ K, or $+4°$ centigrade.

At a point just outside the sun's surface the black-sphere temperature is $\sqrt[4]{\frac{1}{2}}$ times that of the photosphere (5750°), or 4835° K. At a height equal to the sun's radius it is 2924°. At greater heights it is inversely proportional to the square root of the distance from the sun's center (since the area of the portion of the sky covered by the sun is very nearly inversely proportional to the square of this distance). Thus, at a distance R astronomical units it is $277°/\sqrt{R}$.

613. Temperature of the Outer Solar Atmosphere. The temperature of the solar reversing layer and chromosphere is customarily estimated as about 5000° K, or near the black-sphere temperature. Owing to the Fraunhofer lines in the solar spectrum, however, the outgoing solar radiation is weak in the very wave-lengths which the solar atmosphere is able to absorb most strongly; and on this account the temperature of the outer reversing layer and the chromosphere may be considerably less than 5000°. The black-sphere temperature well out in the corona is still as high as 3000°.

The solar atmosphere necessarily gaseous. Since the pressures prevailing are known to be very low, all these temperatures, except perhaps the last, are above the temperature of volatilization of even the most refractory known materials (such as carbon and tungsten). Thus, the important conclusion is indicated that in the solar atmosphere (with the possible exception of the outer regions of the corona) no known substances can permanently exist except in the gaseous state.

It also appears that some special activity must be at work to keep the temperatures of sun-spots as low as they are (§ 599).

614. Theory of Planetary Temperatures. There is no meaning in the phrase "the temperature of space." A material body near the sun is heated by solar radiation to a definite temperature, but this temperature depends on the properties of the body and may be very different in different parts of it.

Consider a "black" planet which, like Mercury, always keeps the same face toward the sun. At the point on its surface where the sun is overhead, radiation is received at the rate corresponding to the solar constant at the planet's distance, that is, at the rate of $1.35 \times 10^6/R^2$ ergs per square centimeter per second, where R is the distance from the sun in astronomical units. Just this amount of heat must be got rid of by radiation; and if T is the absolute temperature of the surface, the rate of radiation is, by Stefan's law, $5.72 \times 10^{-5}T^4$ ergs per square centimeter per second. Equating energy emitted and received, $T = 392°/\sqrt{R}$. This is the highest temperature to which a black planet (with no atmosphere) can be heated by the sun at the given distance. Where the sun's rays strike obliquely, the heat received per

square centimeter is less and the temperature correspondingly less (varying as $\sqrt[4]{\cos z}$, where z is the sun's zenith distance). The dark side of the planet would be exceedingly cold.

For a rotating planet the noonday temperature would not rise so high and the night temperature would be much higher, since the surface would not have time to cool off before the sun rose again. The faster the rotation the smaller would be the difference between day and night. For rapid rotation the average temperature all over the planet would be nearly the black-sphere temperature (§ 612). The average at a given latitude for a given season will vary with the *insolation* (the average rate of reception of heat for both day and night together).

615. Negligible Effect of Conduction of Heat from the Interior. The interior of a planet is presumably much hotter than its surface, so that the surface is somewhat warmed by heat flowing out from within, as well as by solar radiation. Formerly it was supposed that such conduction of heat might keep the surface of a planet appreciably hotter than solar radiation alone; but for the four inner planets, which possess rock surfaces, the effect of such conduction in raising the surface temperature is certainly negligible. This is because the thermal conductivity of rock is so low (about 2500 ergs per square centimeter per second for a temperature gradient of 1° per meter) that even if the interior is molten, heat flows to the surface very slowly.

For example, in the case of the earth the rate of this flow of heat has been determined as about 80 ergs per second per square centimeter, corresponding to a temperature increase of 1° centigrade for every 30 meters of depth. Allowing for the fact that it operates twenty-four hours a day, conduction, all over the earth during a year, furnishes only 1/4200 as much heat as the solar radiation. If conduction were to be 10 per cent as effective as solar radiation, the rate of increase of temperature downward would have to be 420 times its actual present value, giving 1° in 7 centimeters, or 23,000° per mile. So rapid an increase in temperature with depth is absurd.

It follows that if the sun ceased to supply heat, the internal stores of heat in Mercury, Venus, the earth, and Mars would in all probability keep their surface temperatures only a few

degrees above absolute zero (40° at most, in the case of the earth). Consideration of the outer planets, with their cloud surfaces, is more complicated, as convection enters; but observation indicates that their temperatures are low, and it is probable that, even for these, solar radiation is the important source of surface heat.

616. Effects of Atmosphere. The atmosphere of a planet, by carrying heat from the warmer to the cooler parts of the surface, tends to smooth out the temperature differences which would otherwise develop between night and day and between high and low latitudes. The thermal conductivity of air is even less than that of rock; but an atmosphere transfers heat rather effectively by convection, that is, by winds that consist in the movement of warm air into colder regions, and of cool air into warmer regions. Heat is likewise transferred by ocean currents, but the study of these processes leads from astronomy into meteorology.

Greenhouse effect of atmosphere. An atmosphere possesses another important influence upon the surface temperature of a planet, sometimes called the *greenhouse effect.* The glass in a florist's greenhouse is transparent to light-rays but rather opaque to infra-red rays. Most of the solar heat is transmitted, warming the air inside, but a much smaller fraction of the radiation from within is directly transmitted out again through the glass, because it lies in the long wave-lengths. Consequently the glass walls of the greenhouse act as a trap for heat, raising the temperature inside by many degrees. The earth's atmosphere functions in a similar manner, as it is transparent to light but opaque in many regions of the infra-red.

A sufficiently dense atmosphere may raise the surface temperature of the planet by 20 to 25 per cent above the black-sphere temperature. When there is a heavy atmosphere, most of the radiation to space takes place not from the solid surface of the planet but from regions high in the atmosphere; and the actual surface temperature may thus be considerably greater than that of the effective radiating surface.

Surface not a perfect radiator. No planetary surface is perfectly black, and for this reason the temperature may be either greater or less than that computed for a black surface. Actual

surfaces absorb less heat, but also at the same temperature emit less, than a black surface. If a surface is blacker for the short waves, which it absorbs, than for the long waves, which it emits, it will be hotter than a perfectly black surface; in the contrary case, cooler.

617. Application to the Earth. Calculation of the mean temperature of the earth affords an example of these principles. Take the solar constant as 1.35×10^6 ergs per square centimeter per second. Of this incident radiation 37 per cent, according to Abbot, is reflected back into space, mainly by the clouds. The heat available to warm the earth is therefore 8.5×10^5 ergs per second per square centimeter of its cross-section, or 2.1×10^5 ergs per second per square centimeter of its entire surface, from which the radiation takes place. If this surface were black and of uniform temperature, this temperature, according to Stefan's law, would be 247° K, or − 26° C.

The actual mean temperature of the surface is about 287° K or +14° C. Part of this difference may be ascribed to the imperfect radiating power of the surface; but most of it undoubtedly arises from the greenhouse effect. Most of the long-wave radiation from the earth which actually escapes into space comes from the upper parts of the atmosphere, where the temperature is actually not far from the black-body temperature just computed. This confirms the proposition that moist air, which is nearly opaque for long waves, radiates them like a black body.

618. Measurement of Planetary Temperatures. The radiation received from a planet or the moon can be measured with a sensitive bolometer or thermocouple. Indeed, with large telescopes it is possible to distinguish between radiation from different portions of a planet's surface, as center and limb, equator and pole, dark and light markings. Planetary temperatures cannot, however, be deduced by simple application of Stefan's law to such observations, because the planets shine largely by reflected light and because the earth's atmosphere is transparent to their own infra-red radiation only in a comparatively narrow region between wave-lengths of 1.0×10^{-3} and 1.4×10^{-3} centimeters. By means of a water-cell (a small, transparent vessel containing water, placed in the path of the rays), which is opaque to the

low-temperature, long-wave planetary radiation, the fraction of the total radiant energy from the planet which is merely reflected sunlight can be determined.

Then, treating the planet as a gray body, application of Planck's formula to the remaining, planetary radiation leads to an estimate of the planet's temperature. The actual surface temperature will be higher if the planet possesses much atmosphere. Departures from grayness may work either way and are hard to allow for.

Such radiometric measurements have recently been made by various observers. The results thus far are only provisional and are summarized in Table IX.

TABLE IX. PLANETARY TEMPERATURES

	Measured	Calculated	
		A	B
Mercury (sunlit side)	690° K	445° K	631° K
Venus (dark side)	250	—	—
(bright side)	330	327	464
Earth (mean)	287	277	392
Moon (center of illuminated hemisphere)	400	277	392
Mars (warmest portions)	285	222	316
Jupiter	135	122	173
Saturn	120	90	128
Uranus	less than 90	63	89
Neptune	—	51	72

NOTE. All temperatures are given on the absolute scale. To change to centigrade, subtract 273. The column headed "Measured" presents values determined by Coblentz and Lampland, and by Pettit and Nicholson. The column headed "A" gives black-sphere temperatures (§ 614); "B" gives these multiplied by $\sqrt{2}$, or the calculated maximum temperatures of the center of the illuminated hemispheres of atmosphereless black planets. The observed values lie, as expected, between A and B in nearly every case.

The case of Mars merits special mention; it appears well established that the temperature of the equatorial regions during the day, and of the poles after the summer solstice, rises above 0° centigrade. The polar regions in spring are much colder. The morning side of the planet is colder than the after-

noon side, which has been exposed longer to the sun's rays; the dark regions are warmer than the light ones, and seasonal effects due to prolonged insolation have been detected, — those areas where summer is advancing having been observed to rise slowly in temperature.

619. Planetary Spectra. As most of the radiation received from the planets and satellites is reflected sunlight, their spectra closely resemble the normal solar spectrum, showing the dark Fraunhofer lines on a continuous background. In addition the spectra

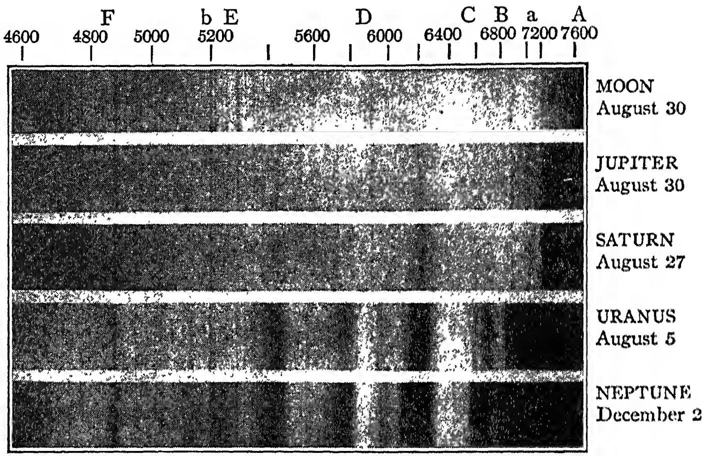

FIG. 208. The Spectra of the Major Planets

The spectrum of the moon is shown at the top for comparison. The strength of the bands in the spectra of Uranus and Neptune is remarkable. (Photographed in 1907 by V. M. Slipher, Lowell Observatory)

of the major planets (Jupiter, Saturn, Uranus, and Neptune) are crossed by heavy absorption bands which have not been duplicated in terrestrial laboratories (Fig. 208). These increase in strength from Jupiter to Neptune (§ 473).

Reflection from a solid planetary surface (if colored) alters the intensity relations of the continuous background of the spectrum of sunlight but does not result in additional dark lines. On the other hand, telluric bands in the spectrum of direct sunlight are known to be produced by oxygen and water-vapor during the passage of the rays down through the earth's atmosphere (§ 578).

There have been numerous attempts to determine whether identical bands are present in planetary spectra, particularly in those of Venus and Mars, which may have atmospheres similar in composition to that of the earth. The observations are very difficult because the telluric bands would conceal the planetary bands of presumably identical wave-lengths, except when the planet is approaching or receding rapidly, when the latter would be shifted by Doppler effect and so rendered separately observable. Long exposures are necessary to obtain photographs under these conditions with sufficient dispersion.

Observations made at Mt. Wilson show that these telluric bands are substantially absent from the spectrum of Venus but faintly present in that of Mars, with intensities estimated to correspond to 5 per cent as much water-vapor and 15 per cent as much oxygen as occurs in the earth's atmosphere. These preliminary numerical estimates may require later revision.

EXERCISE

What change in the sun's temperature would be necessary to change the solar constant by 1 per cent? *Ans.* $\frac{1}{4}$ of 1 per cent, or 15° K.

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CHAPTER XVII

ATOMIC THEORY AND ASTROPHYSICS

THE ELECTRICAL STRUCTURE OF MATTER • THE NUCLEAR ATOM AND THE PERIODIC TABLE • ENERGY STATES • SPECTRAL SERIES • QUANTUM THEORY OF SPECTRA • THERMAL IONIZATION AND EXCITATION • APPLICATIONS OF THE THEORY TO THE SUN-SPOT SPECTRUM AND FRAUNHOFER LINES • RADIATION PRESSURE IN THE CHROMOSPHERE • THE AURORA • THE OPACITY OF IONIZED GAS • EXTREME TENUITY OF SOLAR ATMOSPHERE • PHYSICAL CONDITIONS IN THE SUN'S INTERIOR • AGE OF THE SUN • SOURCE OF SOLAR ENERGY

620. The New Astrophysics. Astrophysics, or the "new astronomy," began in the latter half of the nineteenth century with the employment of the spectroscope and photometer for astronomical observations. The data obtained by the new methods, however, remained but partially interpreted until very recent years. Many of these data, in particular the spectroscopic, involve the behavior of atoms in the stars, and interpretation has waited for the development, by physicists and chemists, of a theory of the nature of matter capable of explaining the behavior of atoms in the laboratory.

We are still far from a complete theory of atomic structure, but much is already known, and application of the new principles to astrophysical matters is leading to results of great interest in astronomy. Here is the most active frontier of the science, where discoveries of importance are continually being made. On that very account the new astrophysics is with difficulty described in a textbook: it is still in a state of flux, and conclusions which today are thought to be well substantiated may be overthrown tomorrow.

THEORY OF ATOMIC STRUCTURE

621. The Electrical Structure of Matter. The atoms of matter (the least particles separable by chemical means) are not ultimate units, but *structures*, which are thought of as composed of far more minute electrified corpuscles. The chemical, spectroscopic,

and other properties of the chemical elements depend upon the manner in which these structures are built up, the constituent corpuscles being similar in all cases.

These corpuscles, or natural units of electricity, are termed *electrons*. They are of two kinds, arbitrarily denoted as positive and negative; the latter are commonly called simply electrons, while the former are often called *protons*. Every positive or negative electron possesses the same electric charge, namely, 4.774×10^{-10} electrostatic units (accurately measured by Millikan at Chicago). The mass of a negative electron is 8.93×10^{-28} grams; that of a positive electron, 1.65×10^{-24} grams, or 1847 times as great. An electron is supposed to be about 2×10^{-13} centimeters in radius, — hundreds of thousands of times smaller than an atom; a proton is probably much smaller.

Electrons of unlike signs attract one another, and those of like signs repel, in accordance with Coulomb's law of inverse squares, which has been shown to hold good for distances as small as 10^{-12} centimeters. They are also subject to forces when moving in a magnetic field, and generate magnetic fields when in motion.

622. The Nuclear Atom. A mass of evidence supports Sir Ernest Rutherford's suggestion (1911) that each atom consists of a positively charged *nucleus* surrounded by a number of negative electrons, which are thought of as attracted by the nucleus and moving about it in orbits, forming a sort of miniature solar system (though more complicated). The positive charge in the nucleus exactly balances the negative charges of the electrons, making the atom as a whole neutral. The nucleus itself is very tiny (of the order of 10^{-12} centimeters in diameter), while the outer electron orbits, which determine the size of the atom, may be 100,000 times as large. The nucleus contains almost the whole mass of the atom and is very probably built up of protons and negative electrons held closely together in a manner not yet understood.

623. Atomic Number. Almost all the properties of an atom depend on the number of electrons outside the nucleus, and on the consequent arrangement of their orbits. This number equals the net positive charge of the nucleus (measured in electronic units) and is called the *atomic number*.

Two atoms with nuclei of the same net charge but of different constitution are indistinguishable by ordinary chemical and spectroscopic means, although they may differ in atomic weight and in radioactive properties. Such atoms are called *isotopes*. Aston and others have shown that most of the heavier chemical elements are mixtures of isotopes.

Hydrogen, the lightest of the atoms, has the atomic number 1, helium 2, lithium 3, and so on. The frequencies of the "characteristic X-rays" emitted by the various elements are connected with the atomic numbers by simple formulæ which make it possible to assign these numbers with certainty for all known elements, the greatest being 92 for uranium. Only two of less atomic number (85, 87) remain to be discovered.

624. Atomic Structure and the Periodic Table. Many atomic characteristics can be explained on the supposition that the electron orbits in the more complex atoms are arranged in successive layers, or *shells*, each composed of orbits considerably larger than the last. When one of these shells is complete, containing all the orbits that the general laws of atomic structure permit it to include, the atom shows no tendency to chemical combination. Since no chemical combinations are formed by the inert gases, — helium (atomic number 2), neon (10), argon (18), krypton (36), xenon (54), and radon (86), — in order to complete the successive shells it seems necessary, therefore, to have 2, 8, 8, 18, 18, and 32 electrons, as was first suggested by Lewis and Kossel.

An atom like sodium (11), which contains one electron outside the complete shells, in a much larger orbit than the rest, has one unit of chemical *valence*, or combining power; one like calcium (20), which has two such electrons, has two units of valence; and so on. This accounts for the repetition of *chemical properties* shown in the well-known *periodic table* of the elements (Table XXXVII, Appendix), and discussed in all treatises on chemistry. The *spectra* of elements which have the same number of valence electrons are likewise very similar. Where the number of valence electrons is large, the spectra are usually complicated and rich in lines.

625. Ionization; Spark Spectra. The process of removing an outer electron from an atom is called *ionization*. This can be

accomplished by various physical means, some of which permit the measurement of the energy required. An ionized atom, or *ion*, has a net positive charge, and its spectrum differs entirely from that of the neutral atom. There is conclusive evidence (some of which will be discussed later) that *arc spectra are produced by neutral atoms, and spark spectra by ionized atoms.* (Compare § 592.)

The spark spectrum of any element strikingly resembles, in general structure, the arc spectrum of the element of next smaller atomic number. For example, the spectrum of ionized magnesium (commonly denoted by Mg+) is so like that of sodium (Na) that corresponding lines may be matched, one by one. Since both Mg+ and Na have eleven electrons outside the nucleus, and only one outside the completed shells, the inference is obvious that the general characteristics of a spectrum depend on the number of electrons remaining in the atom outside the complete shells.

This conclusion has been confirmed by study of the spectra of multiply ionized atoms which have lost two, three, or even six electrons; such spectra are built on the same plan as those of atoms of atomic number less by the number of lost electrons.

626. Excited Atoms; Energy States. Not only can one or more electrons be removed from an atom, in ionization, but also a given atom or ion can be *excited*, that is, the position of one or more of its electrons can be changed within the atom. The change from the *normal state* to the *excited state* requires the *taking up of energy* by the atom. When the same change occurs in the reverse direction, — that is, from the excited state back to the normal state, — the *same amount* of energy is *emitted* by the atom.

The principal experimental method of investigating the excitation of atoms was developed by Franck, in Germany (1913). Atoms of the given element, in a high vacuum, are bombarded with electrons which have been caused, by falling through a measured drop in electrostatic potential (volts), to move at a high velocity with known kinetic energy. After colliding with the far more massive atom the electron may be found moving at the same speed as before; such a collision is called *elastic*. Sometimes the electron is slowed up; such a collision is *inelastic*.

The lost kinetic energy is absorbed into the atom, with presumable rearrangement of its constituent electrons.

Measurements of this sort show that an atom of a given kind always absorbs energy in one or another of a set of *definite amounts*, corresponding to changes from its normal state to definite excited states. If the energy of the bombarding electrons is less than the smallest of these amounts, all the collisions are elastic and the atoms are unaffected.

There is good evidence that even the simplest atom, hydrogen, can exist in more than twenty excited states, while for a complex atom like iron hundreds of such energy states have already been recognized by spectroscopic means; and *molecules*, which may be excited in a similar fashion, exhibit an even greater manifold.

An excited atom gets rid of its contained energy and returns to the normal state in a very short time. In many instances the *life of the excited state* is known to average less than a ten-millionth of a second. For some *metastable states* the life may be as long as a thousandth of a second.

The passage of an atom from one energy state to another is called a *transition*.

THE ORIGIN OF SPECTRA

627. Radiation from Atoms. When an excited atom returns to the normal state or passes to an excited state of less energy, the difference between the energy of the former state and of the latter has to be got rid of somehow. Such a loss of energy may occur in two quite different ways.

If the excited atom collides with a "free" electron or another atom, the energy may be expended in setting the two in motion or in exciting the other atom. Such encounters (which are the reverse of inelastic collisions) are called *collisions of the second kind*.

Of more importance for our purpose is the fact that an excited atom or molecule can discharge its store as *radiant energy*. When the atom is free from external influences, this radiation is *monochromatic* and *gives rise to a spectral line*. It is now believed that all spectral emission lines originate in such transitions.

In a rarefied gas an atom is very much more likely to dispose of its energy as radiation than by collision.

Dark-line ("*absorption*") *spectra* indicate the reverse process, that is, the taking up of radiant energy by atoms, which are consequently excited to states of greater energy. Naturally this reverse process occurs most frequently when intense radiation from a hotter source is passing through a cooler gas, as Kirchhoff's third law implies.

628. The Relation between Energy and Frequency. In every case where a transition occurs in an atom from a state of higher energy to one of lower energy, with emission of radiation, *the frequency of the emitted radiation is exactly proportional to the amount of energy radiated.* This is one of the most important physical principles discovered in the twentieth century. It was first brought to view by Planck, further employed by Einstein, and its application to the origin of spectra is due to Bohr (1913).

If W_1 is the energy of the atom in the initial state concerned, and W_2 that in the final state, their difference, E, will be the energy liberated as the result of this transition as radiation of frequency ν. We have, then,

$$W_1 - W_2 = E, \tag{1}$$

$$E = h\nu. \tag{2}$$

The constant of proportionality, h, in equation (2) is a *universal constant of nature*, called *Planck's constant.* When W_1, W_2, and E are expressed in ergs, and ν in vibrations or cycles per second, the value of h is experimentally found as

$$h = 6.55 \times 10^{-27} \text{ c.g.s. units.}$$

The wave-number n (or number of waves per centimeter) equals ν/c (§ 550), so that

$$n = \frac{W_1 - W_2}{ch}. \tag{3}$$

If W_1 is greater than W_2, energy is radiated; if less, it is absorbed.

Why this extraordinary relation of proportionality between energy and frequency should exist is still quite unknown. There is reason to suspect that it is connected with some property of radiation itself rather than of the atoms which emit it.

These equations have been amply and accurately verified in so many cases that their general applicability over a large field of atomic data is unquestioned. Hence, in studying the structure of a given atom, the *manifold of energy states* may be investigated either directly, by means of experiments involving electronic bombardment, or indirectly, through a study of the frequencies or wave-lengths of the lines in the spectrum of the element when excited in the gaseous form. The latter method is of very much greater precision. Thus, formula (3) may be written $n = W_1/ch - W_2/ch$. Consequently, the wave-number of every spectral line appears as the *difference* of two *spectroscopic terms*, corresponding to the initial and final energy states involved in the transition; and the line is said to arise from the *combination* of these terms. The existence of such terms was recognized some years before the existence of the energy states themselves was realized.

629. The Spectrum of Hydrogen; Bohr's Theory. Long before the development of modern atomic theory, Balmer (1885) discovered that the wave-numbers of the well-known hydrogen lines could be represented with great accuracy by an equation which may be written in the form

$$n = 109{,}678.8\left(\frac{1}{4} - \frac{1}{m^2}\right). \tag{4}$$

Giving m the integral values 3, 4, 5, 6, we obtain values which compare as follows with the observed wave-numbers of the lines:

	Observed n	Computed n
Hα	15,233.22	15,233.16
Hβ	20,564.79	20,564.77
Hγ	23,032.54	23,032.55
Hδ	24,373.06	24,373.07

Twenty-seven lines of this "series" (up to $m = 29$) have been observed in the spectrum of the solar chromosphere, and about twenty in the laboratory. They become steadily fainter, as well as more closely spaced, as the value of m increases.

From what has just been said it is evident that these lines must be produced by transitions between energy states such that $(W_1 - W_2)/ch$ has the values given by Balmer's formula.

This suggests at once that these lines arise from transitions between a set of states, for which $W/ch = R/m^2$ (where R is the constant 109,678), and a single state, in which $W/ch = R/4$.

A simple theoretical explanation of this was given by Bohr in 1913. It has led to developments of great importance, both for the theory of atomic structure and for spectroscopy. The atomic number of hydrogen is 1, indicating that a hydrogen atom consists of one electron and a proton (the nucleus). The two supposedly attract each other according to the electrostatic law of inverse squares (Coulomb's law), and the electron may therefore be supposed to be in orbital motion about the proton (which is so much more massive that it moves very little). Since the two remain together, the orbit must be an ellipse; but, on the classical theory, it might be of any size and any eccentricity. Since the atoms of hydrogen are actually just alike, there must be some conditions which determine the size and shape of the actual orbits. Bohr, by a suitable specification of these conditions, succeeded in accounting for the spectrum.

The quantum conditions. These conditions are known as the quantum conditions, and their formulation is due to Bohr, Wilson, and Sommerfeld. As they are at present supposed to apply to the orbits of the electrons in an atom, they may be stated as follows.

(1) The product of the average angular momentum of the electron in its orbit by the angle swept out during a revolution is equal to the product of an integer, τ_1, and Planck's quantum constant h.

(2) The product of the average radial momentum (that is, momentum of the motion in the direction of the radius vector) by the whole amount by which this radius changes during a revolution (adding the increase and the decrease) is also equal to an integral multiple of h, $\tau_2 h$, which may or may not be equal to the first.

Only orbits which satisfy these quantum conditions (in addition to the equations of mechanics) appear to be possible (or at least to be followed by the electrons long enough to be detected). Each of these orbits is supposed to correspond to one of the energy states described above. The integers τ_1 and τ_2 are called *quantum numbers*.

The sizes of the orbits and the corresponding energy of the atom may be calculated (§ 630). It is found that the mean distance in the elliptic orbit must be 1, 4, 9, 16, $\cdots$ in general, k^2 (when k is a positive integer) times a certain quantity (5.3×10^{-9} centimeters, or 0.53 angstrom, or about one five-

hundred-millionth part of an inch). The energy of an atom in which the electron is in the kth orbit is found to be

$$W = C - \frac{chR}{k^2} \tag{5}$$

(where C is a constant and the other letters have the meanings already given).

Substituting this value in equation (3), it appears that a transition from the kth to the jth orbit will be accompanied by radiation having the wave-number

$$n = \frac{W_1 - W_2}{ch} = \frac{R}{j^2} - \frac{R}{k^2}. \tag{6}$$

Transitions from the third, fourth, fifth, orbits to the second orbit give exactly the wave-numbers of Hα, Hβ, Hγ, . . . , and account for the Balmer series (which includes all the hydrogen lines known in stars). But others are likewise possible. Transitions from the second, third, . . . , orbits to the first give computed positions far in the ultra-violet; those from the fourth, fifth, etc. to the third give lines in the infra-red. Hydrogen lines in exactly the predicted positions have also been discovered and measured by Lyman in the ultra-violet; and by Paschen and Brackett in the infra-red, affording a striking confirmation of the theory.

Many other lines appear in the spectrum of hydrogen gas, but are ascribed with good reason to the hydrogen *molecule*.

An additional postulate of Bohr's theory is that an electron moving in a permissible orbit (or *stationary state*) does not stir up electromagnetic radiation which escapes into space. This is inconsistent with the "classical" theory (which applies to the action of large electric charges), but is demanded by the facts. Why such stationary states are possible, why the quantum conditions should exist, and why the same constant h should be met with in many other diverse physical problems, no one has as yet the least idea. Rutherford may fairly be called the Copernicus of the atom, and Bohr its Kepler; but the Newton of atomic theory has not yet appeared. When a more complete theory becomes available, it may be that present ideas concerning the details of the electron orbits will be much changed, but the reality of the restriction to definite states of energy and motion appears to be assured.

Although still mysterious, these quantum relations are of the greatest importance; in fact, it is only on account of their operation that atoms,

and, indeed, all material bodies, have any definite size and shape, or possess characteristic physical and chemical properties.

630. Calculation of the Orbits. With close approximation a hydrogen atom may be treated as consisting of an electron revolving around a fixed center (the proton). The force between them is eE/r^2 (where e and E are the charges on the two; and r the distance between them). Circular, elliptic, parabolic, and hyperbolic orbits of any size are all possible under the classical dynamics, but few of these satisfy the quantum conditions.

All circular orbits satisfy the second condition, for there is no change at all in the distance, so that τ_2 is zero (which counts as an integer). The first condition applies to the angular momentum. If m is the mass of the electron, and ω its angular velocity, the centrifugal force is $m\omega^2 r$. This equals the attraction, so that

$$m\omega^2 r = Ee/r^2 \qquad \text{or} \qquad m^2\omega^2 r^4 = Eerm.$$

The angular momentum is $m\omega r^2$ or, by the last equation, $\sqrt{Eerm}$. The angle swept out during a revolution is 2π, and the first quantum condition gives $2\pi\sqrt{Eerm} = \tau_1 h$, which may be written

$$r = \frac{\tau_1{}^2 h^2}{4\pi^2 mEe}.$$

Introducing numerical values, $h = 6.55 \times 10^{-27}$, $m = 8.93 \times 10^{-28}$, $e = E = 4.77 \times 10^{-10}$, and

$$r = 0.53 \times 10^{-8} \tau_1{}^2 \text{ cm}.$$

A hydrogen atom may therefore exist with its electron moving in a circular orbit of radius 1, 4, 9, 16, $\cdots$ times a certain fundamental value (0.53 angstrom).

As in the gravitational case (§ 315), if the kinetic energy of the particle moving in the circular orbit were doubled, it would fly off in a parabolic orbit to infinity, and the atom would be ionized. In any state, therefore, the atom has less energy than an ionized atom by an amount equal to the kinetic energy of the moving electron; this is

$$\tfrac{1}{2} m\omega^2 r^2 = Ee/2r = \frac{2\pi^2 mE^2 e^2}{\tau_1{}^2 h^2}. \qquad (7)$$

Here, if C is the energy of the ionized atom,

$$W = C - \frac{ch}{\tau_1{}^2}\left(\frac{2\pi^2 mE^2 e^2}{h^3 c}\right). \qquad (8)$$

When the numerical values are introduced, the factor $2\pi^2 mE^2 e^2/h^3 c$ is found to be 1.09×10^5, agreeing, within the accuracy of the data, with the spectroscopic *Rydberg constant*, R. Setting k instead of τ_1 for the variable integer, we have equation (5) of section 629.

Elliptical orbits. The preceding account of Bohr's theory has been abbreviated for the sake of simplicity. Many other orbits exist, for which the second (or radial) quantum number, τ_2, is not zero. These are ellipses having the major axes equal to diameters of circular orbits, and minor axes which are simple fractions of the major, as $\frac{1}{2}$ for the two-quantum orbit, $\frac{1}{3}$ or $\frac{2}{3}$ for the three-quantum orbits, and so on. The energy W is substantially the same for all the orbits which have the same major axis; so that formula (5) still holds good, except for certain minute corrections. These indicate that the lines should have a very narrow *fine structure,* — that is, should be composed of several very close components. This prediction has been verified by observation with high dispersion.

The latest work. Recent work (especially by Schrödinger) indicates that the whole theoretical picture of electrons moving in orbits within an atom, like that of light moving in rays in empty space, may be supplanted by some more comprehensive, but more intricate, wave-theory. The conception of electron orbits, like that of rays of light, may, however, remain invaluable in elementary discussions of the subject.

631. The Spectrum of Ionized Helium. A helium atom which has lost one electron is very like a hydrogen atom, except that the charge on the nucleus is twice as great. Thus, the numerical value of E must be doubled in (8), giving, for the wave-numbers of the lines in the helium spectrum,

$$n = 4\,R\left(\frac{1}{\tau_1{}^2} - \frac{1}{\tau_1'^2}\right) \tag{9}$$

as compared with

$$n = R\left(\frac{1}{\tau_1{}^2} - \frac{1}{\tau_1'^2}\right)$$

for hydrogen (where R represents the *Rydberg* constant). Lines corresponding to $\tau_1 = 1, 2, 3,$ and 4, and to various values of $\tau_1'^2$, have been observed in the predicted positions in the helium spectrum.

The observed value of the constant R is 109,678 for hydrogen and 109,723 for ionized helium. This difference turns out to be in favor of Bohr's theory, for the nucleus of the atom, in each case, is not fixed but moves in a small orbit about the center of gravity of itself and the electron. The momentum of this motion must, in accurate calculation, be added in working out the orbits. When this is done, it is found that the value of R

for helium (which has the heavier nucleus) should differ from that for hydrogen by exactly the observed amount.

The lines of the He + series, for which $\tau_1 = 4$ and τ_1' has even values, lie very close to the hydrogen lines of the Balmer series but a little to the violet. Both series are observable together in vacuum tubes and in some of the hottest stars. The intermediate lines of the He + series (lying about halfway between the hydrogen lines) were observed in the stars by Pickering in 1896, — sixteen years before Fowler produced them in the laboratory.

632. The Spectra of Other Elements. When there are two or more electrons circulating about the nucleus, all the complications of the problem of three bodies are encountered (since the repulsions between electrons are comparable in strength with the attraction of the nucleus); and the exact calculation of orbits and prediction of spectral lines is not feasible. There is good reason to believe that all the electrons move in orbits quantized according to principles similar to those already described. When one of the inner electrons, belonging to a complete shell, is removed (which can be done by collision with a fast-moving electron from outside), another electron "falls in" from an outer orbit to replace it. The energy radiated is in this case very large, and the frequency of the radiation is high enough to place it in the X-ray region. A study of these *characteristic X-rays* gives information regarding the inner portions of the atom.

The outer electrons are less firmly bound, and changes in their orbits give rise to radiations in the visible region and also in the infra-red and ultra-violet. The energy associated with such orbits varies with their size, shape, and inclination to one another; so that the number of different energy states (or *spectroscopic terms*, as they are also called) is much greater than for hydrogen, and the spectra are much richer in lines.

633. Spectral Series; Rydberg's Formula. Where there is but a single outer electron, as in sodium, the energy for a given orbit can be expressed in the form

$$W = \frac{chR}{(m + \mu)^2}.$$

Here m is an integer (increasing as the radial quantum number increases), and μ is very nearly the same for all orbits which

have the same angular momentum, but changes as this changes. Transitions between various orbits of this type give rise to lines having wave-numbers, n, represented by the formula

$$n = \frac{R}{(m_1 + \mu_1)^2} - \frac{R}{(m_2 + \mu_2)^2}. \tag{10}$$

This formula was discovered in 1889; the constant R is known as Rydberg's constant. The interpretation in terms of electron orbits is due to Sommerfeld. The values of μ cannot yet be predicted theoretically, but can be found very accurately by observation of the spectral lines.

634. Illustration: the Sodium Spectrum. The transitions within the atom which give rise to its spectrum are clearly represented by means of a diagram of the type suggested by Bohr and Grotrian. Fig. 209 gives such a diagram for the sodium arc spectrum. It does not picture the orbits of the electrons, but shows the *amount of energy* which the atom contains in each state, and indicates the transitions between the various states. Each state is represented by a dot. The distance below the heavy horizontal line is proportional to the energy (W) which must be communicated to the atom in this state to pull the valence electron entirely away, ionizing the atom. The lowest dots therefore represent the states of least energy, and the heavy line the ionized condition.

States believed to correspond to orbits of the same angular momentum are plotted in the same column. Thus, the column headed s represents those of the smallest angular momentum, for which the energy is given, approximately, by the formula

$$W = \frac{chR}{(m + 0.63)^2};$$

the states of next greatest angular momentum are plotted under p, and for these the denominator is $(m + 1.12)^2$; for the third set (commonly called d) the angular momentum is again greater, and the denominator is $(m + 0.99)^2$; and so on. Only a few of the states are shown in the diagram, corresponding to the smaller values of m, as indicated. The remaining points would be closely crowded near the upper line.

Transitions between these states occur in accordance with the *selection principle* that during a transition the angular mo-

mentum must change by a unit. Such transitions are represented by lines joining points which lie in adjacent columns of the diagram. An atom in the 1 *s* state (the lowest energy state in the *s* column), for example, can change to any of the *p* states, but not to the others; similarly, an atom in the 1 *p* state can change to any *s* state or *d* state; and so on. (Other transitions, while not quite impossible, are extremely rare, and give faint lines.)

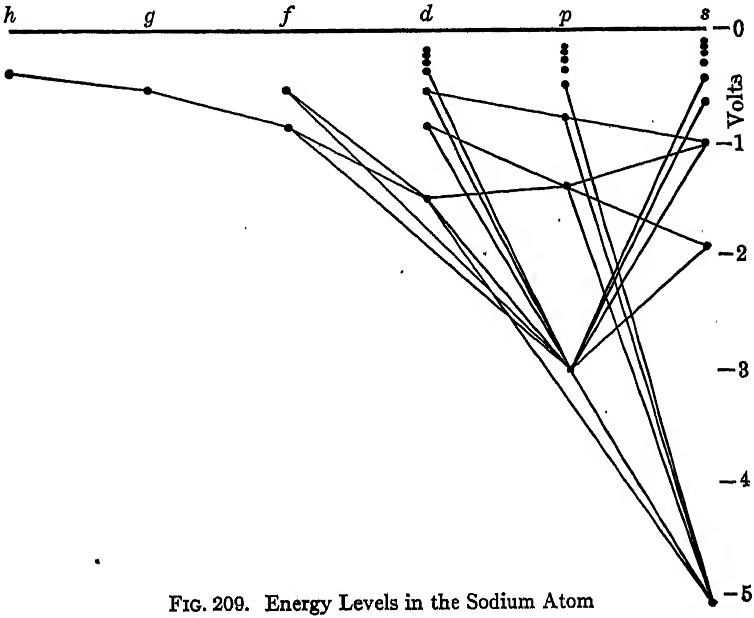

FIG. 209. Energy Levels in the Sodium Atom

The difference of level of any two points in the diagram represents the amount of energy radiated (or absorbed). Thus, small differences of level correspond to lines of small wave-number in the infra-red; larger differences, to visible lines; and still larger, to lines in the ultra-violet.

It should be added that all the *p* states are double, the two components being far too close to show separately on the scale of the diagram. Transitions from these states to others give double lines. It is easy to see that all the pairs which correspond to changes from or to the 1 *p* state (for example) will have the same separation (when measured in wave-numbers), and this is a great help

in interpreting the spectrum. (The latest suggestion regarding the origin of this duplicity is that the electron itself is rotating very rapidly, either forward or backward, as compared with its orbital motion, thereby producing small differences in the energy of the atom. This effect also is governed by quantum laws.)

635. Variation of the Sodium Spectrum with Temperature. Sodium vapor in the electric arc emits all the lines corresponding to the transitions shown on the diagram; but in the Bunsen burner, or in the electric furnace at low temperatures, it emits only lines corresponding to transitions from the p states to the lowest s state. (This series of lines is commonly denoted by $1\,s - mp$; a particular line in the series is denoted as $1\,s - 3\,p$; etc.) Again, if white light is passed through sodium vapor in a furnace at red heat, only the lines of the series $1\,s - mp$ are absorbed; but if the furnace temperature is 2000° or more, absorption lines of the series $1\,p - ms$ and $1\,p - md$ appear, though faintly.

The $1\,s$ state is the normal state of a sodium atom, since it has the lowest energy. If the atom is in the $1\,p$ state, it can return to the normal state, with radiation of a quantum of energy corresponding to the transition $1\,s - 1\,p$; and there is evidence that it is almost sure to do so in less than a millionth of a second. To get it back into the $1\,p$ state the atom must receive the same quantum of energy, either from absorption of radiation or from collisions with other atoms or electrons. In red-hot sodium vapor, collisions between atoms are very rarely violent enough to furnish the required energy, and substantially all the atoms are in the normal state.

An atom in this state can absorb amounts of energy corresponding to the transitions $1\,s - mp$, and no others. Hence only the corresponding lines appear in the absorption spectrum, the transition $1\,s - 1\,p$ giving the familiar pair of (D) lines in the yellow (§ 570), and the others producing lines in the ultraviolet having wave-numbers given by the formula

$$n = \frac{R}{1.63^2} - \frac{R}{(m+1.14)^2}.$$

This set of lines is called the *principal series.* Its members are the strongest lines in the spectrum, and appear *reversed* in the

ordinary arc spectrum, the cooler vapor in the outer parts of the arc absorbing the central portions of the widened line emitted by the hotter and denser inner vapors.

When the temperature is raised to 2000° or so, the more intense heat radiation, together with the more violent atomic collisions, maintains a sensible (though very small) fraction of the atoms in the $1\,p$ state, and the vapor is now able to absorb the lines of the series $1\,p - ms$ (called the sharp series) and $1\,p - md$ (the diffuse series). As the temperature is raised the fraction of the atoms in the $1\,p$ state, though still very small, increases, and these lines become stronger. At the same time the vapor emits the lines $1\,s - mp$ with increasing strength, for the atoms which are put into the $1\,p$ state are continually falling back into the normal state.

At still higher temperatures the fraction of the atoms in the higher energy states, such as $2\,s$ and $2\,d$, becomes perceptible. The lines $1\,p - 2\,s$ and $1\,p - 2\,d$ are emitted, and those of series $2\,d - mf$, $2\,s - mp$, etc. are absorbed.

The manner in which the various sodium lines successively appear, in emission or absorption, as the temperature is raised, is now clear. In the electric arc the vapor is heavily ionized, and many atoms are losing an electron while others are picking one up. In the latter case the electron may first fall into any one of the s, p, d, or f orbits, and from this down to others, so that all the lines are simultaneously emitted by the vapor, though any particular atom, at a given instant, can emit but one.

636. Sharp and Diffuse Series. In the sodium arc all the lines in the series $1\,p - ms$ are fairly sharp, and the lines $1\,p - md$ very fuzzy, whence the names "sharp" and "diffuse" series and the letters s and d. The later members of a series are also often much more diffuse than the earlier ones. If the density of the vapor is very low (vacuum arc or vacuum furnace), all the lines become sharp.

The explanation is that at low densities the atoms are far apart and do not influence one another perceptibly. At high densities the influence of neighboring atoms disturbs the electron orbits, causing their energies to be slightly different from one atom to another. Then the radiations emitted as the result

of similar transitions are not quite of the same wave-number, and combine to form a hazy line rather than a sharp one. These effects should be more pronounced for the larger orbits. Large orbits involve large angular momentum (corresponding to d and f terms) or large radial momentum (corresponding to later members of a series). Lines arising from combinations of such terms are the ones actually observed to be diffuse, and they are the most influenced by *pressure shift* (§ 564).

Photo-electric absorption. In addition to the line absorption a strong *continuous absorption* is observed in sodium vapor, beginning at the limit of the principal series and extending, with diminishing intensity, toward the violet.

This is also easily explained. To shift an electron from the normal state to one of the p orbits requires (in a rarefied gas) a definite amount of energy and gives a sharp line. To pull it out of the atom and ionize the latter demands the energy represented by the limit of this series of lines. The absorption of a still greater amount of energy not only removes the electron but ejects it with a more or less considerable kinetic energy, which, in the absence of quantum restrictions, may have any magnitude. Thus the corresponding frequency of the absorbed light varies continuously from one atom to another. Absorption by hydrogen, due to this process of "photo-electric ionization," has been observed in the stars (§ 942).

637. Ionization and Excitation Potentials. The amount of energy required to ionize an atom from the normal state is called the *ionization potential*, and that required to raise it from the normal state into any given excited state is called the *excitation potential*. These are usually expressed in *volts*.

As a unit of energy a "volt" is the energy acquired by an electron in falling through a difference of potential of one volt. This is 1.59×10^{-12} ergs. The wave-length λ of any line, in angstroms, and the energy V, in volts, required to bring about the corresponding transition are connected by the equation $V = 12{,}345/\lambda$.

For the (D) lines of sodium ($1\,s - 1\,p$; $\lambda\lambda$ 5896, 5890) V is 2.09 volts; this, then, is the excitation potential for the $1\,p$ state. The transition $1\,p - 2\,d$ gives lines at $\lambda\lambda$ 8195, 8183. Hence the additional energy required to produce it is 1.51 volts, and the excitation potential for the $2\,d$ state is $2.09 + 1.51$, or 3.60 volts.

The ionization potential for sodium (corresponding to the transition from 1 *s* to the limit of the *p* series of terms) is found to be 5.11 volts.

638. Complex Spectra. When more than one valence electron is present, the spectra are more complex than that of sodium. There are different systems of energy states; and the individual terms may be not merely double, but triple, quadruple, or even eightfold. Combinations between such complex terms give rise to groups of related lines known as *multiplets*, which

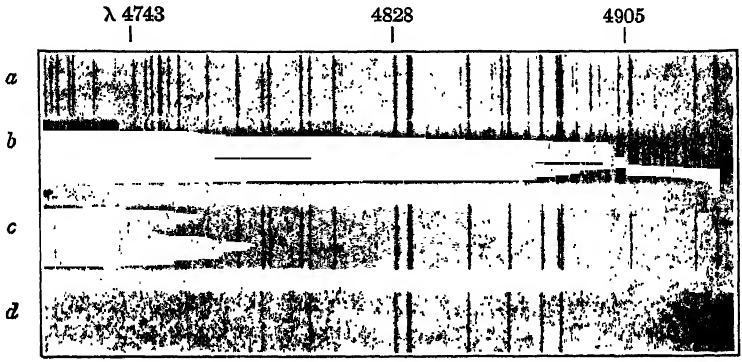

FIG. 210. Temperature Classification of Lines

The four strips show the spectrum of vanadium, in the blue, under different conditions: *a*, in the arc; *b*, in the furnace, at 2900° K; *c*, at 2650° K; and *d*, at 2400° K. At the lowest temperature only those lines are visible which arise from the lowest energy levels in the atom; at the higher temperatures additional lines appear, which arise in higher levels. Carbon bands appear in *b* and *c* at λ 4737 and λ 4715. (From photograph by A. S. King, Mt. Wilson Observatory)

may contain as many as fifteen members, some strong and some weak, the ratio of their intensities being determined by quantum conditions.

In some spectra, as in calcium, series of the type already discussed have long been known; in others, as in titanium, vanadium, and iron, there are literally hundreds of multiplets overlapping one another in bewildering confusion. It was only in 1922 that the key to the analysis of these difficult cases was found. Since then progress has been rapid, and almost all the important lines have already been interpreted; that is, it is known just what energy changes in the atoms produce them.

A good beginning has been made in working out the arrangement of the electron orbits, and it is already possible to predict the general character of a spectrum from the structure of the corresponding atom (neutral or ionized). Sometimes two electrons may change their orbits in the transition connected with the emission or absorption of a single line; but such detailed knowledge is not needed to interpret the energy relations, which are the important thing in astrophysics.

Just as in the case of sodium, the lines which correspond to transitions to or from the lowest energy level (according as absorption or emission is in question) appear at low temperatures in the electric furnace and are placed in Class I of King's temperature classification (§ 592). Lines of higher excitation potential appear successively at higher temperatures and fall into classes II, III, and IV, Class V being reserved for the lines peculiar to the arc.

The lines which arise from the high energy levels are more likely to be diffuse and to show large pressure shifts. The Zeeman effect for a given line depends upon the system and series to which it belongs, and may often be used to help in analyzing the spectrum, as may also the temperature classification.

639. *Raies Ultimes.* The intensity of a line increases with the number of transitions of the kind which produce it. At low temperatures, as has been stated, almost all the sodium atoms are in the normal state, of lowest energy; and even at high temperatures a large majority are in this state. The principal lines which correspond to transitions from this state (in absorption) or back to it (in emission) are therefore the strongest of all, and the last to disappear as the quantity of the element present is decreased. The *observed* last lines of any element are commonly known by the French term *raies ultimes*, introduced by de Gramont. Thus the (D) lines are the *raies ultimes* of sodium. Such lines are of obvious importance in spectrum analysis.

The English term "ultimate lines" is often used to describe all lines, whether strong or weak, which correspond to transitions to or from the lowest level. Lines from other levels are termed subordinate. When the ultimate lines are all at wave-lengths unobservable with ordinary apparatus (as for hydrogen), the observed *raies ultimes* arise from a higher level.

640. Spectra of Ionized Atoms; Enhanced Lines. The spectrum of an ionized element resembles, in general plan (§ 625), the arc spectrum of the element of next smaller atomic number. The principal difference is that since there is a net positive charge on the ionized atom, the energy necessary to remove the second electron or to shift it from one orbit to another is much greater, and the corresponding lines have much shorter wave-lengths. Thus the pair $1\,s - 1\,p$ is at $\lambda\lambda$ 5890–5896 for Na and at $\lambda\lambda$ 2796–2803 for Mg +, while $2\,d - 3\,f$ is at λ 18,460 for Na and at λ 4481 for Mg +.

The Rydberg constant, R, must be replaced by $4\,R$ in the formulæ defining series in the spectra of singly ionized elements (as was found by Fowler in 1914). Bohr's theory explains this (§ 631) and, conversely, shows that all series having the fourfold constant must originate in ionized atoms.

All the lines of the ionized atom are much enhanced in the spark spectrum, compared with the arc; they are therefore commonly called *enhanced lines*.

Ultimate enhanced lines, when at observable wave-lengths, appear in arc spectra and sometimes even in the furnace. For example, the (H) and (K) lines of ionized calcium (Ca +), at $\lambda\lambda$ 3933, 3968 (which are $1\,s - 1\,p$ combinations, like the (D) lines of sodium), are very strong in the arc and show at high temperatures in the furnace. It follows that a large fraction of the calcium atoms are ionized in the arc, and a small fraction in the furnace.

On the other hand, enhanced lines of high excitation potential, like λ 4481 of Mg +, do not appear in the arc at all and can only be brought out under the more disruptive conditions of the spark. To emit a line like λ 4481 the atom must be not only ionized but strongly excited in addition, and so violent a disturbance rarely occurs in the arc.

When the ultimate enhanced lines are inaccessible in the far ultra-violet, and the excitation potential for the others is very high, the enhanced lines are hard to produce, even in the spark, and none of them appear in the arc. This is the case for sodium and lithium.

641. Multiple Ionization. By very violent discharges additional electrons may be removed from the atoms. Each stage

of ionization has its own characteristic spectrum, and presumably its own complete set of electron orbits.

Four successive stages have been detected and fully analyzed by Fowler in the spectrum of silicon. They are denoted, beginning with the arc spectrum, by Si, Si +, Si + +, Si + + +, or, alternatively, by Si I, Si II, Si III, Si IV, etc. All four contain series, with constants R, $4R$, $9R$, and $16R$ respectively. Similarly, Paschen has analyzed three stages in the aluminium spectrum. Still higher stages of ionization (P V, S VI, Cl VII) have been observed by Millikan.

The ultimate lines for the higher stages of ionization lie in the extreme ultra-violet; but some of the subordinate lines lie in the visible region and, for silicon, have been observed in the stars.

642. Thermal Ionization: Effects of Temperature and Density. Atoms are ionized at sufficiently high temperatures in the absence of artificially produced electrical disturbances. The process is essentially similar to the dissociation by heat of a chemical compound into its constituent elements, the neutral atom dissociating into an ionized atom (or ion) and an electron.

As the temperature of a gas rises the average energy associated with each atom increases. A certain fraction of the atoms (very small at low temperatures but steadily increasing with temperature) acquire enough energy to undergo ionization, losing an electron. If the ionized atoms could be kept away from such free electrons, ionization would go on until no neutral atoms were left. Actually, sooner or later, as it flies about in the gas, each ion meets an electron and recombines, becoming neutralized again.

The relative numbers of neutral atoms and ions under any given conditions will depend upon an equilibrium between these two processes. High temperature, which increases the rate at which the atoms break up, favors ionization. High density, on the other hand, favors recombination, since, when ions and electrons are closer together, there is a better chance of their meeting. Thus, at the same temperature, the relative number of ions decreases with increasing density.

For different elements at the same temperature and pressure the degree of ionization is less, the greater the energy required for ionization, that is, the greater the ionization potential.

643. Thermal Excitation. The fraction of neutral atoms in a hot gas which are in any given excited state, with more than the normal minimum content of energy, also increases rapidly with the temperature. This fraction is usually small, and, at the same temperature, is least for states of highest excitation potential. The density of the gas probably has little influence, provided it remains low.

Exactly the same principles apply to the excitation of ionized atoms, when these are present.

There is evidence that after about a hundred millionth of a second an excited atom may discharge its excess energy as radiation. If, while still excited, it collides with an electron or an atom, this energy may instead be expended in setting them both in motion or, partly, in exciting the other atom. At high densities such "collisions of the second kind" are often numerous enough to be important.

644. Importance. Thorough comprehension of the principles of ionization and excitation is essential to any real understanding of solar and stellar spectra.

The idea that enhanced lines were emitted by atoms which had been decomposed into something simpler by high temperature was first propounded by Sir Norman Lockyer. It was only after the development of modern atomic theory that this decomposition was recognized as ionization, and the full importance of the subject was first pointed out by Saha, of Calcutta, in 1920.

645. Quantitative Treatment. Saha's theoretical equation for the fraction, X, of the atoms of any element which are ionized in a gas at the absolute temperature T is

$$\log \frac{X}{1-X} = -\frac{5048\,I}{T} + \frac{5}{2}\log T - 6.5 - \log p_e,$$

where the logarithms are to the base 10, I is the ionization potential in volts, and p_e the electron pressure in atmospheres. The latter is the pressure which the free electrons would exert if acting like a gas, and bears the same ratio to the total pressure that the number of free electrons bears to that of free electrons, atoms, and ions taken together.

The fraction Y of the neutral atoms of any element (or of the ions, as the case may be) which are in an excited state of excitation potential E volts, is given by the equation

$$\log \frac{Y}{1-Y} = -\frac{5048\,E}{T}.$$

More precise equations have recently been given by R. H. Fowler.

646. Enhanced Lines in Furnace Spectra. In the electric furnace, at high temperatures and low pressures, the theory indicates that a small proportion of the atoms are ionized, and this is confirmed by the presence of ultimate enhanced lines in the spectrum. For an element of low ionization potential, like calcium (6.1 volts), the *raies ultimes* of the ionized atom appear at moderate furnace temperatures, falling in King's Class II; for magnesium (7.6 volts) they appear only at high temperatures and are in Class IV. Enhanced lines of iron cannot be obtained

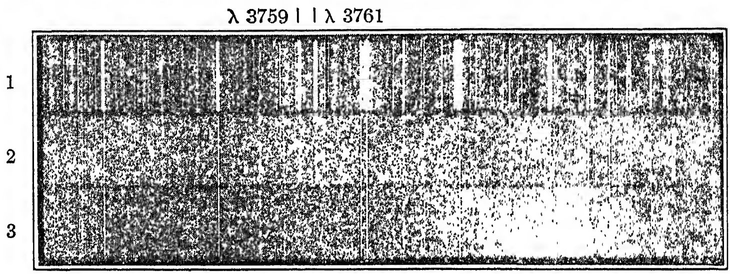

FIG. 211. Ionization in the Electric Furnace

No. 1 is a part of the arc spectrum of titanium in the ultra-violet. The ultimate enhanced lines at λλ 3759, 3761 (marked on the margin) are very strong. No. 2 shows the spectrum of this element in the furnace at a fairly high temperature. The arc lines are strong, and the enhanced lines are easily visible. No. 3 shows what happens when potassium is added (other conditions being unchanged). This element has an ionization potential of 4.3 volts, as against 6.8 for titanium; and at the furnace temperature it is abundantly ionized, setting free large numbers of electrons. These meet the ionized titanium atoms and combine with them, altering the conditions of equilibrium, so that very few Ti + atoms are left and the enhanced lines disappear, while the lines of the neutral atom are unaffected. This is a spectroscopic example of the "principle of mass action" (well known to the chemist) and affords an independent proof that the enhanced lines are really emitted by ionized atoms. (From photograph by A. S. King, Mt. Wilson Observatory)

in the furnace at all, doubtless because the ionization potential is too high for appreciable ionization at furnace temperatures.

When two or more kinds of atoms are present simultaneously, an interesting interaction occurs, which is illustrated by Fig. 211.

647. Band Spectra. In many cases molecules, as well as atoms, may be excited to emit characteristic spectra. These are of quite a different character from those which we have been discussing, and consist of very numerous lines, closely packed into *bands* and concentrated at certain points into prominent *heads* (Fig. 210). These spectra also have been interpreted by

the quantum theory. Their great complexity corresponds to the variety of processes that can go on in a molecule consisting of two or more atoms. An electron in one of the atoms may shift from one orbit to another; the two atoms may vibrate, changing their distance apart; and the whole molecule may rotate about its center of gravity. All these changes are quantized; those of rotation, which alter the energy very little, account for the close-packed lines of a band; changes in the vibratory energy of the atoms, for the various band heads; and various electron transitions, for systems of bands in different regions of the spectrum. Molecules may be ionized without dissociation into atoms, giving new band spectra quite different from those of the neutral molecules. Some of the molecules which give prominent band spectra are chemically incomplete, such as cyanogen (CN) and hydroxyl (OH). These arise from the partial dissociation of ordinary molecules in the luminous gases.

Band spectra produced by the molecules of cyanogen, titanium oxide, etc. occur in absorption in the sun-spots and in the cooler stars. The spectra of comets contain emission bands of N_2+, $CO+$, etc. The conspicuous telluric lines at the red end of the solar spectrum (Fraunhofer's (A) and (B)) belong to bands produced by oxygen in the earth's atmosphere.

APPLICATIONS TO SOLAR PHYSICS

The principles outlined above throw a flood of light upon many astrophysical problems. The discussion in Chapter XV of the absences of certain elements from the solar spectrum may now be somewhat amplified, details of the sun-spot spectrum explained, and a more accurate knowledge obtained of physical conditions in the sun, not only in the photosphere, reversing layer, and chromosphere but also in the deep interior.

648. The Absence of Certain Elements from the Solar Spectrum. If an element occurs in very small proportions in the sun, we should expect to detect it spectroscopically only by its ultimate lines. When these are in the visible region, even very rare elements, such as gallium and germanium, show such evidence of their presence.

For most of the missing elements (listed in Table VIII, p. 503) the ultimate lines are in the unobservable ultra-violet (see Note 4 of the table). Many, although not all, of these elements have other lines in the visible region, but the excitation potentials of the energy states in which these lines originate are so high that only a minute fraction of the atoms are able to absorb them, even at the high temperature of the solar atmosphere. Consequently such subordinate lines appear only if the element in question is abundant there.

Thus, the red triplet of oxygen (λ 7772–75), for which the excitation potential is 9.1 volts, appears in the solar spectrum. The fraction of the oxygen atoms in the reversing layer which are in a state to absorb this triplet is doubtless extremely small, so that oxygen must really be very abundant in order that the lines may show at all.

649. The sun-spot spectrum affords beautiful applications of modern theory. Many lines are strengthened in it; others are weakened (§ 593). The latter are, for the most part, enhanced lines; the strengthened lines are all arc lines, and mostly of low-temperature classes (I and II). It has long been recognized that these facts indicate that the spots are cooler than the photosphere, but the detailed behavior of different elements is only explicable by the theory of ionization. An arrangement of the elements according to the behavior of their lines in the spot spectrum is substantially an arrangement in order of ionization potentials.

For some elements (scandium, vanadium, titanium, yttrium, and, to a less degree, calcium) all the arc lines are strengthened in the spot spectrum, while the enhanced lines are very little affected. These elements all have low ionization potentials, about six or seven volts; and at the high temperature and low pressure of the reversing layer above the photosphere they are, according to Saha's equation, highly, although not completely, ionized. Even above the spots, where the solar atmosphere is probably 1500° or 2000° cooler, the majority of these atoms must be ionized. Thus, passing from photosphere to spot, the number of neutral atoms (using round numbers as an illustration) may increase from 1 per cent to 10 per cent of the whole,

while the percentage of ionized atoms drops from 99 to 90. The relative number of atoms ready to absorb the arc lines, therefore, is greatly increased above the spots, while the number absorbing the enhanced lines undergoes little proportional change. The latter are therefore little weakened, while the former are much strengthened.

This effect is most pronounced for arc lines of low excitation potential. For those of high excitation potential the fraction of the neutral atoms that are in the excited state is less at the lower temperature, and this counteracts the effect of the increase in their total number, and may quite neutralize it, as for some calcium lines.

For elements of higher ionization potential many of the atoms are neutral above the photosphere, and most of them above the spot; and the relative increase of the number of neutral atoms above a spot is much smaller. Consequently, only the ultimate arc lines are much strengthened, while arc lines of high excitation potential are weakened. This happens with magnesium, iron, and nickel, and conspicuously with silicon. The observable enhanced lines of these elements are of high excitation potential, and, as might be anticipated, are greatly weakened in the spot spectrum.

Zinc, which has the high ionization potential of 9.3 volts, represents a further stage. No enhanced lines appear in the sun, and the observable arc lines, which have excitation potentials of four volts and upward, are conspicuously weakened in the spots. Here almost all the atoms are neutral anyway, and the diminution above the spots of the fraction which are excited has full play.

A further stage is represented by the red triplet of oxygen. Here the ionization potential is 13.6 volts and the excitation potential 9.1. This triplet, while fairly prominent in the sun, disappears altogether in the spot spectrum.

The lines of the Balmer series of hydrogen (I. P., 13.5; E. P., 10.3) are also greatly weakened, though too strong to disappear.

650. Behavior of the Alkali Metals. At the other end of the series come some elements of even lower ionization potential than scandium. For sodium (5.1 volts) all the arc lines are

greatly strengthened in the spots, and for potassium (4.3 volts) only the lines of the principal series appear in the sun, and these are enormously strengthened in the spot spectrum. This is clearly a case of almost complete ionization. Rubidium (4.2 volts) does not show in the solar spectrum at all, but its ultimate lines ($\lambda\lambda$ 7948, 7800) appear distinctly in the spots.

This case is of special interest, for Saha predicted that the element, although completely ionized in the normal reversing layer, would show these lines faintly above spots, where they were found a few months later. Lithium (5.4 volts) behaves in the same way, its ultimate line λ 6708 appearing only in the spot spectrum. For cæsium, which has the lowest known ionization potential, 3.9 volts, the ultimate lines are in the infra-red, are absent in the Fraunhofer spectrum, and have not yet been investigated in the spots.

It will be remembered (§ 593) that *band spectra* (due to titanium oxide and to hydrides of calcium and magnesium) are prominent in the spots. Their disappearance outside them clearly means that the vapors of these compounds are dissociated at the higher temperature. The cyanogen and hydrocarbon bands, however, persist in the photospheric spectrum, as do also the OH bands in the ultra-violet, showing that these compounds are harder to decompose.

651. Strong and Weak Lines. When the lines of such an element as titanium are critically studied, it is found that strong lines are much less conspicuously strengthened in the spot than weak ones, even though the lines in question belong to the same multiplet, and are absorbed by atoms in exactly the same state. The increase in the number of these atoms over the spot evidently affects the weak lines the most. Again, among the enhanced lines of iron, the faintest lines of the same group are the most weakened in the spots. There are good reasons for believing that the relative numbers of atoms which make the transitions corresponding to the strong and weak lines are determined by quantum conditions and are the same in the photosphere and spot (being accurately calculable). It appears therefore, that a given proportional increase or decrease in the number of atoms that absorb a given solar line produces a much greater change in the appearance of a weak line than in a strong one, so that in the latter case the absorption is nearly saturated.

652. Winged Lines. The Fraunhofer lines are not all of the same width. Strong lines are wider than weak ones, and the strongest usually show hazy "wings" fading out gradually on each side. All such lines are found to be due to elements which

are abundant in the earth, to arise from atoms in the lower energy states, and to correspond to transitions which atoms in these states are most likely to make. All these circumstances indicate that, to produce a winged line, an unusually great number of atoms must be active.

The most conspicuous widened lines belong to ionized calcium, whose ultimate lines are by far the strongest and widest in the whole solar spectrum; iron, which gives a greater number of widened lines than all the others put together; nickel; and magnesium. These are four of the most abundant metals in the composition of the earth as a whole. The other two most abundant elements in the earth are oxygen and silicon; and the ultimate lines of these are inaccessible in the sun. The only silicon line for which we could expect it is strongly winged. As for oxygen, we have seen (§ 648) that it must be abundant in the sun. Sodium must also be plentiful, for the (D) lines are strongly winged in spite of the high degree of ionization. It is probable that both the strength and the width of a Fraunhofer line depend primarily upon the number of atoms in the solar atmosphere which are active in producing them.

On applying the quantum intensity relations mentioned in section 651 it appears probable that the number of atoms concerned is of the order of a million times as many for the strongly winged lines as for a just visible line.

The hydrogen lines are very wide and look different from the others, being more diffuse and not so dark in the middle. It is probable that additional widening influences are effective here, such as the Stark effect (electrical influence of neighboring charged atoms or electrons) and the Doppler effect, due to the thermal agitation of the atoms. Both should be much larger for hydrogen than for any other element.

The hydrogen lines are also much stronger than would be expected from their high excitation potential. The reason for this is not known.

653. The Nature of Absorption Lines. The presence of a dark line in the solar spectrum shows that energy has been subtracted from the outgoing photospheric radiation. This energy has been taken up by atoms in which the transitions corresponding to the line are occurring. Such an atom may leave its excited state and get rid of its store of energy in several ways. The transition may be reversed, and light of the same wave-length may be radiated, — a process often called *resonance*. Or a transition to

some other energy state may occur, with emission of light of a different wave-length, which is called *fluorescence*. Or, again, the stored energy may be converted into kinetic energy by a collision of the second kind (§ 627) and appear as heat in the gas. All these processes are often loosely called *absorption*, although the term, in its strict sense, applies to such as the last only.

Observations show that atoms, when present in large numbers, are capable of dealing not only with light of a particular wave-length but, to a rapidly diminishing degree, with light of neighboring wave-lengths. The widths and wingedness of Fraunhofer lines can thus be accounted for.

Much of the energy which disappears in a Fraunhofer line still persists as light, but is scattered by the atoms in other directions than that of the rays which reach the observer. Such scattering, especially in a thick atmosphere, may deplete the light of a given wave-length so much as to cause a conspicuous dark line, even in the absence of all true absorption (as Schuster has shown).

654. Enhanced Lines in the Chromosphere. It has long been known that enhanced lines are relatively prominent in the spectrum of the chromosphere, especially in the upper part. For example, Mitchell's photographs of the flash spectrum (§ 580) show that while the ultimate line of neutral calcium (λ 4227) is visible up to a height of 5000 kilometers, those of ionized calcium, (H) and (K), extend to 12,000 kilometers and are visible to the very tops of the prominences. Similar differences are found between the arc and enhanced lines of strontium, scandium, titanium, iron, and other elements.

This used to be very puzzling, for it was supposed that enhanced lines indicated a high temperature, and the top of the sun's atmosphere can hardly be hotter than the bottom. Saha, however, found a very simple explanation in ionization. At the top of the chromosphere the density must be very much less than at the bottom, and this greatly increases the ionization, in spite of the somewhat lower temperature. Above 5000 kilometers, for example, practically all the calcium atoms must be ionized, leaving none to emit the arc lines, though the enhanced lines appear up to the height where the chromosphere thins out

entirely. For less abundant elements the lines do not extend so high, but the enhanced lines still run far above the arc lines. Lines corresponding to transitions which the atoms rarely make of course disappear at lower levels than the ultimate lines.

655. Height of the Chromosphere; Radiation Pressure. The great height to which the chromosphere extends was also formerly inexplicable. In an atmosphere in equilibrium under gravity at the sun's surface, and at a temperature of 5000°, the density should increase tenfold for a vertical descent of 17 kilometers (assuming a mean molecular weight of 20, which is of the right order of magnitude). A height of hundreds (not to mention thousands) of kilometers is therefore possible only if some other force practically neutralizes that of gravity. Such a force is found in *radiation pressure*, which, as calculation shows, is entirely capable of supporting atoms against solar gravity. Only radiation of a very small part of the whole solar spectrum is intercepted by the atom; but, on the other hand, its weight is also very small. An ionized calcium atom would be supported against solar gravity if it took up a quantum of the energy corresponding to the (H) and (K) lines about 20,000 times per second, which is known, from other physical data, to be a reasonable rate of working.

Those atoms which have ultimate lines in the region in which the sun's continuous spectrum is strong will therefore find solar gravity nearly counterbalanced, and may rise to great heights, especially if they are of small atomic weight, while heavy atoms, or those with no ultimate lines in the right spectral region, cannot do so. The upper part of the sun's atmosphere is therefore very deep, and the density within it increases but slowly downward. This is the *chromosphere*.

Milne has worked out a theory of this equilibrium, and shown that if the calcium in the chromosphere were not protected by the heavy absorption of its ultimate lines lower in the sun's atmosphere, it would be driven away from the sun by the radiation pressure.

Electrons are very light and would rise to great heights if it were not that the positively charged ions attract them and keep them down. This attraction on the ions, as Pannekoek has shown, helps to keep the latter at high levels.

The amount of matter that can be held up by radiation pressure is extremely small, probably much less than the whole mass of the sun's atmosphere, so that in the lower atmosphere gravity gets the mastery, and the density increases rapidly downward. This region (one or two hundred kilometers in thickness) is the *reversing layer* and contains atoms of nearly all the elements. The transition between it and the chromosphere is probably gradual. It has been suggested by several authors that the *corona* may be a very rarefied cloud of such isolated electrons and ions, which, by scattering sunlight, produce the continuous spectrum.

Only the narrow dark centers of the stronger Fraunhofer lines are produced in the chromosphere, as is shown by the (H) and (K) lines. The central cores of these can be studied with the spectroheliograph and show high-level objects, like the prominences, in absorption. The reversing layer accounts for the main parts of the strong lines and for their wings, and for the multitude of weaker lines. The fainter lines, which correspond in general to processes that occur very rarely in the corresponding atoms, or to atoms of very uncommon kinds, must be mainly produced low down, where the density is relatively great.

656. Heights of Different Elements in the Chromosphere. The relative heights to which various elements rise in the chromosphere are now intelligible. Sodium, for example, shows the (D) lines only to about 1500 kilometers, which is explicable by the fact that these are absorbed only by the neutral atoms. Ionized sodium atoms absorb practically nothing (except in the remote ultra-violet, where the sun's radiation is doubtless very weak) and are not supported by radiation pressure. Hence there is no sodium in the upper chromosphere.

Calcium, when ionized, still has strong ultimate lines in the visible region, and so ascends to great heights. Atoms of magnesium are lighter, and if their ultimate lines (near λ 2800) could be seen through the earth's atmosphere, we should probably find them extending higher still. Iron atoms are heavier and do not rise so high. Helium rises high in the chromosphere, as is shown by strong lines in the flash spectrum. Its lines do not appear in the ordinary solar spectrum, but sometimes show dark above sun-spots. The reason for their absence is unknown.

657. The Prominences. The quiescent prominences (§ 585) are evidently held up by some force or other, and radiation pressure probably plays a large part. The appearance, changes, and motions of these objects are still imperfectly explained.

Radiation pressure may also have much to do with the extraordinary rapid rise of some eruptive prominences. As has been said, the chromosphere is ordinarily in a balanced state, radiation pressure just counteracting gravity; but if for any reason the radiation from a point of the sun's surface should increase (in the effective wave-lengths, such as the middle of the (K) line), atoms of $Ca+$, for example, would be repelled from the sun and would rise more and more rapidly. As soon as their velocity became considerable the Doppler effect would shift the absorption lines to the violet, out of the dark center of the Fraunhofer line, and on to the bright continuous spectrum (as has often been observed). Thus the radiation pressure would be greatly increased and might blow the prominence clear away from the sun. This is much too simple a statement, however. It does not explain why hydrogen and calcium keep together in these prominences, or why the velocity appears to increase by sudden starts, and not steadily, or why rapid horizontal motions often occur. There is still a great deal to be found out.

658. The Aurora, Solar Activity, and Magnetic Storms. It has long been known that the light of the aurora borealis is produced at great heights in the earth's atmosphere, and that the streamers, which are often conspicuous, are parallel to the earth's lines of magnetic force. The researches of Störmer have shown that the lower ends of the streamers are always at an altitude of about 100 kilometers, and have made it clear that the light is due to excitation of the atmospheric gases by electrified corpuscles, which enter the atmosphere from without and set it shining like the gas in a Geissler tube. Under the laws of electrodynamics these move in spiral paths along the lines of magnetic force, usually reaching the earth in high latitudes.

The spectrum of the aurora shows bands, due to nitrogen, and a very conspicuous green line at λ 5577.35. This line is always present in the spectrum of the night sky, indicating that a weak auroral excitation is always present. It was first obtained in the

laboratory in 1925 by McLennan and Shrum, who regard it as certainly due to oxygen.

Disturbances of the earth's magnetic field (magnetic storms) always accompany an aurora. Lindemann has explained the observed connection between solar activity and the occurrence of magnetic storms and auroras on the earth (§ 241) by supposing that a mass of hydrogen (driven away from the sun by radiation pressure) expands to so low a density that it becomes completely ionized and therefore emits no light at all. When such a cloud of protons and electrons passes near the earth, and the charged particles enter the atmosphere, the heavier protons penetrate to within about 100 kilometers of the surface and produce the aurora, while the lighter electrons are stopped much higher up. Opposite electric charges thus collect in different parts of the upper atmosphere, which is also ionized and rendered conducting; and the effects produce the magnetic storm.

659. The spectra of comets may appropriately be discussed at this point (compare § 510). The emission spectrum consists mainly of bands, due to molecules of the permanent gases. The presence in the comet of these gases, even at low temperatures, need excite no surprise. It has been suggested that their luminosity is due to bombardment by electrified particles, as in the case of the permanent terrestrial aurora. Recent studies of band spectra, however, show that many of the most prominent cometary bands, including those of cyanogen in the head, and of nitrogen and carbon monoxide in the tail, are absorbed by molecules in the normal energy states, and thus correspond to the *raies ultimes* of atoms. The great majority of the molecules of the appropriate kind must be in a position to absorb and emit these bands, and it now appears probable that in comets' tails their luminescence is due mainly to resonance (§ 653) — sunlight being absorbed and re-radiated in the same wave-length. The whole amount of gas which would be required to account for the line emission of a comet is small, and not too great to be consistent with the transparency of the comet (§ 506). The appearance of the resonance lines of sodium when the comet gets near the sun is similarly explicable. The fact that the bands of *neutral* CN appear in the head, and those of the *ionized* molecules $CO+$ and

N_2+ in the tail, is evidently connected with the very low density of the tail, which is such that an ionized molecule would have practically no chance of picking up an electron. How the gases become ionized is a puzzle. The ionization potentials are high (14.2 volts for CO and 16.5 for N_2), and it is not easy to understand how so much energy can be fed into the individual molecules.

660. The Opacity of the Solar Photosphere. Until recently it was supposed that the photosphere consisted of clouds floating in the sun's atmosphere, but this belief is now quite untenable. There is direct evidence that the density in the region of the photosphere is much less than that of atmospheric air (§ 590). The temperature being 6000° K, all known substances — even tungsten — would be vaporized, and the solar gases, therefore, probably do not contain suspended solid or liquid particles (such as cause terrestrial clouds and haze).

Why, then, do these gases become opaque at the depth of the photosphere? Direct observation of the sharpness of the sun's limb shows that almost complete transparency is replaced, when the line of sight runs not more than 75 kilometers deeper, by substantially complete opacity. Such rays running tangentially through the solar atmosphere encounter a column of gas 10,000 kilometers or more in length, so that complete opacity could be produced by even a thin solar haze, through which objects would be seen at distances as great as 1000 kilometers.

Abbot suggested that the scattering of light by the gas molecules (§ 602) would suffice to produce this, but on this hypothesis we should be able, at the center of the disk, to see down through a layer containing at least as much gas per square mile as the earth's atmosphere. At the base of this the pressure would be 27 atmospheres, which is more than a hundred times as great as the observations indicate.

There must be some other explanation for the high opacity of the photospheric gases, and it now appears very probable that this is to be found in the fact that they are highly ionized.

661. Evidence for the Opacity of Heated Gas. Direct observational evidence that an ionized gas is actually opaque has been obtained by Anderson. When the discharge of a very large condenser is passed through a fine wire, the wire explodes into a

mass of intensely heated gas (certainly far too hot to contain solid particles) emitting a continuous spectrum and opaque to the light of another spark behind it. The continuous spectrum is crossed by dark lines produced in the cooler outer portions of the highly luminous gas, which thus resembles a miniature star. The presence of enhanced lines shows that even the cooler parts are ionized.

Theory of the opacity of an ionized gas. The suggestion that the observed opacity, both in the laboratory and in the sun, might be due to the ionized condition was first made by Stewart, who pointed out that the free electrons would pick up energy from the electric field associated with the light, and transform it by collisions to random motions of electrons and molecules, that is, to heat in a gas. Calculations indicated that this process would make the solar atmosphere hazy enough to appear opaque at a depth where the pressure was only 0.01 atmosphere.

More recently Milne has calculated the opacity due to photo-electric ionization of the atoms (§ 636), taking into account the fact that for excited atoms this opacity should be great in the visible region. He concluded that the solar gases should be effectively opaque at a pressure of 0.0001 atmosphere.

662. Extreme Tenuity of the Solar Atmosphere. The theories are somewhat provisional, but are sufficiently trustworthy to make it probable that the solar gases begin to be effectively opaque at depths where the pressure is 0.001 atmosphere. The haziness is proportional to the square of the pressure, so that 20 kilometers higher the atmosphere should be nearly transparent. The photosphere, therefore, merges gradually into the reversing layer, although at the sun's distance the transition appears sharp.

A pressure of 0.001 atmosphere, under solar gravity, would be produced by a layer of ordinary air only one foot thick. If such a layer were expanded to be 50 or 100 miles deep, it would be of the average density of the reversing layer. The chromosphere must be far more tenuous.

It appears incredible at first that so small a quantity of gas produces the thousands of lines of the solar spectrum, but in the laboratory the sodium vapor in the outer flame of an arc (only a few millimeters thick and at a partial pressure less than one atmosphere) produces dark reversals in the centers of the

bright (D) lines, and these are found, on direct comparison, to be much wider and more intense than the Fraunhofer (D) lines, which are among the strongest of all.

It follows that we see only a thin superficial layer of the sun. In sun-spots, for example, the deep-seated vortex must escape direct observation, although, on account of the lowered ionization, we may see a few kilometers deeper than usual. Where the *surface* gases are cooler than the average, a dark marking appears; where they are hotter, a bright one.

663. Vertical Temperature Gradients; Radiative Equilibrium. The lower layers of the solar atmosphere must be hotter than the upper, for heat is continually flowing from the former to the latter. Schwarzschild has pointed out that nearly the whole of this flow of heat must be carried by radiation, even at depths where the gases are highly opaque. In empty space, radiation proceeds without interruption; but inside the photosphere it is relayed on, so to speak, from atom to atom; and a steady rise of temperature inward is necessary to keep it going.

In an atmosphere in equilibrium this internal temperature gradient is proportional to the outward flux of energy, and also to the opacity of the material. If the manner in which the opacity depends on the temperature and density were known, the temperature, density, and pressure at any depth could be precisely calculated; and a good general idea of the situation can be derived from existing data. This theory of "radiative equilibrium" shows that, starting in the interior, the temperature, density, and pressure fall rapidly till the photosphere is reached, which may now be further defined as the region from which a considerable part of the emitted radiation escapes directly into space, without being absorbed or scattered on the way. This is a layer of moderate depth, much hotter at the bottom than at the top. Its effective temperature (§ 605) is an average value. From its bottom but a small fraction of the emitted radiation escapes directly into space; from its top, nearly the whole; but neither boundary is sharp. Just above the photosphere the temperature is about 85 per cent of the effective temperature. It falls very slowly in the thin outer parts of the atmosphere, where the density and pressure decrease toward zero.

664. Spectrum of the Sun's Limb. The principle of radiative equilibrium has important consequences. For example, in observing the sun's limb the line of sight passes very obliquely through the atmosphere and is relatively very long, so that the effect of the haziness is greater and we cannot see down so far as at the center of the disk (Fig. 212 *A*). Thus the light from the limb comes, on the average, from higher layers, which are cooler and at lower pressures. Hence it is fainter and redder than the light from the center. The degree of darkening at the limb, and the law of its change from the center to the edge of the sun, are fairly well accounted for by this theory.

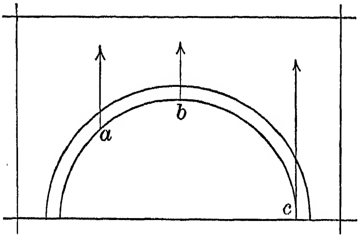

FIG. 212 *A*. Cause of the Darkening of the Sun's Limb

Near the limb *c* the line of sight has a much longer path through the sun's atmosphere than near the center *b* of the visible disk

In the spectrum of the limb (Fig. 212 *B*) ultimate arc lines are slightly strengthened and arc lines of high excitation potential are weakened, as would be expected from the lower temperature. The wings of wide lines are greatly reduced. Enhanced lines are much weakened and, when the light is taken from points very close to the limb, with good seeing they actually appear as bright lines, probably because they are so brilliant in the overlying chromosphere.

The differences in wave-length between center and limb (§ 591) are not yet satisfactorily explained.

665. Levels of Origin of the Fraunhofer Lines. For wave-lengths corresponding to the Fraunhofer lines the opacity of the solar atmosphere is enormously greater than for others, — probably owing chiefly to the intense scattering of the light out of the direct beam, as resonance radiation (§ 653). Light near the center of a Fraunhofer line, then, comes from regions high in the sun's atmosphere and of relatively low temperature. Hence the Fraunhofer lines are dark, but only by comparison with the background.

In the strongest lines, which are produced by the largest number of atoms, the level down to which we see will be highest. In the faint lines we can see down to lower levels. Observations

of the strong lines therefore reveal only phenomena in the upper part of the sun's atmosphere. This accounts for the difference in behavior of strong and weak lines in the Evershed effect (§ 589), and confirms the interpretation of the observed differences of velocity as belonging to different levels.

The wings of the strongest Fraunhofer lines may be accounted for by the fact that a gas scatters light of wave-lengths near those of its own lines more strongly than light of wave-lengths remote from them. Since this scattering is very small compared with that at the center of the line, the wings should appear only

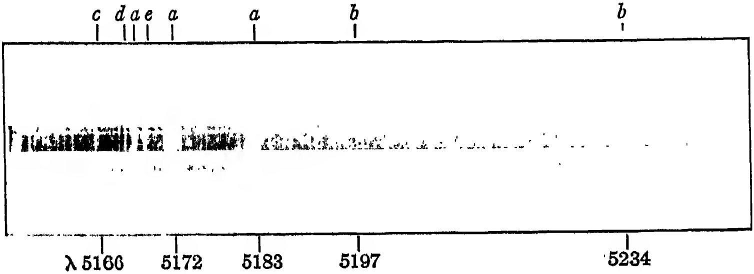

FIG. 212 *B*. Spectra of Center and Limb of the Sun

The narrow central strip gives the spectrum of the sun's center, in wave-lengths near λ 5200. The strips on each side give that of the limb, taken with a tangential slit just overlapping the sun's disk, so that the spectrum of the sky shows faintly at the top and bottom. The lines of the Mg triplet (*a* in the figure) lose their wings completely at the limb. The enhanced lines of Fe, at *b*, show bright at the extreme edge and are obliterated just inside. The Fe line marked *c* is a high-temperature line, Class IV, and *d* a low-temperature line, Class I; the latter is relatively stronger at the limb. At *e* an iron arc line of Class I and an enhanced Fe line occur very close together; the latter is bright at the limb. All the lines in the spectrum of the limb are slightly shifted toward the red, some more than others. (From photograph by Mt. Wilson Observatory)

for strong lines, and should be produced at much lower levels in the sun's atmosphere than the cores of the lines, the outer parts of the wings corresponding to the lowest level. Observation confirms these conclusions.

CONDITIONS INSIDE THE SUN

666. The Interior of the Sun. Modern atomic theory is nowhere more important than in the consideration of the sun's interior. Elementary considerations show that the pressure, density, and temperature are greatest at the sun's center. The central density

is probably considerably greater than the mean density. The central pressure, owing to the weight of the overlying material, must exceed 10^9 atmospheres, or 1,000,000 tons per square centimeter. The corresponding central temperature, if the ordinary gas laws are applicable, must be at least 10,000,000° K.

Until within the past ten years, however, it was very doubtful whether the laws derived from our experimental knowledge of the properties of matter could safely be extended so as to allow us to predict those properties under such very different conditions.

The situation has now changed. Though a temperature of millions of degrees cannot be produced artificially, a "target" in an X-ray tube can be bombarded with electrons which have fallen through a potential difference of hundreds of thousands of volts, and which have thus acquired very high speed and kinetic energy, — high enough, indeed, to deliver, to those atoms of the target which they hit, blows as violent as would be produced by collisions in a gas at a temperature of more than a billion degrees. (In falling through one volt of accelerating potential an electron acquires the average thermal energy of an atom in a gas at 7600° K.) Thus the behavior of atoms can be studied under disturbances of this type, which are more violent than they are likely to be subjected to anywhere in the sun. The knowledge thus obtained is invaluable in the analysis of solar problems.

667. Properties of Matter in the Sun's Interior. Under the temperatures and pressures that prevail in the interior of the sun (and of the stars in general) it now appears that

(1) The material is gaseous throughout, notwithstanding the high pressure.

(2) The atoms must be stripped of most (in some cases all) of their orbital electrons. Present knowledge of atomic structure permits calculation of the energy required to remove successive electrons. Applying the theory of ionization, it is found that, under the conditions which prevail at the sun's center, atoms of less atomic number than carbon or nitrogen would lose all their outer electrons, being thus reduced to bare nuclei. Atoms such as calcium would retain only the two innermost electrons; iron would retain two or three.

(3) The mean molecular weight of the resulting mixture of ions and free electrons is almost independent of the chemical composition of the material. Each free electron counts as a molecule, and the number of electrons released from an atom is usually a little less than half the atomic weight. The resulting mean molecular weight at the sun's center comes out between 2 and 2.5 for almost any composition (unless the proportions of hydrogen and helium are large). Toward the surface it rises slowly.

(4) The resulting ions are very much smaller than normal atoms. The size of the outermost electron orbits determines the diameter of an atom. For some reason, doubtless connected with the quantum laws, atoms refuse to approach one another so closely that these outer orbits interlock, and no available pressure will make them do so, whether in the solid, liquid, or gaseous state. But under conditions here discussed the largest remaining orbits are not more than 1/30 the diameter of those at the periphery of neutral atoms. It follows that the intensely hot ionized gas should be compressible to densities far exceeding those of ordinary solids, and that even at these very high densities the ionized material should still behave like a perfect gas, for the space occupied by the denuded atoms and the free electrons is still small in comparison with the total volume. This very important conclusion was first stated by Eddington in 1924.

(5) The atoms, though highly ionized, are in no sense destroyed or changed from one element into another. The atomic nuclei, being themselves structures, might be disintegrated by violent collisions. Indeed, Rutherford has actually accomplished this in a number of cases, by collisions with the very fast alpha particles (helium nuclei) emitted by radium. But the energy of such particles is several millions of volts and enormously greater than that of any collision which is in the least likely to occur in the sun's interior. The nuclei (even when unprotected by any orbital electrons) will therefore maintain their identity. If a sample of material at the sun's center should be removed and allowed to cool, the nuclei would automatically reclothe themselves with electrons, and the familiar atoms would appear once more.

668. Radiation in the Sun's Interior. The interior of the sun must be filled, not only with electrons and ions, but with radia-

tion streaming in all directions through the hot gas. At all points below the photosphere black-body conditions substantially prevail (since no radiation escapes directly into space from the interior), and the corresponding laws can be applied.

Wien's law (§ 608) indicates that an overwhelming proportion of the radiation is of very short wave-length. At a temperature of 10,000,000° K the strongest radiation is of wave-length 2.94 angstroms, that is, of the kind ordinarily called soft X-rays. Long-wave radiation, such as visible light, though actually thousands of times stronger than at the surface, contributes but a very small part of the total energy.

Stefan's law (§ 607) indicates that the total flux of radiation is enormously great. At 10,000,000° the flow in any given direction is 8×10^{12} times as great as the outward flow from the photosphere.

This tremendous stream of energy is continually passing through the gas from one portion to another in all directions, being absorbed by the atoms and reëmitted, so that, if we isolate in imagination a particular quantum of radiant energy, we find it flying back and forth from atom to atom in all directions, working its way very slowly from the hotter toward the cooler regions.

The radiation pressure inside the sun is very great. It is proportional to T^4, and at 10,000,000° K is 25,000,000 atmospheres, which is a sensible fraction of the total pressure. Indeed, in some of the stars, though not in the sun, the radiation pressure, according to calculation, does more to sustain the huge weight of the overlying material than does the gas pressure (§ 952). Like the latter, it is very nearly the same in all directions, the flying radiant energy acting like the flying atoms and electrons in the gas.

669. The Internal Constitution of the Sun. Very recently Eddington has developed a theory of the internal constitution of the sun and stars, based on these principles, which, though admittedly an approximation, represents the known facts so well that it is undoubtedly a good one. This matter is discussed in detail in Chapter XXV. According to this theory, radiation pressure inside the sun is everywhere, except close to the surface, the same fraction (about 5 per cent) of the total pressure. The central temper-

ature is about 40,000,000° K, and the central density 20 times the mean density, or 28 times that of water. The surface temperature depends on the rate at which heat escapes from the interior, and this again on the opacity of the deep-lying material which prevents the intense internal radiation from escaping faster.

670. The Age of the Sun. The sun has been shining, much as at present, for a very long time indeed. The best evidence for this is of a geological nature. The earth's surface temperature is maintained almost entirely by the sun's radiation. If the solar constant should be doubled, the earth's mean temperature would (according to the principles of § 612) be multiplied by approximately $\sqrt[4]{2}$, that is, raised from about 290° K to about 350°. Similarly, if the solar constant should be halved, the temperature would drop below 250° K, — well below freezing. Alterations in cloudiness, etc. might reduce these changes somewhat; but it seems probable, in either event, that substantially all terrestrial life would be wiped out.

The geological record exhibits (in the fossils) an unbroken course of organic evolution since Pre-Cambrian times. Thus, it is reasonable to suppose that throughout this great interval of time the sun's radiation has never risen to twice or dropped to half its present amount, even for a few years.

There is good evidence that the time elapsed since the Pre-Cambrian is to be measured at least in hundreds of millions of years (§ 155). During this vast interval, then, the sun has been continually pouring out substantially the same flood of energy as at present.

671. The Source of Solar Energy. The sun is doubtless losing heat into space *in all directions* at the rate indicated by the solar constant. The reason for believing this is that the earth certainly (§ 617) gets rid of heat by radiation in all directions into space at about the rate which corresponds to its effective temperature, and there is no known reason whatever why the sun should not do the same.

Thus, the sun is radiating 1.44 calories, or 6.03×10^7 ergs, per year for each gram of its mass (§ 604). If this rate has held for a billion years, the sun has lost 1440 million (1.44×10^9) calories per gram.

This quantity of dissipated energy is so great as to raise serious questions regarding its origin. There is no known way in which it could have been put in continuously from outside. The hypothesis that it is supplied by the *falling of meteors* into the sun is untenable, for the number of meteors striking the earth should in that case be far greater than is observed. Consequently this energy seems to have been present in the sun as some form of potential energy, and to have been gradually liberated. But in what form was it stored?

The energy of *chemical combination* is ridiculously inadequate, amounting at best to a few thousand calories per gram of reacting materials. A satisfactory explanation once appeared to have been reached in Helmholtz's suggestion that the sun was slowly contracting. The work done by the gravitational forces in pulling the attracting particles together would reappear as heat, generated by the compression of the gaseous mass. A diminution of the sun's diameter by only 280 feet per year would suffice, in this manner, to supply its radiation. During historic times this contraction would diminish the sun's diameter quite imperceptibly, but when extended over geological epochs the theory breaks down entirely. The whole store of *gravitational potential energy* liberated by the sun (corresponding to the work done by the attractive forces in bringing its parts together from an indefinitely great distance) amounts to only about 2.3×10^{15} ergs, or 5.5×10^{7} calories, per gram. The sun's temperature is so high that fully half of this must be stored as heat in its interior, and the remainder would have kept it shining at the present rate for a beggarly fifteen million years or so.

The requisite amount of energy is at least a hundred times as great as this, and the only places in the known universe small enough to hold so much are the tiny *nuclei of the atoms*. The word "small" is used advisedly; for, to result in so great an amount of potential energy, the forces must be exceedingly great, and they will not be great unless the electrons and protons are extremely close together.

Observations show that the nuclei of the heavy *radioactive atoms* contain huge stores of potential energy, liberated during radioactive transformations, in which portions of the nuclei

(electrons or alpha particles) are expelled with great velocity. The radioactive elements are thus transformed into more stable elements. The whole amount of energy emitted by a gram of uranium, before it settles down into an isotope of lead, is about 5×10^9 calories, which is of the right order of magnitude.

There is, however, no evidence that uranium or other radioactive elements are abundant in the sun or even present there, although it is possible that these elements — or, as Jeans has suggested, other, heavier ones, not known on earth, — may exist below the photosphere. It may be that, far in the sun's interior, atoms not ordinarily radioactive are transformed, in somewhat similar fashion, — not all at once, or by wholesale, but very rarely, yet, in the aggregate, releasing a great amount of energy.

672. Possible Conversion of Mass into Energy. According to the theory of relativity (§ 360) any change in the energy content of a body alters its mass. A loss of m grams of mass corresponds to the liberation of mc^2 ergs, that is, of 9.00×10^{20} ergs, or 2.15×10^{13} calories, per gram. Thus, it is as legitimate to speak of a pound of heat as of a pound of iron; but a pound of heat would be sufficient to raise twenty million tons of rock (of specific heat 0.2) by more than 2500° C., and convert it into incandescent lava.

Reckoning the energy radiated by the sun (3.8×10^{33} ergs per second) as mass in the same fashion, the amazing conclusion follows that *the sun is losing by radiation 4,700,000 tons of heat every second*. But if the sun's mass is diminishing at this rate, the suggestion seems reasonable that processes occur within it which lead to a loss of mass in some of the atoms of which it is composed, with the liberation of the corresponding quantity of energy.

Two such processes have been suggested.

(1) *The transformation of hydrogen into other elements.* The atomic weight of a given element (or of a particular isotope, if there are more than one) is usually, probably always, less than the weight of the number of hydrogen atoms which, according to accepted theory, contain the same number of electrons and protons. This indicates that, in packing together electrons and protons to form the nucleus of a heavy atom, energy corresponding to the loss of mass would be liberated.

This loss of mass is in most cases about one part in 130 — which would correspond to the liberation of 1.6×10^{11} calories per gram.

(2) *The annihilation of matter.* It is imaginable that an electron and a proton meeting under suitable (and very unusual) circumstances may neutralize one another's charges and annihilate one another. The energy corresponding to the masses of the two would presumably appear first as some sort of radiation, but would soon be transformed into heat in the medium. We know nothing of the conditions under which this might happen (which is not surprising, since we are still ignorant of the reason why electrons and protons all have the unit charge). Certain data described in Chapter XXVI indicate that the stars, and the sun among them, lose a very considerable fraction of their masses during their existence as luminous bodies. This indicates that the annihilation of matter is the main source of solar (and stellar) energy. Radioactive processes, and the building up of helium or of heavier elements, appear to be, in comparison, side reactions demanding relatively little energy.

The time scale opened by this conception is bewilderingly long. At the present rate of loss of mass by radiation the sun, during the past billion years, has lost only 1/15,000 of its whole mass, and it is improbable that its size or other characteristics have changed perceptibly in that interval. Its whole life as a luminary has probably endured for thousands of billions of years; and the sun is likely to shine, with gradually decreasing splendor, for even longer in the future.

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Progress in this subject is so rapid that it cannot be closely followed in textbooks, and the interested student is advised to consult the current physical periodicals.

CHAPTER XVIII

THE STARS

THEIR NATURE, NUMBER, AND DESIGNATION · STAR CATALOGUES AND CHARTS · COLLECTIONS OF PHOTOGRAPHS · STELLAR SPECTRA: OBSERVATION AND CLASSIFICATION · BRIGHTNESS OF THE STARS: MAGNITUDE; VISUAL, PHOTOGRAPHIC, AND PHOTO-ELECTRIC PHOTOMETRY; COLOR-INDEX AND COLOR EQUATION; RADIOMETRIC MEASUREMENT AND HEAT-INDEX · NUMBER OF STARS OF EACH MAGNITUDE AND THEIR TOTAL LIGHT · DISTRIBUTION OF THE STARS · PROPER MOTION · RADIAL VELOCITY · STELLAR PARALLAX · DISTANCE · TANGENTIAL VELOCITY · ABSOLUTE MAGNITUDE AND LUMINOSITY · TABLES OF STARS OF GREATEST APPARENT BRIGHTNESS, OF LARGEST PROPER MOTION, AND OF LARGEST PARALLAX · STATISTICAL DISCUSSION

673. Our solar system is an island in space, surrounded by an immense void occupied only by meteors and comets. If there were any body a hundredth part as massive as the sun within a distance of a thousand astronomical units, its presence would be indicated by disturbances of Uranus and Neptune, even if it were itself invisible.

The nearest star, so far as known at present, is at a distance of about 272,000 astronomical units, — so remote that, seen from it, our sun would look like one of the brighter stars, and no telescope ever yet constructed would be able to show a single one of all the planets of the solar system. More than half the stars visible to the naked eye are a hundred times as far away as this, and many of those visible with great telescopes are ten thousand times more remote.

Newton was the first astronomer to form any true idea of the distances of the stars. From the theory of gravitation he showed that the stars must be many hundreds of times as remote as Saturn, since otherwise they would either fall into the sun or describe orbits about it. At this distance they could not be visible, like the planets, by reflected light. Therefore they must be self-luminous bodies like the sun, and are probably comparable with it in brightness. By comparing Sirius with Jupiter he showed

that the sun would have to be put at about 100,000 times its present distance to appear like Sirius. The actual distance of Sirius is still greater, but Newton's reasoning, which was as ingenious as it was sound, led to a quite correct estimate of its order of magnitude.

The stars are *suns* (that is, bodies of the same nature as our own sun), each with an incandescent photosphere surrounded by a gaseous envelope. This is shown conclusively by their spectra, which are crossed by dark absorption lines similar to those in the solar spectrum.

Small as the stars appear to us, they are many of them much larger and hotter than the sun; others, however, are smaller and cooler, and some hardly shine at all.

The stars seem to be fixed in the heavens, but this is only because of their enormous distances. They are actually moving through space in all directions at speeds which often equal and sometimes greatly exceed that of the earth about the sun.

674. Number of the Stars. Those that are visible to the eye, though numerous, are by no means countless. If we examine a limited region, as, for instance, the bowl of the Dipper, we shall find that the number we can see within it is not very large, — hardly a dozen, even on a very dark night.

In the whole celestial sphere the number of stars bright enough to be distinctly seen by an average eye is between six and seven thousand, and that only in a perfectly clear and moonless sky; a little haze or moonlight will cut down the number by fully one half. At any one time not more than two thousand or twenty-five hundred are fairly visible; one half are below the horizon, and near it the fainter stars (which are vastly the most numerous) disappear. The total number bright enough to be observable by the ancient astronomers with their instruments is not quite eleven hundred.

With even the smallest telescope the number is enormously increased. A field-glass brings out 50,000 or more, and with a $2\frac{1}{2}$-inch telescope Argelander observed 324,000 in making his *Durchmusterung* of stars from the north pole to 2° south of the equator. The Yerkes telescope, 40 inches in diameter, probably reaches over one hundred million, and, finally, there are probably

well over a billion stars which, sending their light to the 100-inch Mt. Wilson reflector, are able to record themselves on a photographic plate of long exposure.

675. Constellations. A glance at the sky shows little order in the arrangement of the stars. But when one attempts to point them out to others or to identify them for future reference, various characteristic groups and configurations are noticed. With more or less imagination the outlines of familiar objects may be seen in these groups. The shepherds of old whiled away the wakeful hours in tracing the forms of animals and of mythological heroes. A life in the open air and a vivid imagination gave rise to the "constellations." Some of the constellations are of extremely ancient origin, especially those of the zodiac and those near the north pole. Many of these bear the names of animals. They must have been named in Mesopotamia (for the animals are those of the Bible), not in India (there is no tiger or elephant) or in Egypt (there is no crocodile or hippopotamus). The majority have names drawn from the Greek and Roman mythology, many of them being connected with the Argonautic expedition.

Of the eighty-eight constellations now generally recognized, forty-eight have come down from Ptolemy, while the others have been formed since 1600 by later astronomers, in order to embrace stars not included in the old constellations, and especially to provide for the stars near the southern pole. The best known of these is the Southern Cross. Many other constellations have been proposed at one time or another, but have since been rejected as useless or impertinent, though a few of them have obtained partial acceptance and still hold a place upon some star maps.

The modern meaning of a constellation is a *region* of the sky, and the entire surface of the celestial sphere is divided up among these regions. Their boundaries are arbitrary and irregular, and some little confusion (now past) has resulted from the assigning of a star to two constellations. The exact definition of the boundaries is now in the hands of an international committee.

A thorough knowledge of these artificial star groups and of the names and places of the stars that compose them is not at all essential, even to an accomplished astronomer; but it is a matter of real interest to an intelligent person to be acquainted with the principal constellations and to be able to recognize at a glance the brighter stars, from fifty to one hundred in number. This amount of knowledge is easily obtained in a few evenings by studying

the heavens in connection with a good star map, taking care, of course, to select evenings in different seasons of the year. A list of constellations will be found in the Appendix (Table XXXV).

676. Methods of Designating Individual Stars. Hipparchus, Ptolemy, — in fact, all the older astronomers, including Tycho Brahe, — indicate a particular star by its place in the constellation, speaking, for instance, of "the star in the head of Hercules," or in the "right knee of Boötes" (Arcturus). This method has been abandoned along with the attempt to trace the imaginary figures.

(1) *By names.* About sixty of the brighter stars have names in more or less common use. A good many of these names are of Greek or Latin origin (for example, Capella, Sirius, Regulus, etc.); others have Arabic names (Aldebaran, Vega, Rigel, etc.). For the fainter stars the names are almost entirely Arabic. A list of the brightest stars is given on page 637, Table XIV.

(2) *By constellation and letter.* In 1603 Bayer, in publishing his star map, adopted an excellent plan, ever since followed, of designating the stars in a constellation by the letters of the Greek alphabet. The letters were usually (not always) applied in the order of brightness, α being the brightest star of the constellation and β the next brightest; but sometimes (as in the case of the Dipper) they are assigned to the stars in their order of position rather than in that of brightness. Thus, Sirius, the brightest star in Canis Major, is α Canis Majoris (the genitive case being always used).

When the naked eye stars of a constellation are so numerous as to exhaust the letters of the Greek alphabet, the Roman letters are sometimes used; or the numbers are employed which Flamsteed assigned a century later, — for example, 61 Cygni.

At present almost every naked-eye star can be referred to and identified by its letter or Flamsteed number in the constellation to which it belongs. A set of abbreviations for the constellation names, which saves much space, was adopted by the International Astronomical Union in 1922.

(3) *By catalogue number.* The preceding methods all fail in the case of telescopic stars. To such we refer as number so-and-so of someone's catalogue; thus, "Ll., 21185" is read "Lalande,

21185," and means the star so numbered in Lalande's catalogue. At present more than a million different stars are contained in our numerous catalogues, so that (except in the Milky Way) every star easily visible in a 3-inch telescope can be found and identified in one or more of them. The many millions of still fainter stars are identified when necessary either by giving the star's right ascension and declination or, better, by marking it on a photograph of the region.

Double stars and variables have their special designations (§§ 762, 831).

Synonyms. Of course all the bright stars that have names have letters also, and are sure to be found in nearly every catalogue that covers their part of the heavens. A star that is notable for any reason, therefore, usually has many "aliases," and sometimes care is necessary to avoid confusion.

677. Star-Catalogues. These are lists of stars, arranged in some regular order, giving their positions (that is, their right ascensions and declinations, or longitudes and latitudes) and usually also indicating their so-called magnitudes, or brightness.

The oldest star-catalogue (epoch 137 A.D.) that we possess is contained in Ptolemy's *Almagest*. It gives the longitudes and latitudes of 1025 stars. The still older catalogue (epoch 129 B.C.), compiled by Hipparchus, is unfortunately lost. The next of the old catalogues of any value was that of Ulugh Beigh, made at Samarkand about A.D. 1450. It was followed in 1580 by the catalogue of Tycho Brahe, containing 1005 stars; his was the last catalogue constructed before the invention of the telescope.

The modern catalogues are numerous, — already counted by the hundred. They may be divided into several classes, based largely on their accuracy and the purpose which they serve.

(1) *Durchmusterungen*. These give the positions and magnitudes of all stars in the region surveyed, down to a certain limiting brightness, with sufficient accuracy for their identification. Such a catalogue and the set of charts on which the positions are plotted enable an observer, for example, to identify readily a comparison star for the observation of a comet or of a star that has been announced to be variable. To this class belongs Argelander's *Durchmusterung* of the northern heavens, completed at

Bonn in 1862, which is known as the B.D. and which contains, between the north pole and 2° south declination, over 324,000 stars. A continuation of this work to 23° south declination, by Schönfeld, contains over 133,000 stars. Thome, at Córdoba, continued the survey as far as declination − 61°. Identification of approximately 580,000 stars is provided by the C.DM., as this catalogue is called. The extension to the south pole is being carried on at Córdoba. The *Cape Photographic Durchmusterung* (C.P.D.) also serves for the identification of nearly 455,000 southern stars between declination − 18° and − 90°. The *Durchmusterung* catalogues contain all stars to about the tenth magnitude. The stars are arranged in the catalogue in zones of declination 1° wide and numbered in order of right ascension in each zone. Thus, B.D. + 30° 3639 is number 3639 in the zone between 30° and 31° North.

(2) *Catalogues of precision.* These are of two classes.

(*a*) Fundamental catalogues, containing the positions of a few hundred or a few thousand stars, well distributed, determined with the highest possible accuracy and independently of previous work, that is, fundamentally (§ 87).

(*b*) Differential catalogues. Fundamental star positions serve as standards to which the positions of other stars may be referred by measuring *differences* of right ascension and declination. Such measurement is usually made in a zone of declination, sufficiently narrow to make it possible to observe all the stars desired as they come through the field of view of the meridian circle.

Accurate observation of all the B.D. stars (to − 23° declination) was started in 1863 under the auspices of the Astronomische Gesellschaft. This great undertaking required the coöperation of many observatories in various latitudes. All catalogues have been published with the exception of the zone − 18° to − 23°. The Córdoba and La Plata catalogues form a continuation into the southern sky. Still more accurate star positions may be obtained by *photography*, by measuring the star-images on the plate and using the standard stars, which appear on it, to determine the constants of reduction.

The *Astrographic Catalogue*, planned in 1887 by coöperation among eighteen observatories, and now about half finished, will

contain the positions of three or four million stars derived from 44,000 plates. Many of the published volumes give only the star positions as measured on the plates, with tables for reducing these to right ascension and declination when required.

The Yale Observatory is engaged in the reobservation of the A.G. zones by photography, and one catalogue is already published. The total labor and cost of observation will be very much less than by the older method. The meridian circle, however, must still be used in determining the positions of the fundamental stars.

(3) *General catalogues*. These are formed by collating the results of all the most accurate catalogues, with special attention to systematic errors and proper motions. The latest authority is Boss's *Preliminary General Catalogue*, containing 6188 stars and including nearly all that are visible to the naked eye. A more extensive catalogue is in preparation.

(4) *Special catalogues*. Catalogues are also compiled to give certain characteristics, such as photometric brightness and spectral type, or for special classes of stars, such as variable or double stars.

678. Use of a Star-Catalogue. A modern star-catalogue gives the *mean* right ascensions and declinations of the stars, that is, positions corrected for aberration and nutation and referred to the mean equator and equinox of a specified date or *epoch*. Thus, the epoch of the B.D. is 1855.0; of the A.G. catalogues, 1875.0; and of most modern catalogues, 1900.0.

To obtain the right ascension and declination at any other time the catalogue place must be "brought up" as follows:

First, the position must be referred to the mean equator and equinox of the beginning of the current year. All catalogues give the precessions, or annual changes of right ascension and declination, and the secular variations, or amounts by which the precessions change in a hundred years.

To obtain the total corrections the precessions at the middle of the interval are to be found and multiplied by the number of years elapsed. Allowance must also be made for the proper motion of the star (§ 708), if this is known.

Secondly, the mean place must then be transformed into the *apparent* place by allowing for precession since the beginning of the year, for nutation, and for aberration. Formulæ and data

for this purpose are given in the nautical almanacs. (Compare *American Ephemeris*, 1926, p. 200.)

The resulting place should then agree with the observed position of the star corrected for refraction and parallax. When the difference of right ascension and declination between an object and one or more comparison stars has been determined by micrometer observations or by photography, the catalogue places of the stars may be brought up in full as described above. Applying the measured differences, the apparent right ascension and declination of the object are determined. This is necessary if the position of the moon, for example, is to be compared with the predictions of the *Nautical Almanac*.

If the star places are brought up only to the beginning of the year, the "mean place for the beginning of the year" of the object is obtained. This saves labor, and by international agreement (1925) observations and ephemerides of comets and asteroids will in future be given on this system.

679. Star Charts. Before the advent of photography, star charts had to be made by laboriously plotting the observed right ascensions and declinations.

For the naked-eye stars this is an easy matter, and atlases showing these stars are inexpensive and should be a part of the equipment of every serious student.

The more comprehensive charts, on which all the *Durchmusterung* stars are plotted, form a necessary part of the equipment of every observatory. Those so far published cover the region from the north pole to $-43°$ declination.

In comparing star charts with the sky, either with the naked eye or the telescope, care must be taken to hold them at the proper angle so that a north-south line points toward the celestial pole. In telescopic work it should be remembered that the field is inverted, and that red stars appear much fainter on photographs than they do to the eye. It is convenient to draw on tracing paper a circle corresponding to the size of the field of view, on the scale of the chart. The stars are then identified by the configurations which they form with their neighbors. Their coördinates may then be read from the chart and the stars identified in a catalogue.

More comprehensive charts can now be made much better and more rapidly by photography. The "Harvard Sky" covers the entire heavens on 55 plates and shows stars to about the twelfth

magnitude. The Franklin-Adams charts, 206 in number, likewise cover the entire heavens and go down to about the sixteenth magnitude. The Wolf-Palisa charts were reproduced chiefly to aid asteroid hunters and are centered about the ecliptic. They show still fainter stars. A very extensive system of charts was also planned in connection with the *Astrographic Catalogue*, and a considerable number of them have already been published.

680. Collections of Photographs. A single plate may record the position and brightness of many thousands of stars at the time when it was exposed, and gives a permanent record upon which any object of interest may be looked up later. In this way important observations of comets, asteroids, and variable stars have been made before their discovery, and afterward utilized. The greatest collection of such photographs is at the Harvard Observatory. At their two stations in Cambridge, Massachusetts, and Arequipa, Peru, the whole sky has been systematically and repeatedly photographed, with instruments of large field, as often as possible during the last three decades, and the Harvard photographic library contains a wealth of information, — so great that only a part of it has yet been utilized. There is another large collection at Heidelberg.

STELLAR SPECTRA

Our detailed discussion of the stars may well begin with their spectra, which have been found to be of fundamental importance.

681. Principal Characteristics of Stellar Spectra. *The spectra of the stars* were observed as early as 1824 by Fraunhofer, and were studied extensively by Huggins and Secchi in 1864, as soon as the principles of spectroscopy were known. Huggins studied a few bright stars carefully and identified lines of sodium, magnesium, calcium, iron, hydrogen, etc.; Secchi examined a great number of stars (nearly four thousand) in less detail, with a view to classification. These early investigations brought to light three main facts.

(1) *The spectra of the stars*, like the spectrum of the sun, are essentially *dark-line spectra*, showing that the stars too possess incandescent photospheres overlain by gaseous atmospheres. A small proportion of the stars (less than 1 per cent) show *bright*

lines also in their spectra, but these very rarely contribute any large fraction of the whole light.

(2) *The lines in the spectra of the stars can be identified as those of known elements.* At the present time only a very few lines of any importance remain unidentified, and these are found mainly in exceptional stars.

This is one of the most impressive phenomena in the whole range of physical science, showing, as it does, that the same atoms exist, and follow the same laws, throughout the vast extent of the visible universe. It is also noteworthy that with the means available in the laboratory it is possible to duplicate almost all the conditions of spectroscopic excitation which occur in the stars.

(3) Though the stars are so numerous, *their spectra may be grouped into a remarkably small number of classes.* Secchi found that practically all the spectra which he observed belonged to one or another of four types: (I) with heavy dark lines of hydrogen; (II) with numerous, though less intense, lines of the metals; (III) with bands sharp toward the red (now known to be due to titanium oxide); (IV) with bands sharp toward the violet (due to carbon or some carbon compound).

Later investigation, especially at Harvard, has shown that *the spectra of nearly all the stars can be arranged in a continuous sequence,* different stars exhibiting every stage in the transition, by almost imperceptible degrees, from one type to another throughout the whole range.

682. Methods of Observation. The earlier observations were made visually; but starlight, when spread out into a spectrum, is very faint, and such methods are now practically abandoned in favor of photography, which was first introduced about 1880 by Huggins in England and by Henry Draper in America.

Two principal methods are now employed.

(1) The *slit spectrograph* is a prism spectrograph designed so as to economize light, and mounted so that its slit is in the focal plane of an equatorial telescope. The image of a star is thrown on the slit and kept there, by continuous "guiding," during an exposure which may extend for hours, while a photographic plate in the camera records the spectrum.

(2) The *objective prism*, or slitless spectroscope, consists simply of a large prism (usually of rather small angle) placed in front of the objective of a photographic telescope. No slit is needed, since the star is a point-source, and no collimator, since its rays are parallel. The spectra appear as narrow streaks on a plate placed in the focal plane of the telescope.

683. Relative Advantages of the Slit Spectrograph and the Objective Prism. The spectrograph has the enormous advantage that a *comparison spectrum* (from an arc or spark close to the instrument) may be photographed on each side of the stellar spectrum, thus making possible precise measurements of wave-length. It also gives excellent definition of the image and, when suitably designed, permits very long exposures upon faint objects, even extending over many nights. On the other hand, it is very wasteful of light; for the telescopic image of the star, owing to "bad seeing," is continually dancing on and off the slit, and on poor nights is so big that the slit cuts out only a part of it. Moreover, *but one star at a time* can be observed.

The objective prism utilizes a much larger fraction of the light, and thus reaches fainter stars. In addition, the spectra of dozens, and sometimes of hundreds, of stars in the same field can be photographed on one plate, which is an overwhelming advantage when the object is to record and classify the spectra of stars by wholesale. The most serious disadvantage of the objective prism is the impracticability of introducing a comparison spectrum, making it impossible to detect those small differences in wave-length which arise from radial velocity, for example; and in this field the slit spectrograph is supreme. Attempts have been made to meet this difficulty by introducing in front of the plate a screen of some substance (such as a solution of neodymium chloride) which has well-defined absorption lines; but these lines are not as sharp as the stellar lines, and the method, though promising in certain cases, falls far behind the slit spectrograph in precision. Bad seeing, moreover, which slows up exposures with a slit spectrograph, blurs and spoils photographs with an objective prism.

Objective prisms, which must be of fully as good optical quality as the best lenses, are very costly; but against this must be set the fact that a slit spectrograph loses so much light that it

can be used only with a large and costly telescope, most of its present work being done with 25 inches aperture or more, while important work with objective prisms is done with no more than 10 inches aperture.

The stellar spectrum on a slit-spectrogram is very narrow, but not too narrow to show the lines clearly as interruptions in it, under a suitable magnifying power. Objective-prism spectra are usually widened by setting the prism so that the linear spectra run north and south on the plate and letting them drift a little to the east or west. This makes the spectra more "legible," but of course it lengthens the time of exposure.

684. Spectroscopic Observations. Almost all great telescopes are provided with slit spectrographs, and much work has been done, especially at the Lick, Mt. Wilson, Dominion, and Yerkes observatories. The objective prism has been used with great success at Harvard by Pickering and his coworkers, notably Miss Cannon. Many thousands of photographs have been taken with instruments of various dispersions. The spectra of all the brighter stars (to below the fifth magnitude) have been studied and described in detail; while on plates of smaller dispersion, which suffice for purposes of classification, the spectra of nearly a quarter of a million stars have already been observed. This great work led to the discovery of the continuous sequence of spectra, mentioned above, and its principal results are expressed in the system of classification developed at Harvard and called the Draper Classification, in memory of the first American observer of stellar spectra. The *Henry Draper Catalogue*, in nine volumes, 1918–1924, gives the approximate positions, magnitudes, and spectra of 225,300 stars, and extensions are still being published.

685. The Draper Classification. In this system the principal classes of the stellar spectra are described by the letters O, B, A, F, G, K, M, R, N, and S. The apparent disorder of this sequence arises from the facts that some of the letters originally employed were dropped out (being found to represent nothing but "bad photographs," as Professor Pickering put it) and that the rational order turned out to be different in some instances from that in which the letters were first assigned.

The classes from B to M inclusive form a continuous sequence, and types intermediate between the main ones are denoted by a decimal classification. Thus B5 signifies a spectrum about halfway between the classes B and A; F8, one much nearer to G than to F; and K0, one typical of Class K. Class O undoubtedly precedes Class B, but its subdivisions, pending further work, are denoted by the letters Oa, Ob, etc. (The spectra of certain stars of this class are alternatively designated O5, O6, . . . O9). Classes R and N appear to form an offshoot branching from the main sequence near Class K, and Class S forms another such offshoot.

Secchi's first type includes classes B to F2; his second type, F5 to K2; his third, K5 and M; and his fourth, classes N and R. The O-stars are sometimes called the fifth type.

Spectra toward the head of the list are commonly described as "early," and those toward the end as "late." It should be borne in mind that these terms denote merely *position in the sequence of spectra* and have nothing to do with the relative age of the stars.

686. The Spectral Sequence. The various spectral classes are distinguished by the prominence of different characteristic lines. A given line is seldom confined to a single class, but appears with less intensity in the neighboring classes; only the lines of hydrogen run through the whole sequence from end to end (Figs. 213–216).

The characteristic lines in Class O are those of ionized helium and of doubly and trebly ionized atoms of oxygen, nitrogen, etc. From classes B0 to B5, neutral helium and singly ionized oxygen and nitrogen are characteristic. These lines weaken in B8 and disappear in A0, while lines of ionized metals (Ca, Mg, Fe, etc.) appear. The hydrogen lines are at their strongest in this class. As Class F is approached, the hydrogen lines weaken and arc lines of the metals appear, — a few as early as A2. In Class G (to which the sun belongs) the arc lines of such metals as iron are stronger than their enhanced lines, and the hydrogen lines are no longer prominent. The lines of ionized calcium are very strong.

In Class K the enhanced lines, in general, are weaker and the arc lines stronger, especially the low-temperature lines. In Class K5, bands due to titanium oxide appear, and grow steadily stronger in M0, M5, and M8. The low-temperature lines of the metals are very strong, and the high-temperature lines are weak.

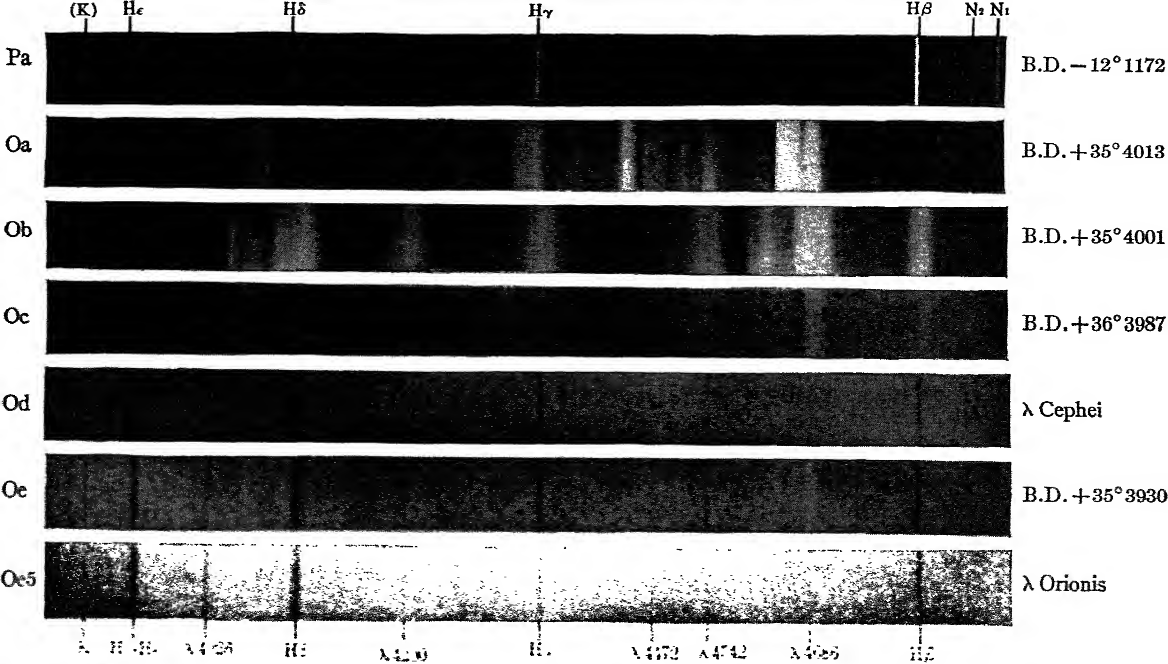


FIG. 213. The bright-line spectrum of a nebula (Pa), the bright-band spectra of the Wolf-Rayet stars (Oa, Ob, Oc), and the dark-line spectra of classes Od, Oe, and Oe5 (which immediately precedes B0). The name or catalogue designation of the star is printed at the right of its spectrum. Of the spectral lines marked at the bottom, $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, are due to hydrogen; λλ 4686, 4542, 4200, to ionized helium; λλ 4472 and 4026 to neutral helium; and (K) to ionized calcium. The pair just to the left of $H\delta$ in Class Oe5 are due to doubly ionized nitrogen

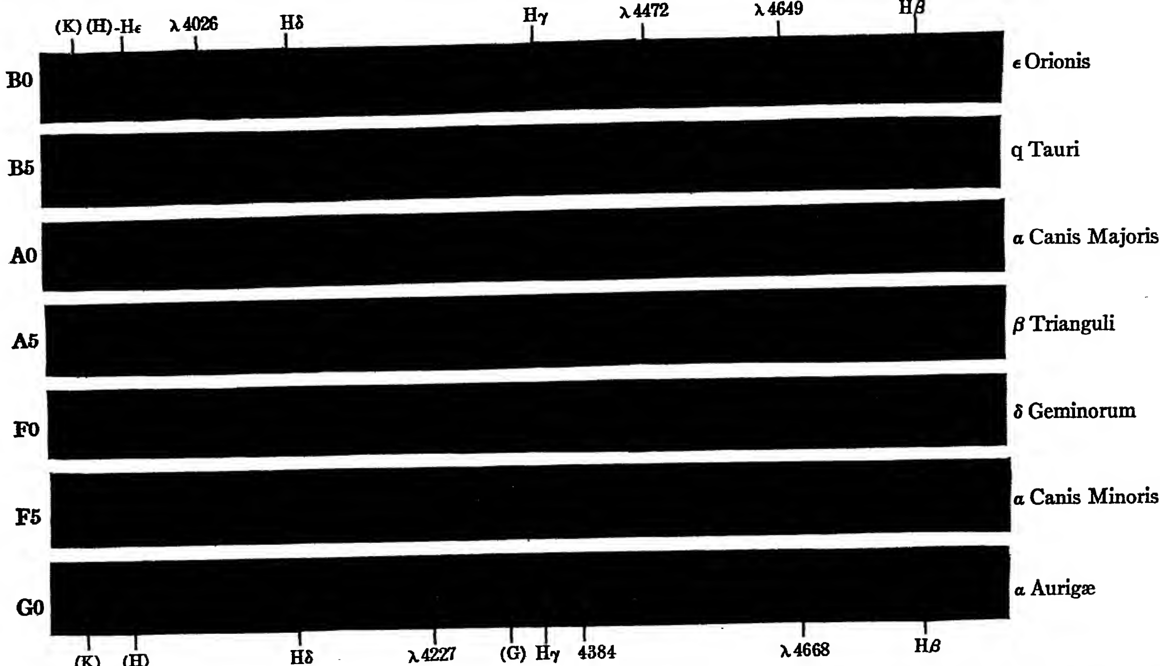


FIG. 214. Spectra of classes B0 to G0. In the first, λ 4649 is due to ionized oxygen. The helium line λ 4472 fades out below B5, and a line of ionized magnesium at 4481 comes out, a little to the right of it. λλ 4384 and 4668 are due to neutral iron, λ 4227 to neutral calcium. The last is a low-temperature line and grows steadily stronger all the way to Class M. (H) and (K) are lines of ionized calcium, and are very strong in Class G. (G) is a band due to some hydrocarbon compound, probably the fragmentary molecule CH

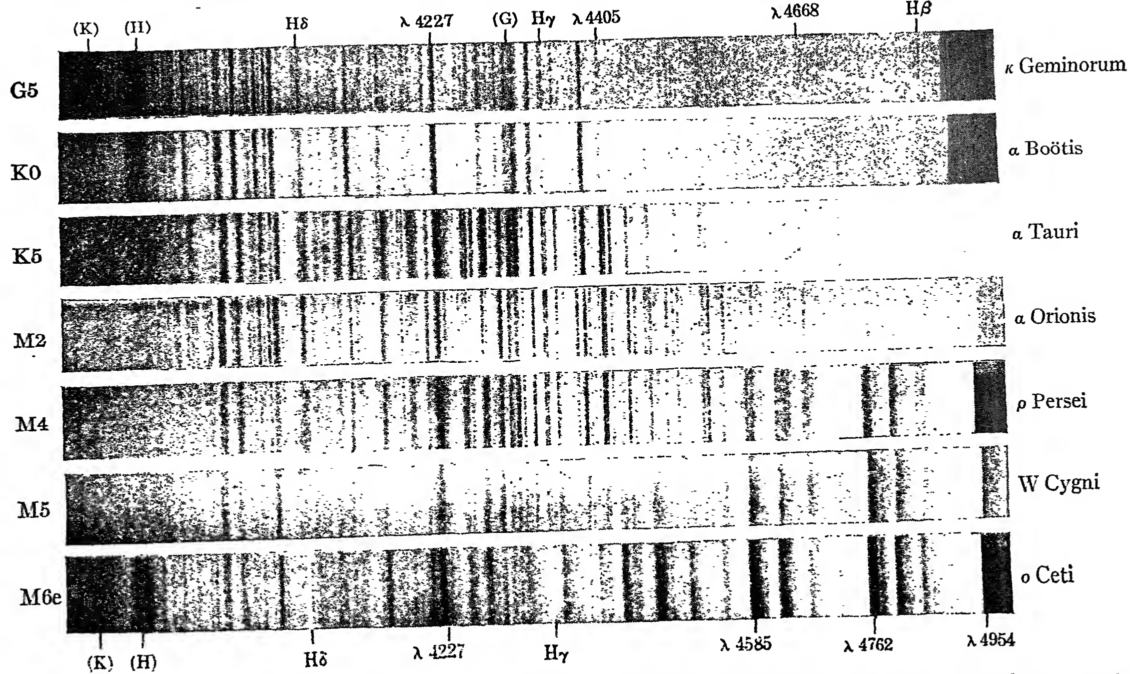


FIG. 215. This figure, including spectra of classes G5 to M, shows spectra full of arc lines of the metals. λ 4405 is a low-temperature line of iron; λλ 4585, 4762, 4954, mark the heads of bands due to titanium oxide. The "e" in M8e denotes the presence of bright emission lines (of hydrogen). Classes M0, M3, M8, were formerly called Ma, Mb, Mc

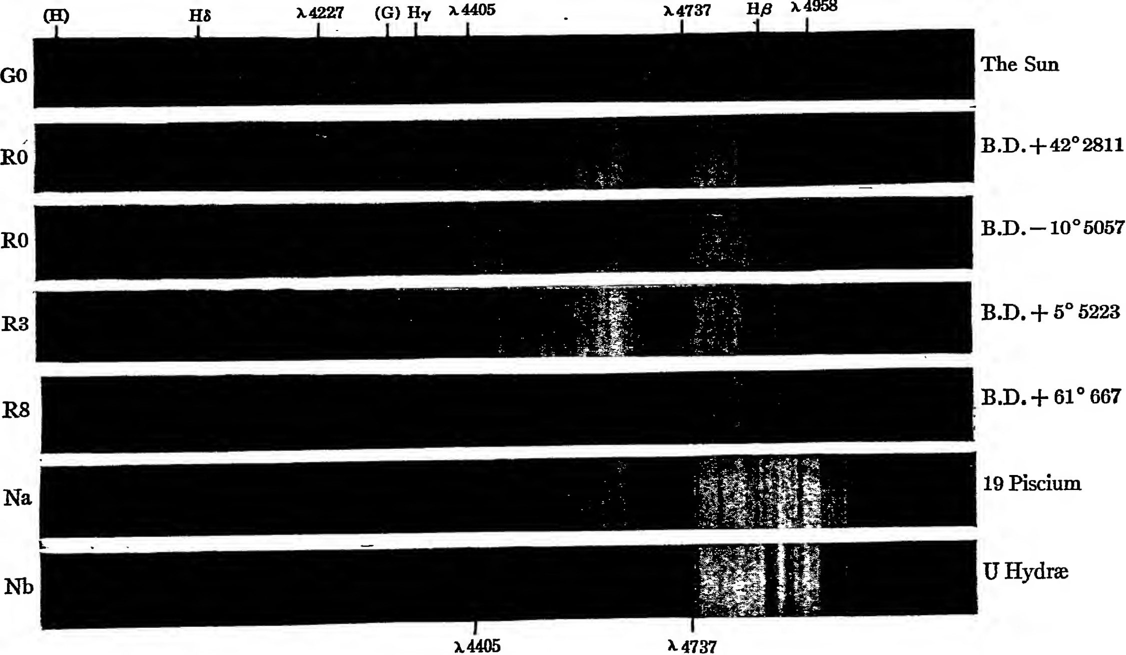


FIG. 216. The "branch sequence," which includes classes R and N. The line at λ 4958 is due to iron. The bands at λλ 4405 and 4737 are due to a carbon compound, while that at λ 4214, conspicuous in Class R, is due to cyanogen. The extreme redness of the stars of Class N is shown by the absence of violet light. (*The sequence of stellar spectra*, illustrated in the preceding Figs. 213, 214, 215, and 216, is reproduced from the admirable photographs of Curtiss and Rufus at the University of Michigan)

Classes R and N are characterized by bands of carbon and cyanogen (which increase in strength from R to R5, N0, and N3), and Class S by bands due to zirconium oxide. The "underlying" line spectrum shows strong low-temperature arc lines.

Emission lines, brighter than the continuous background, appear in many of the stars of Class O, in some of Class B, and again in certain stars of classes M, N, and S. In the intermediate classes bright lines are very rare.

In some stars of Class O (often called the Wolf-Rayet stars, after their discoverers) bright emission lines, widened into bands, form the most conspicuous feature of the spectrum. The spectra of the gaseous nebulæ, which consist entirely of isolated bright lines, are assigned to Class P, and those of temporary stars (§ 854) to Class Q.

Along with these differences in the spectral lines go conspicuous differences in *color*. Stars of classes O and B (as most of the stars in Orion) are bluish-white; those of Class A, white (Sirius, Vega); of Class G, yellow (Capella, the sun); of Class K, orange (Arcturus); of Class M, red (Antares). Those of classes R and S are also red; and of Class N, very red.

We shall see later (§ 696) how these colors may be accurately measured. When this is done, it is found that the differences in color between the principal spectral classes are nearly equal.

687. Meaning of the Spectral Sequence. The most remarkable property of the spectral sequence is that it is *linear*; that is, there is but one set of intermediate steps by which the transition occurs between any two types, such as A0 and G0. This shows at once that the characteristics of a star's spectrum are determined by some *single* physical condition on its surface, other conditions producing at most only minor and inconspicuous differences. In particular, it proves that the differences in the spectra cannot arise primarily from differences in chemical composition; for in this case the properties of the various elements could vary independently, and we might have, for example, iron lines strong and calcium lines nearly absent, or vice versa, which never happens.

The student who has mastered the theory of ionization (Chapter XVII) will see at once, from an inspection of the list of lines characteristic of the various classes, that this physical

condition is the *degree of excitation* of the atoms in the star's atmosphere. In the O-stars most of the lines are produced by doubly or trebly ionized atoms, and even helium is ionized, in spite of the great amount of energy required to do this. The degree of ionization diminishes steadily through classes B and A, while in Class F the arc lines appear (that is, neutral atoms are present), and in Class G, and still more in K, they predominate. Finally, in Class M, bands appear, showing the presence of undissociated compounds, and at the same time the ultimate arc lines, characteristic of low excitation, are strong. The same is true of classes R, N, and S. These differences of excitation may be attributed with entire security to *differences of temperature* in the stellar photospheres and atmospheres, which, as we have seen, are competent to produce the observed effects. The concomitant changes in color afford a strong confirmation of this view and, as described in Chapter XXI, make it possible to determine the actual effective temperatures of stars of the various classes. Stellar spectra are further discussed in Chapter XXV.

BRIGHTNESS OF THE STARS

688. Stellar Magnitudes. The apparent brightness of a star has for many centuries been described by the term "magnitude." This term, as applied to a star, refers simply to its brightness and has nothing to do with apparent angular diameter. Hipparchus and Ptolemy arbitrarily graded the visible stars into six magnitudes, the stars of the sixth being the faintest visible to the eye, while the first magnitude comprised about twenty of the brightest.

After the invention of the telescope the same system was extended to the smaller stars, but without any general agreement, so that the magnitudes assigned by different observers to telescopic stars in the early part of the nineteenth century differed widely. Sir William Herschel, especially, used very high numbers.

The stars which were classed together under one magnitude are not exactly alike in brightness, but shade from the bright to the fainter, so that exactness requires the use of fractional magnitudes. It is now usual to employ decimals, giving the brightness of a star to the nearest tenth of a magnitude. Thus, a star of 4.3

magnitude is a shade brighter than 4.4, and so on. In more accurate photometric work two decimal places are employed.

689. Light-Ratio and Scale of Magnitude. It was found by Sir John Herschel, about 1830, that an average star of the first magnitude is just about *one hundred times as bright as one of the sixth*, and that for the naked-eye stars a corresponding ratio had been roughly maintained by former observers through the whole scale of magnitudes, the stars of each magnitude being approximately two and one-half times as bright as those of the next fainter.

This depends upon the general law of sensation (Fechner's law), that differences in intensity which correspond to the same fractional part of the whole are equally perceptible by the senses, whether the whole intensity is great or small. The smallest difference in brightness which can be detected by a trained eye under favorable conditions is about 1 per cent. To harmonize the arbitrary scales adopted in the earlier catalogues, Pogson proposed, in 1850, a fixed scale of *stellar magnitudes*, adopting the *fifth root of one hundred*, 2.5119, as the uniform "light-ratio" and adjusting the zero of the scale so as to secure as good agreement as possible with the Bonn *Durchmusterung* at the sixth magnitude. This scale has now been generally adopted. According to it Aldebaran and Altair are very nearly typical "first-magnitude" stars, while the two "pointers" and Polaris are of about the second magnitude. With this principle as a guide, the scale of magnitudes is carried forward to the faintest telescopic stars and backward to stars brighter than the first magnitude. Thus, twenty-first-magnitude stars can be photographed with the 100-inch telescope at Mt. Wilson. The magnitude of Capella is 0.1; of Sirius, − 1.6; of Venus, about − 4; of the full moon, − 12.5; and of the sun, − 26.7. A standard candle at a distance of one meter is of magnitude − 14.2; at one kilometer, + 0.8.

The magnitudes of the B.D. are not far from the photometric scale down to $8^{m}.0$. Below this they make the stars too bright. Argelander's 9.0 should be 9.3; 9.3, 9.9; 9.4, 10.2; and 9.5, 10.6.

690. Relative Brightness of Different Magnitudes. On the standard scale, therefore, the *light-ratio* (that is, the ratio between

the light of two stars standing just one magnitude apart in the scale) is exactly $\sqrt[5]{100}$, or *the number whose logarithm to the base 10 is 0.4000*, that is, 2.512 approximately.

A star of the first magnitude ($1^m.0$) is therefore

$(2.512)^1$, or 2.512, times as bright as a star of $2^m.0$,
$(2.512)^2$, or 6.30, times as bright as a star of $3^m.0$,
$(2.512)^3$, or 15.84, times as bright as a star of $4^m.0$,
$(2.512)^4$, or 39.8, times as bright as a star of $5^m.0$,
$(2.512)^5$, or 100.0, times as bright as a star of $6^m.0$,
$(2.512)^6$, or 251.2, times as bright as a star of $7^m.0$,

and so on.

If l_m is the apparent brightness of a star of magnitude m (strictly speaking, the luminous flux (§ 568) received from it), and l_n that of a star of magnitude n, then, in general,

$$l_n/l_m = (2.512)^{m-n}. \tag{1}$$

Taking logarithms, we find

$$\log (l_n/l_m) = (m - n) \log (2.512) = 0.4\,(m - n), \tag{2}$$

whence

$$m - n = 2.500 \log (l_n/l_m) = 2.5\,(\log l_n - \log l_m). \tag{3}$$

Setting $n = 0$, we have

$$m = 2.500 \log (l_o/l_m) = -\,2.5 \log (l_m/l_o), \tag{4}$$

where l_o is the brightness of a zero-magnitude star; whence

$$\log (l_m/l_o) = -\,0.4\,m. \tag{5}$$

As examples of the use of these equations:

(1) A certain variable star rises seven magnitudes between the minimum and maximum; how much does its brightness increase?

From equation (2) $\log l_m/l_n = 0.4 \times 7 = 2.8$. Hence $l_m = 630\ l_n$. (A slide-rule or three-place logarithms will give abundant accuracy in these problems.)

(2) Nova Persei increased 25,000 times in brightness between February 20 and February 22, 1901; how many magnitudes did it rise?

By equation (3) $m - n = 2.5 \log 25{,}000 = 2.5 \times 4.40 = 11.0$ magnitudes.

(3) The star α Centauri is double, the components being of magnitudes 0.33 and 1.70. Of what magnitude does the star appear when viewed as a single object with the naked eye?

If the amounts of light received from the components are l_1 and l_2, we have, by (5),

$$\log (l_1/l_0) = -\,0.4 \times 0.33 = -\,0.132 = 9.868 - 10,$$

whence $l_1 = 0.738\, l_0$,

$$\log (l_2/l_0) = -0.4 \times 1.70 = -0.68 = 9.320 - 10,$$
$$l_2 = 0.209\, l_0.$$

Hence $l_1 + l_2 = 0.947\, l_0$. The corresponding magnitude, m, is found by (4) to be

$$m = -2.5 \log (0.947\, l_0/l_0) = -2.5\,(9.976 - 10)$$
$$= -2.5\,(-0.024),$$

or $m = 0.06$.

691. Telescopic Power required to show Stars of a Given Magnitude. If a good telescope just shows stars of a certain magnitude, then, since the *light-gathering power* of a telescope depends on the *area* of its object-glass (which varies as the square of its diameter), we must have a telescope with its aperture larger in the ratio of $\sqrt{2.512}$ (or 1.59) : 1, in order to show stars one magnitude fainter; that is, the aperture must be increased 1.6 times (nearly). A tenfold increase in the diameter of an object-glass theoretically carries the power of vision just *five* magnitudes lower.

Assuming (what seems to be nearly true for normal eyes and good telescopes) that the *minimum visibile* for a 1-inch aperture is a star of the *ninth* magnitude, we obtain the following little table of apertures required to show stars of a given magnitude, the formula being $m = 9 + 5 \times$ log of aperture (in *inches*).

Magnitude	7	8	9	10	11	12	13	14	15	16	17	18	19
Aperture, in inches. .	0.4	0.6	1.0	1.6	2.5	4.0	6.3	10	16	25	40	63	100

But large telescopes, on account of the increased thickness of their lenses, which causes considerable absorption of light, never quite equal their theoretical capacity as compared with smaller ones.

Thus the Yerkes telescope (40 inches aperture) will barely show stars of the seventeenth magnitude, not quite one magnitude fainter than the smallest star visible with the 26-inch telescope at Washington. (The *number* visible in the larger instrument is probably fully doubled — but that is another matter.)

The foregoing discussion of telescopic light-gathering power refers to stars observed visually. The photographic limit, with long exposures, is fainter, reaching the twenty-first magnitude with a 100-inch telescope.

692. Photometric Catalogues. The results of accurate photometric measurements of the stars are to be found in catalogues, such as *The Revised Harvard Photometry*, which contains the magnitudes of 45,792 of the brighter stars all over the sky. These measurements, which number over a million, were made chiefly by E. C. Pickering, a pioneer in this field, who devised the "meridian photometer" with which the observations were made. The *Potsdamer Photometrische Durchmusterung* contains the results of a similar measurement (with a Zöllner photometer) of all the stars in the northern hemisphere to the 7.5 magnitude, 14,199 in number.

For the fainter stars the corrections required to reduce Argelander's estimates of magnitude to the photometric scale have been determined by measures of some 25,300 B.D. stars at Harvard. Magnitudes of still fainter stars in the "selected areas" have also been measured.

There are similar though less extensive catalogues of photographic magnitudes (§ 695).

693. Visual Methods and Instruments. The relative brightness of two stars may be found by comparing them directly or by comparing each with a third source of light. For accurate measurement it is necessary that the two stellar images should be so close to each other that they may be seen at the same time; it should be possible to reverse the two star-images in order to eliminate the common tendency of an individual observer to see the star on the right or on the left comparatively brighter than the other. It is desirable, also, in order to eliminate errors arising from a difference in the paths which the two rays of light follow, that the instrument itself should be reversible.

The measurement is made by diminishing the brightness of one or both of the images by known amounts until they appear *equally bright.* This may be accomplished by the use of some polarizing device or of a "wedge," which is a strip of neutral tinted material of graduated absorption, increasingly dense toward one end.

An "artificial star," formed by a suitable combination of lenses and diaphragms, and brought, by reflection, into the field of view close to the image of the star to be measured, is employed

in the wedge photometer, and in some forms of polarizing photometers. It is never quite possible to make an artificial star look exactly like a real star, and photometers which avoid its use should give more accurate results. Even on the clearest nights there is considerable loss of light as it passes through the earth's atmosphere (§ 116). In photometric work this is called the *extinction*. A star 45° from the zenith, on a clear night at sea-level, appears $0^m.06$ fainter than it would in the zenith. The average "visual extinction" is $0^m.23$ at 30° altitude; $0^m.45$ at 20°; $1^m.0$ at 10°; $1^m.7$ at 5°. For the photographically active rays the extinction is greater. Hazy weather may increase it very much. When stars at some distance from each other are compared, the uncertainty of the necessary extinction corrections adds to the inaccuracy of the observations.

Difference of color is another source of error. It is impossible to make a red star look identical with a blue one by any mere increase or decrease of brightness, and the measurements of different observers will differ considerably. The relative sensitiveness of the eye to differing colors varies also with the intensity of the light. This is known as the *Purkinje effect*: two colored sources which appear equally bright at a certain intensity appear unequal when the intensity of each is cut down in the same ratio, the bluer then appearing the brighter. It is a common experience that in a dark room blue objects can be seen more distinctly than red ones.

In systematic measurement of all the stars to a certain magnitude the comparison of differing colors cannot be avoided. The differences in the Harvard and Potsdam measures of the same stars, which amount in some cases to 0.3 magnitude, are probably due largely to a difference in color sensation. The Harvard scale is now generally adopted as a standard. In the observation of a single star — a variable, for example — it is usually possible to select a comparison star of the same color.

694. The Meridian and Zöllner Photometers. The Harvard meridian photometer is placed in an east-west direction in a horizontal position. Two mirrors or prisms feed light to two objectives of equal aperture and focal length. The polestar is kept in the field of view by means of one mirror. The other may be tilted so as to bring in light from any star on or near the

meridian. The two beams of light pass through a polarizing prism, thence through the eyepiece, and finally through a Nicol prism (§ 565). The eyepiece and Nicol prism are attached to a graduated circle which is read when, by rotation of the Nicol prism, the ordinary ray from one star appears equally bright with the extraordinary of the other. The other two rays are stopped by a diaphragm. The brightness ratio of two stars may be deduced from the angle between two readings of equality.

An optically similar photometer, which has proved to be very efficient in the observation of variable stars, has been devised by Pickering for use with the equatorial.

In the Zöllner photometer the light from an artificial star is polarized and a Nicol prism is turned until it appears equal in brightness to the unpolarized light of a star.

695. Photographic Methods and Instruments. The relative photographic brightness of stars may be deduced from the diameters of images made in good focus, or from the densities (degree of blackness) of the images, which for this purpose are often made intentionally out of focus. The images to be compared should have exposures of the same length, on plates of the same batch, and should be developed together. The diameters may be measured with a micrometer or estimated by comparison with a graduated scale of star-images; the densities may be found by comparison with a wedge of graduated density or by the effect on a thermopile of light passing through the stellar image.

In order to find what diameter or density corresponds to a particular magnitude it is necessary to vary the incident light by a measurable amount. This may be done by making additional exposures, cutting down the aperture by diaphragms, sectors, or screens, or by changing the size of extrafocal images. If a coarse wire grating is placed over the objective, the image of each star is flanked by a series of diffraction images, which are fainter than the principal one in fixed ratios (easily calculable). Such images are shown in Fig. 264. The scale is extended to fainter stars by using one of these devices to diminish the intensity of the images of the brighter stars, thus making them comparable with those of the fainter stars.

A sequence of stars near the north pole, and others well scattered over the sky, have been determined with the greatest care to serve as standards. One of these sequences may be

photographed (with the same exposure) on a plate which has just been exposed to the field to be measured, and the unknown magnitudes can be found from the known.

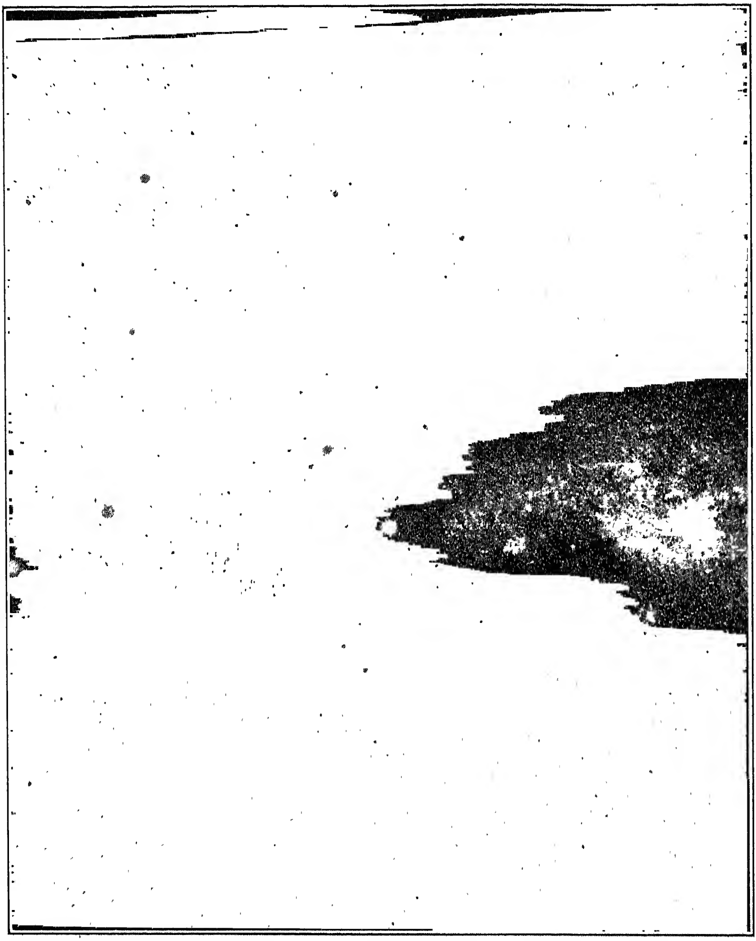


FIG. 217. Southern Milky Way, Southern Cross, and the Globular Cluster ω Centauri

Photographed with a 1-inch telescope by Margaret Harwood, at Harvard College Observatory Station, at Arequipa

The diameters of the images of stars of the same brightness are usually different at different distances from the center of the plate. Corrections for this must be determined and applied. The

curacy of photographic photometry is limited mainly by small regular variations in sensitiveness from one point of the film to other, which introduce probable errors amounting to 3 or 4 r cent. Measures of density are somewhat more accurate than ose of diameter.

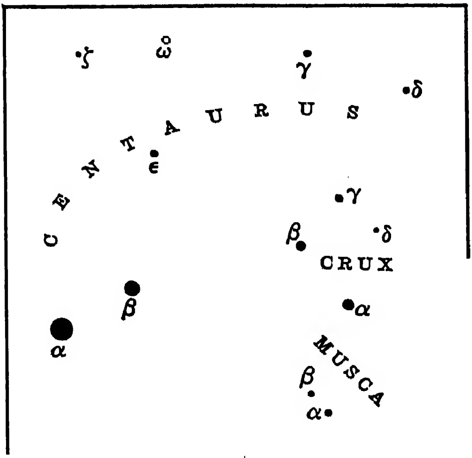


FIG. 218. Identification Chart for Fig. 217

e sizes of the spots as printed on the chart indicate the *visual* magnitudes of the stars. e appearance of the Southern Cross on the photograph (Fig. 217) is entirely different. is is a consequence of the large color-index of the very red star γ Crucis. The color-dex of α Centauri also is large. The following table gives the name, the spectral type, nd the visual and the photographic magnitude of each of the stars charted in Fig. 218

NAME	SP.	VIS.	PH.	NAME	SP.	VIS.	PH.
Cen . . .	G0	0m.06	0m.63	α Cru . . .	B1	1m.05	0m.58
Cen . . .	B1	0 .86	0 .46	β Cru . . .	B1	1 .50	1 .11
Cen . . .	A0	2 .38	2 .28	γ Cru . . .	M5	1 .97	3 .57
Cen . . .	B1	2 .56	2 .19	δ Cru . . .	B3	3 .08	2 .83
Cen . . .	B3p	2 .88	2 .66	α Mus . . .	B3	2 .94	2 .69
Cen . . .	B2p	3 .06	2 .43	β Mus . . .	B3	3 .26	3 .01

696. The Visual and Photographic Magnitude Scales; Color-ndex. Light which produces the maximum effect on the or-inary photographic plate is of shorter wave-length than that to hich the eye is most sensitive. Red stars, fairly bright to the ye, are relatively inconspicuous on the plate, as is well illustrated y comparing Fig. 217 and Fig. 218.

The "zero point" of the photographic scale has been established, by definition, so that the photographic and visual magnitudes are the same, on the average, for stars of Class A0 and magnitudes between 5.5 and 6.5.

The difference between the visual magnitude and the photographic magnitude of any star affords an accurate measure of the color of its light, and for this reason it is called the *color-index*. It is taken in the sense of photographic magnitude *minus* visual magnitude, and is therefore positive for red stars (classes K and M) and negative for bluish stars (classes O and B). The negative values are always small, rarely exceeding $-0^m.3$. The positive values often approach $+2^m.0$, and are occasionally much greater. We shall see in Chapter XXI that these color-indices afford the simplest means of measuring the temperatures of the stars.

697. Photo-visual Magnitudes, etc.; Color Equation. With isochromatic plates and a "color filter" to cut off the blue and violet light and let through the green and yellow, magnitudes may be obtained which agree very closely with the visual scale. These are called *photo-visual magnitudes*. By comparing them with ordinary photographic magnitudes, color-indices may be determined entirely by photography, thus eliminating most of the Purkinje effect.

The color sensitiveness of various combinations of telescopes, plates, and filters is very different, and the color-indices found by comparing the results of different sets of observations with visual magnitudes differ widely. It is found, however, that such color-indices can be closely represented by multiplying the ordinary color-indices, as defined above, by a factor which is constant for each instrumental combination. The color sensitiveness is therefore sufficiently described by giving the observed color-index of stars of Class K0. This is called the *color equation* of the method of observation.

For ordinary photographic plates the color equation is about $+1^m.1$ for both focal and extrafocal observations. For observations with orthochromatic plates and a red filter, red stars photograph brighter than white ones and the color equation is negative.

Every difference in the methods of observation — the absorption of the objective, the peculiarities of the observer's eye, the method of measurement — may introduce a small color equation. Thus, the color equation of the Potsdam visual measures, relative to the Harvard scale, is about $-0^m.25$, while different telescopes, with the same plates, may show differences of color equation exceeding $0^m.1$. These corrections must be determined and applied in refined photometric work.

698. Physical Photometers. When light falls on a suitably prepared "cell" of selenium the electrical resistance of the cell is diminished; when it falls on a surface of alkali metal (potassium, rubidium, etc.) in vacuum, electrons are emitted from the surface. Either effect is proportional to the intensity of the light and may be measured precisely by suitable electrical devices. The "photo-electric" photometer has been developed, by patient attention to detail, into the most accurate of all astronomical photometers, the probable error of a single measure on bright stars being only a few thousandths of a magnitude. (For faint objects, however, visual or photographic methods have the advantage.) These "physical photometers" have their own color sensitiveness. The color equation of the selenium photometer is much like that of the eye. That of the photo-electric photometer, when potassium is used, resembles that of an ordinary photographic plate; with rubidium or cæsium it approaches closer to the visual scale.

699. Radiometric Magnitudes. Heat-measuring devices, such as the thermocouple, differ widely from the photometers so far described, in that they are equally sensitive to radiations of all frequencies. If it were not for the selective absorption of our atmosphere they would measure the total energy which the stars send us, and as it is they give a fair approximation to this for the redder stars (§ 814). Measures with such instruments are becoming increasingly important. They are expressed in a scale of *radiometric magnitudes*, which, like the photographic, are adjusted to agree with the visual for Class A0. The difference obtained by subtracting the radiometric magnitude from the visual magnitude is called the *heat-index*. For red stars (which send us a great deal of heat in proportion to their light) this is positive, and sometimes very large.

The meaning of heat-indices and color-indices is further discussed in Chapter XXI.

700. The Number of Stars of Each Magnitude and their Total Light. Knowledge of the number of stars of each magnitude is necessary to an understanding of the extent and form of the sidereal universe. The *Durchmusterung* catalogues corrected to the photometric scale furnish the material for the brighter stars. With the more numerous fainter stars the task of counting all

those of each magnitude over the whole sky is too stupendous to be undertaken. Herschel therefore selected a number of small areas as samples and estimated the total number of stars from the counts in these selected areas. This method of securing data from a relatively small number of stars, which should be typical for the whole sky, was recognized in Kapteyn's *Plan of Selected Areas*, put forward in 1906. He chose 206 areas distributed uniformly over the sky and proposed that observatories should coöperate in finding the number, proper motions, magnitudes, color-indices, radial velocities, etc. of the stars in these areas.

Table X gives the number of stars brighter than the indicated magnitude, both visual and photographic, according to Seares and van Rhijn (1925).

TABLE X. NUMBER OF STARS BRIGHTER THAN A GIVEN MAGNITUDE

Magnitude Limit	Number of Stars		Magnitude Limit	Number of Stars	
	Photographic	Visual		Photographic	Visual
4.0	360	530	13.0	2,720,000	5,700,000
5.0	1,030	1,620	14.0	6,500,000	13,800,000
6.0	2,940	4,850	15.0	15,000,000	32,000,000
7.0	8,200	14,300	16.0	33,000,000	71,000,000
8.0	22,800	41,000	17.0	70,000,000	150,000,000
9.0	62,000	117,000	18.0	143,000,000	296,000,000
10.0	166,000	324,000	19.0	275,000,000	560,000,000
11.0	431,000	870,000	20.0	505,000,000	1,000,000,000
12.0	1,100,000	2,270,000	21.0	890,000,000	

The photographic results are based on all the available material (about 300,000 stars in the "selected areas," 1,400,000 in the *Astrographic Catalogue*, etc.). As far as magnitude 18.5 they depend on direct observation; the values for the twentieth and twenty-first magnitudes are extrapolated.

The data for visual magnitudes have been derived from the photographic results by allowing for the color of the stars. Very few stars are bluer than those of Class A0, for which the visual and photographic magnitudes are equal; but many stars are redder and have color-indices of $+1^m$ or more. A list of stars brighter visually than the tenth magnitude, for example, will contain many red stars which are photographically of the eleventh

magnitude or fainter, and a great number which are photographically fainter than the tenth magnitude. On the other hand, a list of stars to the tenth photographic magnitude will contain a few blue stars which are visually below the tenth magnitude, but not many. The difference in the numbers in the two columns is thus explained. The effect increases for the fainter stars, which are, on the average, redder than the brighter ones.

701. Number of Stars in Different Parts of the Sky; Galactic Concentration. The stars are much more numerous in the region of the Milky Way than remote from it. This is illustrated by Table XI (after Seares and van Rhijn), which gives the average number of stars, per square degree, brighter than a given photographic magnitude, for different galactic latitudes (§ 25), and also, for comparison, the average over the whole sky.

TABLE XI. STAR DENSITY AND GALACTIC CONCENTRATION

Limiting Magnitude	0°		20°		40°		90°		Whole Sky		Galactic Concentration
	Number	Ratio of Increase	Number	Ratio of Increase	Number	Ratio of Increase	Number	Ratio of Increase	Number	Ratio of Increase	
5.0	0.045		0.028		0.0175		0.013		0.025		3.4
		8.0		7.9		8.0		7.9		8.0	
7.0	0.36		0.22		0.14		0.103		0.20		3.5
		7.8		7.6		7.7		7.0		7.6	
9.0	2.80		1.67		1.07		0.72		1.51		3.9
		7.4		7.0		6.8		6.0		6.9	
11.0	20.8		11.6		7.3		4.3		10.5		4.8
		7.0		6.4		5.8		5.0		6.3	
13.0	146		74		42		21.4		66		6.8
		6.2		5.4		4.7		4.1		5.5	
15.0	910		400		199		87		362		10.4
		5.3		4.5		3.7		3.3		4.7	
17.0	4,780		1,820		744		288		1,710		16.6
		4.3		3.7		2.9		2.7		3.9	
19.0	20,750		6,860		2180		770		6,670		27.0
		3.7		3.2		2.3		2.2		3.2	
21.0	73,600		21,200		5000		1670		21,600		44.2

The ratio of the number of stars of the same sort in equal areas at the galactic equator (0° galactic latitude) and galactic pole is called the *galactic concentration.* For the brighter stars this quantity is between 3 and 4; but for the fainter ones it increases greatly, as the numbers in the last column show.

702. Distribution of the Stars in Space. Statistical material of this sort conveys much more information than might at first be

supposed. For example, the numbers of Table XI prove that the stars are not uniformly distributed throughout space, but are more thinly scattered at great distances from the sun.

To show this, consider stars of any given real brightness and let D be the distance at which such a star would appear to us as of magnitude m. A similar star at a distance 2.512 D will send us $1/(2.512)^2$ times as much light (since the light we receive from a star varies inversely as the square of its distance) and would appear of magnitude $m + 2$. Hence, if we make a list of all the stars which appear to us to be brighter than the magnitudes m and $m + 2$ respectively, we shall include all the stars of the kind under consideration which lie within the distances D and 2.512 D, that is, within spheres described about the sun with these radii. The volumes of these spheres are in the ratio $1 : (2.512)^3$, or $1 : 15.86$. Hence, if the stars were uniformly distributed throughout space, we should find nearly sixteen times as many stars of the sort considered in our second list as in our first. For stars of any other real brightness the distance D would be different, but it would still be true that we ought to find 15.86 times as many of this sort too in our second list as in our first. Hence it follows that if the stars were distributed uniformly in space, and with the same proportions of intrinsically bright and faint ones everywhere, the number of stars apparently brighter than a given limiting magnitude would increase by a factor of 15.86 for a change of the limit by two magnitudes (or by a factor of $(15.86)^{\frac{1}{2}}$, or 3.98, for a change of one magnitude).

The actual ratios of increase for different magnitudes and galactic latitudes are given in Table XI. Even among the brighter stars these ratios are only about 8, while for the faintest ones they decrease to 3.7 in the Milky Way and 2.2 at the galactic pole. It is evident, therefore, that the hypothesis of uniform distribution of the stars cannot be true and that the "density" with which the stars are distributed in space must fall off steadily at great distances, and more rapidly in the direction of the galactic pole than in that of the galactic equator. The fact that this conclusion can be drawn from mere counts of the numbers of stars of a given magnitude, without depending on the measurement of the distance of even a single star, is a good illustration of the importance

and power of the *statistical methods* which are so often used in modern sidereal astronomy.

An alternative explanation of the facts is possible, namely, that space is not perfectly transparent, so that the light of distant stars is weakened. This might account for the observed diminution in the ratios of increase, even if the stars were uniformly distributed. It is impossible to determine, from star-counts alone, which of these hypotheses is true, or whether each one accounts for a part of the observed effect; but there is evidence of other sorts, as we shall see later (§ 883), that there is little or no obstruction of light in its passage through interstellar space, except in certain "obscured regions," and the conclusion stated in the last paragraph may therefore be accepted. It is further confirmed by certain gravitational considerations (§ 884).

703. The total number of the stars can be only very roughly estimated, for even at the twentieth magnitude the number is still very rapidly increasing. Allowing for this, Seares and van Rhijn, from the data of Table X, estimate it as about 30,000,000,000. According to their reckoning, in the region of the galactic pole the number of stars too faint to be seen with any telescope is three times as great as that of those telescopically visible. In the Milky Way it is no less than seventy times as great. The latter estimate is of course very uncertain.

704. The total light of the stars can easily be calculated from the data of Table X, and it is found that the combined light of all the stars in the heavens equals that of 1092 stars of (visual) magnitude 1.0. This estimate of light is far more secure than that of the numbers; for the multitude of very faint stars, though they are so numerous, contribute little to the total light. About one fifth of the light comes from the stars that are brighter than the sixth magnitude and individually visible to the eye. The stars brighter than the eleventh magnitude send us half the total light, and those above the fifteenth magnitude send fully three quarters.

The observed light of the sky on a clear, dark night is much greater than this; but the investigations of van Rhijn show that only about one sixth of this light comes from the stars, the rest being due to the zodiacal light and to a permanent auroral luminosity of the earth's atmosphere.

After allowing for these he finds the total visual light of the stars to be equal to 1440 stars of the first magnitude, — a tolerable agreement with the value given above, in view of the difficulty of correcting exactly for the light from other sources.

705. Number of Stars of Different Spectral Classes. More than 99 per cent of the stars belong to the six principal spectral classes, B, A, F, G, K, and M. The percentages of stars of different magnitudes which belong to these classes, among the 225,000 stars listed in the *Henry Draper Catalogue*, are given in Table XII.

TABLE XII. PERCENTAGES OF STARS OF THE VARIOUS SPECTRAL CLASSES

VISUAL MAGNITUDE	B (B0 to B5)	A (B8 to A3)	F (A5 to F2)	G (F5 to G0)	K (G5 to K2)	M (K5 to M8)
Brighter than 2.24 .	28	28	7	10	15	12
2.25 to 3.24 . .	25	19	10	12	22	12
3.25 to 4.24 . .	16	22	7	12	35	8
4.25 to 5.24 . .	9	27	12	12	30	10
5.25 to 6.24 . .	5	38	13	10	28	6
6.25 to 7.24 . .	4	30	12	14	33	7
7.25 to 8.24 . .	2	26	11	16	37	8
8.25 to 9.24 . .	1	27	10	21	34	7
Below 9.25 . .	1	33	8	25	29	4
All together	2	29	9	21	33	6

The percentages for stars of all magnitudes together are given in the lowest line. The fainter stars, on account of their great numbers, count for much the most here. The stars of classes A and K are the most numerous and about equally abundant, the two together accounting for about five eighths of all the stars. Class B contains a large proportion of the brightest stars, but the percentage belonging to it decreases with remarkable rapidity among the fainter ones. The helium lines, the presence of which differentiates Class B from Class A, are hard to see in the spectra of faint stars, and this may account for some part of the diminution; but most of it is undoubtedly real, and suggests that these stars "thin out" rapidly. The percentage of Class G increases among the fainter stars. That of Class F is nearly constant, and that of Class M diminishes slowly. Stars of the remaining spectral

classes are much less numerous. Among the stars brighter than $6^m.25$ (more than 6000 in number) only 20 are recorded at Harvard as of Class O, and 8 of Class N (not including a few variable stars which are brighter than the sixth magnitude at maximum). The brightest stars of Class O are γ Velorum ($2^m.22$) and ζ Puppis ($2^m.27$); that of Class N is 19 Piscium ($5^m.30$). Spectra of classes R and S are of still rarer occurrence. Only about 70 of the former and 20 of the latter are known. The brightest star of Class R is $-10°5057$, of magnitude 7.04; of Class S, π_1 Gruis ($6^m.65$), omitting again certain variable stars.

706. The galactic concentration for the various spectral classes is very different. For the six principal types it is illustrated by Table XIII (abbreviated from Harvard results), which gives the average number of stars in 100 square degrees near the galactic equator and in regions remote from it.

TABLE XIII. STAR DENSITY FOR DIFFERENT SPECTRAL CLASSES

	B	A	F	G	K	M	ALL
Above $7^m.0$							
40° to 90°	0.2	6.6	3.0	3.4	10.2	1.5	24.9
0°	10.8	21.1	5.1	5.1	15.1	3.9	61.1
$7^m.0$ to $8^m.25$							
40° to 90°	0.1	6.6	9.5	16.4	32.8	6.1	71.5
0°	18.9	75.8	13.6	20.9	53.9	13.6	196.7

The B-stars are almost confined to the Galaxy. Only a few stragglers are found within 40° of the galactic pole, and most of these are bright stars and are probably relatively near us. The A-stars show a strong galactic concentration, especially for the fainter magnitudes. Classes F and G show very little concentration; classes K and M, somewhat more. Counts of fainter stars show that a decided concentration sets in for the latter at about the eighth magnitude.

Stars of Class O are entirely confined to the Milky Way, none at all being found more than 18° from the galactic equator, except for a few in the Magellanic Clouds (§ 870). Those of Class N also show a very strong concentration toward the Galaxy, and the like is true for certain classes of variable stars (§§ 836, 850).

MOTIONS AND DISTANCES

707. Stars not Fixed. In contradistinction to the planets, or "wanderers," the stars are sometimes called "fixed," because they keep their relative positions and configurations sensibly unchanged for centuries. Delicate observations, however, separated by sufficient intervals of time, show that the fixity is not absolute. More than two hundred years ago (in 1718) it was discovered by Halley that Arcturus and Sirius had changed their places since the days of Ptolemy, having moved southward, the first by a full degree and the other about half as much.

Modern observations show clearly that the stars are really all in motion, "drifting" upon the celestial sphere. Not only so, but the spectroscope now makes it possible to measure their rate of motion toward or from the earth, and it appears on the whole that their velocities are of the same order as those of the planets; they are flying through space with incomparably greater swiftness than cannon-shot, and it is only because of their vast distances from us that they seem to go so slowly.

708. Proper Motion. If we compare a star's position (that is, its right ascension and declination), as determined today by a meridian circle, with that observed fifty or a hundred years ago, it will be found to have altered considerably. Most of this difference is due to precession, nutation, and aberration, shared alike by all stars in the same region of the sky. But after allowance has been made for these "common" apparent motions, it will usually be found that the stars have also changed their positions *with reference to each other*, each having a *proper motion* of its own on the celestial sphere. The proper motion of a star is its apparent angular rate of motion on the celestial sphere. It is measured in seconds of arc per year, or per century. Strictly speaking, it should be defined as the motion *as seen from the sun*, and thus cleared of the effects of annual parallax (§ 710).

Proper motions may also be determined by comparing photographs taken a decade or two apart, and this has led to the discovery of many rapidly moving stars. The largest known motion belongs to a tenth-magnitude star discovered by Barnard, and amounts to $10''.25$ per year. At this rate it would take the star

352 years to move 1°; but photographs taken only a week or two apart, with a large telescope, show the displacement readily.

709. Radial Velocity (Motion in the Line of Sight). Observations of the proper motions of stars furnish no information as to the rate at which the stars are receding or approaching; but if a star is bright enough to give an observable spectrum, its radial velocity can be determined by means of the spectroscope and the application of the Doppler principle (§ 564). If the star is receding, the lines of its spectrum will be shifted toward the red; if it is coming nearer, toward the violet. Measures of the stellar wave-lengths can easily be made upon slit-spectrograms provided with a comparison spectrum. The Doppler shift is found by comparing the results with laboratory determinations of the positions of the same lines.

Huggins, in 1867, was the first to attempt such observations (visually). The first reliable results were obtained photographically by Vogel in 1888. Several thousand stars have now been observed with great telescopes.

The radial velocity, like the proper motion, must be referred to the sun, by correction for the earth's motion. It is counted positive when the star's distance is increasing, and is usually measured in kilometers per second.

Proper motion and radial velocity are discussed in detail in the next chapter.

710. Stellar Parallax. The *distances* of the stars can be directly determined by measurements of parallax (§ 111). The radius of the earth, which is used as a base line in measuring the distances of the moon and the nearer planets, is hopelessly short for this new purpose. But fortunately we are provided with a more suitable base line by the revolution of the earth around the sun. At one time we are on one side of the sun, and six months later we are on the opposite side, 186,000,000 miles from the former position.

An imaginary observer on the star would see the sun moving uniformly over the celestial sphere with a certain proper motion, and the earth revolving about it in a small apparent orbit (really almost circular, but apparently elliptical, owing to foreshortening) once a year. The combination of these motions would give the

earth a wavy track on the celestial sphere (Fig. 219). Now the direction of the star from the earth is always exactly opposite to that of the earth from the star, so that the star will appear to a terrestrial observer to move over the celestial sphere in a wavy path of just the same shape, obtained by combining a uniform proper motion with motion in a small "parallactic ellipse" in which the star keeps opposite to the earth's true position. (For a star at the pole of the ecliptic this ellipse becomes a circle; for one in the ecliptic the earth's orbit is seen edgewise and the ellipse becomes a straight line.) The maximum radius of this ellipse (Fig. 219) is evidently equal to the angle subtended

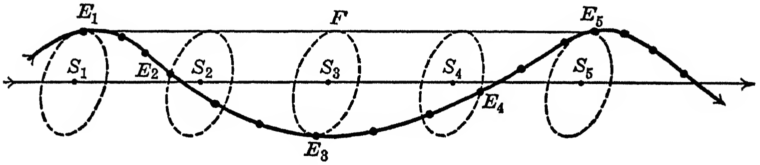


FIG. 219. Tracks of the Earth and Sun as seen from a Star

S_1, S_2, $\cdots$ S_5 are successive positions of the sun at intervals of three months, and $E_1 \cdots E_5$ those of the earth at the same dates. The apparent orbit of the earth about the sun as seen from the star is shown by the dotted ellipses. Reversing the directions in space, $S_1 \cdots S_5$ may be regarded as representing the successive positions of the star as seen from the sun, and $E_1 \cdots E_5$ its positions as seen from the earth. S_1S_5 is the star's annual proper motion, S_1E_1 its annual parallax. The actual proper motion from month to month (as shown by the dots along the star's apparent path) is neither rectilinear nor uniform. The east is at the top of the drawing

at the star by the radius of the earth's orbit, and this is, by definition, the *heliocentric parallax*, or "annual parallax" of the star.

If D is the distance from the sun to the star, R the radius of the earth's orbit, and p the parallax, then (§ 109)

$$D = R/\sin p = 206{,}265\ R/p \tag{6}$$

if p is measured in seconds of arc.

We have, for simplicity, neglected the eccentricity of the earth's orbit. When this is taken into account, the "parallactic orbit" is still an ellipse, but the sun is not quite at its center. Equation (6) still holds true if R represents the earth's mean distance.

711. The determination of stellar parallax by the direct, or *trigonometric*, method is theoretically quite simple. If, for example, we have three observations, made at intervals of six

months, when the earth is farthest to one side or the other of the line joining the sun and the star, the parallactic displacement will be in one direction for E_1 and E_5 (Fig. 219) and opposite for E_3. If F is the middle point of E_1E_5, E_3F will be twice the parallax. The proper motion E_1E_5 is obtained as a by-product. Observations made at any other times may be used, with slight geometrical complications.

In practice the observations are delicate. Even the largest stellar parallaxes are less than a second of arc, and small errors of observation become serious. The "absolute method," in which observations of right ascension and declination are employed, is of little or no value, for certain of the observational errors have a period of one year (changing with the seasons) and become inextricably confused with the true effect of parallax. It is far better to make *differential observations* of the position of the "parallax star" relative to faint stars, apparently close by in the heavens but actually very remote, and derive the parallax from these.

The "comparison stars" are themselves shifted by parallax, in the same direction, at any given instant, as the parallax star, but by a much smaller amount. The differential method gives, therefore, not the absolute parallax of the star but the difference between this and the average parallax of the comparison stars. This was once regarded as a serious drawback; but modern investigations (Chapter XIX) show that the mean parallax of the comparison stars is usually only a few thousandths of a second, and even this small correction can be estimated and allowed for with substantial accuracy.

If the parallax star were remoter than the comparison stars, the relative parallax would be negative, although practically too small to measure. Good measurements, however, occasionally lead to small negative parallaxes when the true parallax is smaller than the inevitable errors of observation. In such a case the negative observed value must be retained when taking averages with the results of other observers. Otherwise, cases where the observational errors made the parallax come out too small would be rejected, while those where they made it come out too large would be included, and the mean value would be vitiated.

Observations for parallax were formerly made with the heliometer, and it was with this instrument that Bessel, in 1838, made the first reliable measure of parallax, for the fifth-magnitude star 61 Cygni, which has a very large proper motion. Henderson, in the same year, obtained by the absolute method a fairly good determination of the parallax of α Centauri, which is large enough to be detected even in this way.

712. Photographic observations of parallax have now practically displaced all other methods. Large telescopes of long focus (including most of the greatest instruments) are employed, and exceptional precautions must be taken. The observations are all made at the same hour angle, for otherwise the effects of refraction and especially of atmospheric dispersion (§ 116) would be different at different parallax "epochs" and might vitiate the results. Great care is taken with the guiding; despite this it is found that the images of stars of different brightness, on the same plate, cannot be compared with the highest accuracy. The light of the parallax star is therefore cut down, so that its image is comparable with the images of the comparison stars. This is usually done by an opaque rotating shutter with a narrow adjustable slit, through which the light of the bright star comes in a succession of short flashes. In this way even Sirius has been successfully compared with stars more than ten magnitudes fainter.

From twelve to twenty plates of a given region are obtained, the image of the parallax star and the images of about half a dozen comparison stars are measured, and the parallax and proper motion of the former are found by least squares (§ 124). The probable error of a single determination of parallax from such a series with the best instruments averages about $\pm 0''.009$, and the results of different observers agree very closely, the systematic differences from the general mean being usually $0''.005$ or smaller.

For stars within five million times the sun's distance (parallax $0''.04$) the mean of two or three good determinations of parallax gives a value of the distance with a probable error of 10 or 15 per cent. Even for twice this distance tolerable values can be obtained. For the remoter stars the trigonometric method is practically useless, but fortunately other more powerful methods are available for exploration to greater distances.

713. The number of measured parallaxes has greatly increased in recent years, thanks to the active work of several observers. Schlesinger, in his catalogue (1924), gives parallaxes for 1870 stars, of which all but 200 were determined by the trigonometrical method. The older observing lists were made up mainly of stars of large proper motion (and presumably, therefore, not very distant) and of stars that are bright, double, variable, or otherwise of special interest; but the observations are now being extended to include a sufficient number of stars of all sorts to give good average values. Observations in the southern hemisphere, which are much needed, have been begun at a station established at Johannesburg by the Yale Observatory.

714. Units of Stellar Distance: the Parsec and the Light-Year. When the distance of a star is expressed in miles, or even in astronomical units, the figures become cumbersome to handle, and larger units are more convenient. The one used in scientific work is the *parsec*, which is the distance at which a star would have a *par*allax of one *sec*ond of arc. This is 206,265 astronomical units and equals 3.083×10^{13} km., or 1.92×10^{13} miles. In more popular discussion the *light-year* is usually employed. This is the distance which light travels in one year, and may be found by multiplying the velocity of light, 2.998×10^5 km./sec., by 3.156×10^7, which is the number of seconds in a year. The result is 9.461×10^{12} km., or 5.88×10^{12} miles (5880 billion miles). A light-year is 63,310 astronomical units and would be represented almost exactly by one mile on a map on which the earth was one inch from the sun. One parsec equals 3.258 light-years.

By equation (6), page 630, we find, for the distance D of a star of parallax p,

$$D = 1/p \text{ parsecs} = 3.26/p \text{ light-years.} \tag{7}$$

715. Tangential Velocity of a Star. If the proper motion μ and the parallax p of a star are known, its velocity at right angles to the line of sight can be found. This "tangential velocity" (§ 734), so called because it is perpendicular to the radial velocity, is given, if measured in astronomical units per year, by the simple equation

$$T = \mu/p, \tag{8}$$

which follows at once from the definitions of the quantities involved — since the angles are small.

The tangential velocity in kilometers per second is given by the equation

$$T = 4.74\ \mu/p; \tag{9}$$

for $\frac{1 \text{ ast. unit}}{1 \text{ year}} = \frac{149{,}450{,}000 \text{ km.}}{31{,}556{,}926 \text{ sec.}} = 4.738 \text{ km./sec.}$

716. Real Brightness of the Stars; Absolute Magnitude. The observed stellar magnitude of a star depends both on its real brightness, or "luminosity," and its distance. If all the stars should be brought to the same standard distance, the magnitudes which they would then appear to possess would indicate their luminosities. These quantities, which are called "absolute magnitudes," can readily be calculated if the apparent magnitude and the parallax are known.

Let m, p, and D be the observed magnitude, parallax, and distance of a star, and let M be the absolute magnitude corresponding to the standard distance D_0 and parallax p_0. If l and L are the amounts of light which would be received from the star at the actual and standard distances, then, since the apparent brightness varies inversely as the square of the distance,

$$\frac{l}{L} = \frac{D_0^2}{D^2} = \frac{p^2}{p_0^2}.$$

By equation (4), section 690,

$$m = 2.5 \log \frac{l_0}{l}, \text{ and } M = 2.5 \log \frac{l_0}{L}.$$

Hence

$$M - m = 2.5 \log \frac{l}{L} = 5 \log \left(\frac{D_0}{D}\right) = 5 \log p - 5 \log p_0. \tag{10}$$

The standard distance is taken as 10 parsecs (International Astronomical Union, 1922).

The *absolute magnitude* thus defined *is the magnitude which a star would have if brought to a distance of ten parsecs.*

In this case $p_0 = 0''.1$, and equation (10) becomes

$$M = m + 5 + 5 \log p. \tag{11}$$

The relative luminosity of any two stars is then given by the equation

$$\log \frac{L_2}{L_1} = 0.4\,(M_1 - M_2)\,. \tag{12}$$

Absolute magnitudes may be visual, photographic, radiometric, etc., according to the way in which the observed, or "apparent," magnitude m is measured.

717. Brightness compared with the Sun. The sun's apparent visual magnitude is -26.72, according to the mean of accordant determinations by four observers. To find the absolute magnitude of the sun we must use equation (10), setting $D_0/D = 2{,}062{,}650$. This gives $M - m = +31.57$, whence $M = +4.85$. A star as bright as the sun would therefore appear, at a distance of 10 parsecs, a little brighter than the fifth magnitude, and on a clear moonlight night would be just comfortably visible to the naked eye.

The (visual) luminosity of any other star is then given by the equation

$$\log \frac{L}{\odot} = 0.4\,(4.85 - M), \tag{13}$$

where $\odot$ here indicates the sun's luminosity.

As illustrations of these equations we may find the absolute magnitude and luminosity compared with the sun for a few stars.

(1) Sirius, for which $m = -1.58$; $p = 0''.371$.

$M = -1.58 + 5.00 + 5(9.570 - 10) = 3.42 + 5(-0.430) = +1.27.$

$\log \frac{L}{\odot} = 0.4\,(4.85 - 1.27) = 1.43$; $L = 27\,\odot$.

(2) Antares, for which $m = 1.22$; $p = 0''.009$.

$M = 1.22 + 5.00 + 5(7.954 - 10) = 6.22 - 10.23 = -4.01.$

$\log \frac{L}{\odot} = 0.4(4.85 + 4.01) = 3.54$; $L = 3500\,\odot$.

(3) Barnard's star, for which $m = 9.67$; $p = 0''.538$.

$M = 9.67 + 5.00 + 5(9.730 - 10) = 14.67 - 1.35 = 13.32.$

$\log \frac{L}{\odot} = -3.39 = 6.61 - 10$; $L = 0.00041\,\odot$.

NOTE. On account of the uncertainty of the data it is never worth while to calculate the luminosity to more than two significant figures.

The most luminous star for which fairly reliable data exist is Rigel (β Orionis), for which $m = 0.34$, $p = 0''.0069$ (according to Kapteyn), which makes $M = -5.5$ and $L = 14{,}000\,\odot$.

Canopus and α Cygni may be as bright or brighter, but their parallaxes are too small to measure with any precision. The faintest[1] is an eleventh-magnitude star discovered by Innes, which is about 2° from α Centauri and shares its very large proper motion and parallax. For this star $M = + 15$, approximately, and $L = 1/11{,}000 \odot$. The known range of luminosity among the stars is therefore enormously great, about twenty-one magnitudes, or more than two hundred million fold. We conclude this chapter with some facts regarding certain of the more interesting stars.

718. The Brightest Stars. Table XIV gives data for the twenty-two stars which are brighter than the visual magnitude 1.6 and are commonly called "stars of the first magnitude," though the range in brightness among them is fully three magnitudes. The first column gives the usual designation of the star and its proper name; the latter is lacking for the southern stars unknown to the ancients. The second column gives the visual magnitude, according to the *Revised Harvard Photometry*. In several cases the stars are double or multiple. The magnitudes of the components are then given separately, the first value in the column representing the combined light. The third column gives the spectral class and the fourth the proper motion, according to Boss, which applies to all the members of a double or triple system, since these are moving together. Then follow the observed parallax and its probable error, taken from Schlesinger's catalogue of trigonometric determinations, except for the stars in Virgo, Scorpius, Crux, and Centaurus, for which the very precise values found by Kapteyn (§ 739) are given. The sixth column gives the radial velocity, mainly from the Lick observations. A colon (:) is employed here, according to the well-established usage, to denote values of low precision. For one or two stars the spectral lines are so diffuse and poor that no good determinations of radial velocity have been made. In the seventh column is the tangential velocity, computed by equation (9); in the eighth, the absolute magnitude, by equation (11); and, finally, the real brightness of the star in terms of the sun's light as a unit, by equation (13). In certain cases, when the probable error of the observed parallax is relatively great, these quantities are marked with colons.

[1] But see § 722.

Table XIV. The Brightest Stars

Name	Magnitude m		Spectrum	Proper Motion μ	Parallax p	$p.e.$	Rad. Vel. V	Tang. Vel. T	Abs. Mag. M	Light
							Km./Sec.	Km./Sec.		$\odot = 1$
α Canis Majoris (Sirius)	− 1.58	− 1.58 8.44	A0 F	1″.315	0″.371	± 0″.004	− 8	17	+ 1.3 + 11.3	26 0.003
α Carinæ (Canopus)	− 0.86		F0	0 .022	0 .005	± 0 .010	+ 20	21 :	− 7.4 :	80,000 :
α Centauri	0.06	0.33 1.70	G0 K5	3 .682	0 .758	± 0 .010	− 22	23	4.7 6.1	1.12 0.32
α Lyræ (Vega)	0.14		A0	0 .348	0 .124	± 0 .004	− 14	13	0.6	50
α Aurigæ (Capella)	0.21		G0	0 .439	0 .069	± 0 .004	+ 30	30	− 0.6	150
α Boötis (Arcturus)	0.24		K0	2 .287	0 .080	± 0 .005	− 5	135	− 0.2	100
β Orionis (Rigel)	0.34	0.34 6.66	B8 B8	0 .005	0 .006	± 0 .007	+ 23	4 :	− 5.8 : 0.2 :	18,000 : 80 :
α Canis Minoris (Procyon)	0.48	0.48 13 :	F5 —	1 .242	0″.312	± 0 .006	− 4	19	3.0 15.5 :	5.4 0.00005 :
α Eridani (Achernar)	0.60		B5	0 .093	0 .049	± 0 .015	+ 19	9	− 0.9	200
β Centauri	0.86		B1	0 .039	0 .011	± 0 .002	+ 4 :	17	− 3.9	3,100
α Aquilæ (Altair)	0.89		A5	0 .659	0 .204	± 0 .006	− 20 :	15	2.4	9.2
α Orionis (Betelgeuse)	0.92 (var.)		M0	0 .032	0 .017	± 0 .004	+ 21	9	− 2.9	1,200
α Crucis	1.05	1.58 2.09	B1 B1	0 .048	0 .014	± 0 .002	+ 7	16	− 2.7 − 2.2	1,000 650
α Tauri (Aldebaran)	1.06	1.06 13	K5 —	0 .205	0 .057	± 0 .005	+ 54	17	− 0.1 12	90 0.001
β Geminorum (Pollux)	1.21		K0	0 .623	0 .101	± 0 .010	+ 4	29	1.2	28
α Virginis (Spica)	1.21		B2	0 .051	0 .014	± 0 .003	+ 2	27	− 3.1	1,500
α Scorpii (Antares)	1.22	1.23 5.5	M0 B3	0 .032	0 .009	± 0 .002	− 3	17	− 4.0	3,400
α Piscis Australis (Fomalhaut)	1.29		A3	0 .367	0 .137	± 0 .014	+ 7	13	2.0	13.5
α Cygni (Deneb)	1.33		A2	0 .004	0 .005	± 0 .008	− 4 :	20	− 5.2 :	10,000 :
α Leonis (Regulus)	1.34		B8	0 .244	0 .058	± 0 .009	+ [illegible]	16	0.2	70
β Crucis	1.50		B1	0 .054	0 .016	± 0 .003	+ 13	16	− 2.5	850
α Geminorum (Castor)	1.58	1.99 2.85 9.03	A0 A0 M1	0 .201	0 .076	± 0 .004	+ 3	13	1.4 2.2 8.4	23 11 0.04

719. Statistical Discussion of Brightest Stars. Many interesting relations appear upon examination of Table XIV.

(1) All the principal spectral classes and all the halfway classes, except G5, are represented in this short list, showing that there is little correlation between apparent brightness and spectral type. This is a fortunate circumstance for the observer, who is thus enabled to study spectra of all classes with high dispersion.

(2) No less than eight of the twenty-two stars are double, and one is triple. It may be added that the spectroscope (§ 777) shows that α Aurigæ, β Orionis, α Virginis, and all three components of α Geminorum are double stars too close to be separated by the telescope, and that photographs show that α Centauri, α Aurigæ, and α Leonis have remote companions (at distances much greater than for the double stars in the table). It appears that duplicity is very common among the stars.

(3) The proper motions of the stars show an extremely wide range, being five hundred times as great for α Centauri and Arcturus as for Rigel and α Cygni.

(4) The parallaxes also differ greatly. Alpha Centauri is the nearest of the stars; Canopus, Rigel, and α Cygni are more than a hundred times as far away, so far as the present observations indicate. Apparent brightness is therefore a very poor criterion of nearness in space.

(5) There is a very marked correlation between proper motion and parallax. Grouping the stars in order of proper motion and taking means, we find

Limits of p.m.	$>1''$	$1''.0$ to $0''.3$	$0''.3$ to $0''.1$	$0''.1$ to $0''.04$	$<0''.04$
Mean μ	$2''.13$	$0''.49$	$0''.22$	$0''.062$	$0''.024$
Mean p	$0''.380$	$0''.127$	$0''.064$	$0''.023$	$0''.009$
Number	4	5	3	4	6

Proper motion is therefore a far better criterion of distance than is magnitude.

(6) Why this is the case becomes evident on examination of the tangential velocities. The mean of them all is 22 km./sec. Seventeen of the twenty-two stars have tangential velocities which lie between half and twice this value. The assumption that all the stars had tangential velocities of 22 km./sec. would there-

fore lead to estimates of parallax which, though obviously crude, would be twice too great or too small for less than one quarter of the stars. To double or halve the parallax would change the absolute magnitude by $1^m.5$. The mean computed absolute magnitude for the twenty-two stars is -1.1, but only six of the individual values are within $1^m.5$ of this. If we should try to estimate parallaxes on the assumption that the absolute magnitude of all the stars was -1.1, the results would be twice too great or too small for almost three quarters of them.

The greater value of proper motion as a criterion of distance depends, therefore, on the fact that the stars are more alike in velocity than in real brightness.

(7) The mean radial velocity, regardless of sign, is 14 km./sec. for the twenty stars for which it has been observed. It can be shown that if the motions of the stars were at random in all directions, the mean tangential velocity should be $\pi/2$, or 1.57, times as great as the radial (not equal to it, as might at first be supposed, because a star has more ways of moving at right angles to the line of sight than along it). The observed ratio for these stars is 23/14, or 1.6. For the stars with $\mu > 0''.1$ this ratio is 29/17, or 1.7; for those with $\mu < 0''.1$ it is 15/11, or 1.4. This indicates that the observed parallaxes cannot, on the average, be seriously wrong; for if they were, the computed tangential velocities would come out too high or too low.

(8) The real luminosities of the stars show a very great range. Making all allowance for uncertainties of parallax, Canopus and Rigel are probably fully ten thousand times as bright as α Centauri, and this again is twenty thousand times brighter than the faint companion of Procyon. There is no conspicuous relation between absolute magnitude and spectral class among these stars.

It should be explicitly stated that a study of so small a number of stars can furnish only suggestions as to the relations which really exist. To establish the conclusions stated above as representing properties of the stars in general a much larger number of stars would have to be considered, but all the propositions which have been stated are confirmed by more extensive study. They are given here in detail as an example of the statistical method of investigation.

720. The stars of largest proper motion are listed in Table XV, which is arranged like the preceding table. Here most of the stars are faint and are designated by their numbers in star-catalogues or known by the names of their discoverers. Six of them (indicated by asterisks) have been discovered photographically (§ 726). One of these is so faint that its spectrum and parallax have not yet been observed.

TABLE XV. THE STARS OF LARGEST PROPER MOTION

NAME	MAG.	SP.	μ	p	V	T	M
					Km./Sec.	Km./Sec.	
* Barnard's star .	9.67	M5	10″.25	0″.538 ± .004	− 117	90	13.3
* Kapteyn's star .	9.2	M0	8 .76	0 .317 ± .016	+ 242	131	11.7
Groombridge 1830	6.46	G5	7 .05	0 .101 ± .006	− 97	330	6.5
Lacaille 9352 . .	7.44	M0	6 .90	0 .292 ± .016	+ 12	112	9.8
Cordoba 32416 .	8.3	M3	6 .11	0 .220 ± .014	+ 26	132	10.0
61 Cygni	5.57 6.28	K7 K8	5 .20	0 .300 ± .003	− 64	82	8.0 8.7
* Wolf 359 . . .	13.5	M6e	4 .84	0 .404 ± .009	− 90	57	15.5
Lalande 21185 .	7.60	M2	4 .78	0 .392 ± .006	− 87	58	10.6
ε Indi	4.74	K5	4 .70	0 .281 ± .040	− 39	79	7.0
Lalande 21258 .	8.6	M2	4 .52	0 .177 ± .009	+ 65	121	9.8
o^2 Eridani A . . o^2 Eridani B . . o^2 Eridani C . .	4.48 9.7 10.8	G5 A0 Me	4 .08	0 .203 ± .008	− 42	95	6.0 11.2 12.3
* Wolf 489 . . .	13	—	3 .94	— —	—	—	—
* "Proxima Centauri" . .	10.5	M ?	3 .85	0 .765 ± .021	—	24	14.9
α Centauri A . . α Centauri B . .	0.33 1.70	G0 K5	3 .68	0 .758 ± .010	− 22	23	4.7 6.1
μ Cassiopeiæ . .	5.26	G5	3 .76	0 .130 ± .006	− 97	137	5.8
Washington 5583. Washington 5584.	9.1 8.9	G5 G0	3 .68	0 .034 ± .008	+ 307 + 295	510	6.8 6.6
Cordoba 29191 .	6.65	M0	3 .53	0 .253 ± .028	+ 13	66	8.7
e Eridani	4.30	G5	3 .16	0 .161 ± .017	+ 87	93	5.3
* van Maanen's star	12.3	F	3 .01	0 .255 ± .008	—	56	14.3

721. Discussion of Table XV. (1) Table XV, unlike the preceding one, is undoubtedly *incomplete*. There is certainly no star brighter than Sirius; but there is a fair chance that photographic observations may detect some faint star with a proper motion greater than Barnard's star, and a practical certainty that they

will in future add a good many stars to the list. Neglecting the fainter components of double and multiple stars, there are five stars between the fourth magnitude and the sixth magnitude, four between the sixth and the eighth, and five between the eighth and the tenth. The four stars fainter than the tenth magnitude have been discovered by photography, and there are probably many more.

(2) There are two double stars and one triple system in the list. In addition, the stars Washington 5583 and 5584, though $5' 6''$ apart, have exactly the same proper motion in the same direction and substantially the same radial velocity, and must be physically connected. Alpha Centauri and the faint star often called "Proxima Centauri," though $2° 11'$ apart, bear a similar relation to one another. The abundance of double and multiple star-systems is thus confirmed.

(3) There is no obvious relation between magnitude and parallax among these stars. The eight systems of largest proper motion have a mean $\mu = 6.72$ and $p = 0''.305$; and the other eight, $\mu = 3''.68$ and $p = 0''.247$.[1]

(4) The parallaxes are much greater than those of the first-magnitude stars (the mean being $0''.276$ as against $0''.113$) and differ less *inter se*. Twelve of the sixteen parallaxes lie between half and twice this mean value, whereas this is the case for only nine of the twenty-two bright stars.

(5) The tangential velocities are great, the average for the sixteen systems being 132 km./sec. Two of the individual values are greater than twice the mean, and two are less than half of it. The mean value itself is six times as great as for the first-magnitude stars, which is not surprising, since a selection of stars of very large proper motion evidently favors high tangential velocities. The wide pair Washington 5583–5584 have the extraordinary velocity of 510 km./sec. in the tangential direction and 300 km./sec. in the radial direction, or almost 600 km./sec. in space.

(6) The mean radial velocity for the fifteen systems for which it is known is 88 km./sec. (regardless of sign). The mean tangential velocity is 1.5 times as great. This is rather surprising, since one might expect that in picking out stars of large proper

[1] The inclusion of Wolf 359 (§ 722) modifies many of these numbers.

motion we would unconsciously give preference to those moving nearly at right angles to the line of sight and so find a ratio considerably exceeding 1.57 (§ 719).

(7) The spectra are mainly of the "later" classes. Counting as usual only the brightest component of a system, only one star is earlier than G, and more than half are of classes K5 and M.

(8) The absolute magnitudes are all faint, the average being 8.7. Only five of the sixteen individual values are within $1^m.5$ of this, showing that here, too, brightness is a poor criterion of distance.

(9) There is, however, a noteworthy correlation between spectral class and absolute magnitude. Still counting only the brightest member of a system, we have

Class	F	G0	G5	K5–K7	M0	M2–M5
Mean *M* . . .	14.3	5.7	5.9	7.5	10.1	10.9
Number of stars	1	2	4	2	3	4

This shows a steady decrease in absolute brightness for the later types, with the exception of the one anomalous star of Class F.

The conclusions here described are again confirmed by studies of much larger numbers of stars.

722. The Nearest Stars. Finally, we give a table showing all the stars which are known to have a distance less than 4 parsecs. Most of these stars appear also in one or another of the preceding tables. There are seventeen of these stars (counting α Centauri and its attendant as a single system) within a sphere of 4 parsecs radius and a volume of 268 cubic parsecs, that is, one star to every 16 cubic parsecs. The actual "star density" must be considerably greater, since the list is doubtless incomplete as regards the fainter stars. With the observed density there should be seven stars nearer than 3 parsecs, and two within 2 parsecs of the sun. The observed numbers are five and two.

Further discussion may be left to the reader.

As the book goes to press, van Maanen reports a parallax of $0''.350$ for the tenth-magnitude star B.D.$-12°$ 4523, and also $0''.404 \pm 0.009$ for Wolf 359 (Table XV). The latter is the faintest star known (up to 1928). Its color-index is $+2^m.0$.

It is probable that other faint stars of large parallax may soon be added to the list.

TABLE XVI. THE NEAREST STARS

NAME	MAGNITUDE	SPECTRUM	μ	p	DISTANCE Light-Years
"Proxima" . . .	10.5	M ?	3″.85	0″.765 ± .021	4.3
α Centauri . . .	0.3 1.7	G0 K5	3 .68	0 .758 ± .010	4.3
Barnard's star .	9.7	M5	10 .25	0 .538 ± .004	6.1
Lalande 21185 .	7.6	M2	4 .78	0 .392 ± .006	8.3
Sirius	− 1.6	A0	1 .32	0 .371 ± .004	8.8
Innes' star . . .	12	—	2 .69	0 .340 ± .020	9.6
Kapteyn's star .	9.2	M0	8 .76	0 .317 ± .016	10.3
τ Ceti	3.6	K0	1 .92	0 .315 ± .009	10.3
Procyon	0.5	F5	1 .24	0 .312 ± .006	10.4
ϵ Eridani . . .	3.8	K0	0 .97	0 .310 ± .008	10.5
61 Cygni	5.6 6.3	K7 K8	5 .20	0 .300 ± .003	10.9
Lacaille 9352 . .	7.4	M0	6 .90	0 .292 ± .016	11.2
*Σ 2398	8.7 9.4	M4 M4	2 .31	0 .287 ± .004	11.4
Groombridge 34 .	8.1	M2	2 .89	0 .282 ± .006	11.6
ϵ Indi	4.7	K5	4 .70	0 .281 ± .040	11.6
Krüger 60 . . .	9.6 11.4	M3 M	0 .87	0 .257 ± .004	12.7
van Maanen's star	12.3	F	3 .01	0 .255 ± .008	12.8
Cordoba 29191 .	6.6	M0	3 .53	0 .253 ± .028	12.9

* This symbol denotes the number in Struve's *Catalogue of Double Stars.*

EXERCISES

1. In the double star γ Andromedæ the visual magnitudes of the components are 2.28 and 5.08. What is the magnitude of the pair seen as a single star? *Ans.* 2.20.

2. The spectrum of the bright component is K0 and that of the fainter B9. What are the photographic magnitudes? (Use the table of color-indices for various spectral classes in § 812.)
Ans. Brighter component, $2.28 + 1.12 = 3.40$;
fainter component, $5.08 - 0.04 = 5.04$.

3. What is the ratio of the brightness of these stars visually? photographically? *Ans.* 13.2 : 1, visual; 4.5 : 1, photographic.

4. Find the total visual amount of light given by all the stars between the tenth and the eleventh magnitude, assuming the average brightness to be that of a star of magnitude 10.5.

Ans. By equation (5) for a star of magnitude 10.5, log $l_m/l_o = -4.20 = 5.80 - 10$. Hence $l_m = 0.000063\ l_o$. By Table X there are 546,000 such stars, so that their combined light equals thirty-five times that of a zero-magnitude star.

5. What is the total light of the stars between the nineteenth and the twentieth visual magnitude?

Ans. Seven times that of a star of magnitude zero.

6. Find the tangential velocity of Krüger 60 and the absolute magnitude of the components. *Ans.* $T = 16.0$ km./sec.; $M = 11.6$ and 13.4.

7. What is the luminosity of these stars if the sun is taken as the unit?

Ans. For the brighter component, $\log L/\odot = 0.4 \times (4.85 - 11.6) = -2.70 = 7.30 - 10$; $L = 0.0020\,\odot$. For the fainter component, $L = 0.00038\,\odot$.

8. What is the absolute visual magnitude of a standard candle?

Ans. At a distance D of one kilometer a standard candle has a visual magnitude $+0.8$ (§ 689). The standard distance D_o (10 parsecs) is 3.083×10^{14} km. (§ 714). Hence by equation (10) (§ 716), $M - m = 5 \log (3.083 \times 10^{14}) = 5 \times 14.492 = 72.45$. Hence $M = +73.2$.

REFERENCES

For the recognition of the constellations and the naked-eye stars, and as an aid in setting a small telescope on the brighter objects, a good star map is essential. From the large variety that can be procured the following are mentioned:

Sky-Map (Camp-Fire Outfitting Co.), a map of the planisphere type which can be adjusted to any latitude between 25° N. and 44° N., and can be set for the day and hour.

Schurig's *Himmels-Atlas* (Ed. Gaebler, Leipzig), containing eight charts which together cover the whole sky and show all the naked-eye stars, nebulæ, and clusters.

W. T. Olcott, *A Field Book of the Stars* (G. P. Putnam's Sons).

Kelvin McKready, *A Beginner's Star Book* (G. P. Putnam's Sons). These last two contain, in addition to star charts, many interesting notes and suggestions for observations with an opera-glass and a small telescope.

For the identification of telescopic stars the B.D. charts will be found most useful. The Beyer-Graff *Sternatlas* is a series of charts prepared for the amateur as a substitute for the B.D. charts. It may be purchased of M. Beyer, Altona (Elbe), Germany, Tresckowallee 6.

The standard works on the magnitudes, spectral types, and exact positions of the stars are:

The Revised Photometry, Harvard Annals No. 50.

The Henry Draper Catalogue, Harvard Annals Nos. 91–99.

Lewis Boss, *Preliminary General Catalogue* (Carnegie Institution).

CHAPTER XIX

THE MOTIONS OF THE STARS

DETERMINATION OF PROPER MOTION AND OF RADIAL VELOCITY · MOTIONS OF THE STARS IN SPACE · MOVING CLUSTERS · THE SOLAR MOTION · STATISTICAL DETERMINATIONS OF PARALLAX · STAR STREAMING · THE ASYMMETRY OF STELLAR MOTIONS

Proper motion and *radial velocity* have been defined in Chapter XVIII (§§ 708, 709). The present chapter discusses the motions of the stars in more detail.

723. Determination of Proper Motion. Proper motions are usually determined by comparing precise observations (made many years apart) of the right ascensions and declinations of the same stars. The differences in the observed coördinates are always large; but much the greater portion of the differences arises from precession, nutation, and aberration; that is, from changes in our coördinate-system or from effects of the earth's motion, none of which indicate real alterations in the directions of the stars. These effects are substantially the same for all the stars in the same region of the sky, and change from one part of the heavens to another according to well-known geometrical relations. If we have observations of stars in all parts of the heavens (or at least over a hemisphere or so) and are therefore able to compare extensive catalogues of stars made at the two epochs, we can determine the constants of precession, and so forth, in such a way as to account for as large a part of the observed differences as can be explained in this fashion. When correction has been made for these, the outstanding differences represent the motions of the stars themselves.

In practice, of course, the catalogue positions have already been corrected for aberration and nutation, and the constant of precession is determined from a special and very careful study of all available catalogues of precision. The proper motions are determined from all the observations of a given star by plotting

the corrected right ascensions (or declinations) against the dates of observation, thus obtaining points on a straight line, whose slope gives the annual change. The equivalent algebraic process takes longer but is slightly more accurate.

In refined work a careful study is made of the "residual" differences of the positions given in any one catalogue from those deduced, for the corresponding date, from the whole set of observations. There is usually a tendency for the values in the catalogue considered to be too high or too low, on the average, by a fraction of a second of arc, and "systematic corrections" are applied to remove these discrepancies. The discordances then outstanding arise from the accidental errors of observation (§ 124) and show what weight is to be assigned to the catalogue. Work of this sort requires great skill and experience.

724. Components of Proper Motion. Such a discussion gives separately the *proper motions in right ascension and declination* (commonly denoted by μ_α and μ_δ). The motion may equally well be represented by means of the *total proper motion* μ (or "proper motion in a great circle") and the position angle in the sky (§ 764) toward which this motion takes place (commonly called ψ). The relations of these quantities are shown in Fig. 220.

725. Accuracy of Determination. When the observations cover a century or more, proper motions may be very accurately determined, since any error in the observed displacement is divided by the number of years in the interval. The most accurate proper motions are those of Boss's *Preliminary General Catalogue*. The probable errors of the proper motion in each coördinate range from $\pm\, 0''.001$ to $\pm\, 0''.010$; for the better-observed stars they are less than $\pm\, 0''.005$. The total proper motion is determined with the same accuracy, while its position angle is known within a fraction of a degree if the proper motion is large, but may be uncertain by many degrees if it is small.

When but few catalogues are available, the accuracy is much less, and proper motions derived from only two catalogues are open to suspicion, since errors of some kind may have crept into one of them. A third observation greatly diminishes this danger.

726. Photographic Determination of Proper Motion; the Blink Microscope. By comparing two photographs of the same field taken ten or twenty years apart, the relative proper motions of the stars can be found with great accuracy. (It is desirable to have photographs with the same telescope, to minimize instrumental errors.) The results can be reduced to absolute proper

motions if the proper motions of some of the stars — at least three or four, and the more the better — are known from meridian observations. This is undoubtedly the method of the future for the fainter stars. The meridian circle will still be required to observe the "reference stars," which serve as standards.

Photography also has special advantages in the discovery of the more rapid proper motions, — indeed, the largest known proper motions have been detected photographically. The most effective device for this purpose is the *blink microscope*, in which two plates of the same field may be viewed alternately at rapid intervals, through microscopes with the same eyepiece, by moving a lever. When the plates are properly adjusted, the images of most of the stars appear fixed when the change is made, while those of the stars which have moved seem to jump, or blink. The farther apart the dates of the two plates, the smaller are the proper motions which can be detected. Motions of 0″.1 per year can be found in this way, but the main value of the method is in making complete lists of the larger proper motions, down to a given limiting magnitude.

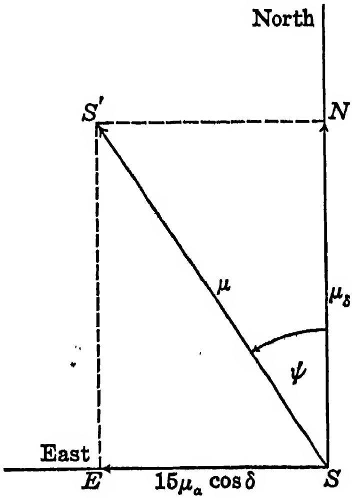


FIG. 220. Components of Proper Motion

Here SS' is the total proper motion, μ, of the star, and NSS' is its position angle, ψ; SN is its northward motion μ_δ, and SE its eastward motion, which is $15\,\mu_\alpha \cos\delta$ — the first factor appearing because μ_α is measured in seconds of *time*, and the last because of the convergence of the hour circles toward the pole. (The relation of the northerly and easterly directions in the sky is the opposite of that which is found on a terrestrial map)

The blink microscope is also unrivaled in detecting changes in brightness. One object which has changed can be picked out at once among thousands of ordinary stars.

727. Size of Proper Motions. The greatest known proper motion, 10″.25 per year, belongs to a star of the tenth magnitude discovered photographically by Barnard; the next greatest, 8″.7, to one found similarly by Kapteyn; the third, 7″.0, to one

discovered by meridian observations. All these three rapidly moving stars happen to be invisible to the naked eye.

About 50 stars are known to have total proper motions exceeding 2″, and 200 exceeding 1″, — the number increasing rapidly as the limit decreases. Our lists are, however, incomplete, as stars below the tenth magnitude have been but partially investigated.

The average proper motion of the first-magnitude stars is 0″.56, and of the sixth magnitude, 0″.06. These values do not mean much, being averages of a few large and many small motions.

728. Proper Motion and Spectral Class. More valuable information can be derived from Table XVII, after Boss, giving the number of stars in his catalogue brighter than $6^m.5$ which have proper motions between given limits.

TABLE XVII. PROPER MOTIONS OF STARS OF DIFFERENT SPECTRAL CLASSES

LIMITS OF P.M.	O	B	A	F	G	K	M	N
0″.00 to 0″.02 . . .	13	238	392	97	107	218	48	3
0 .02 to 0 .04 . . .	6	164	533	115	91	327	54	4
0 .04 to 0 .10 . . .	1	88	476	231	168	393	99	2
0 .10 to 0 .20 . . .	—	—	160	245	70	242	27	1
0 .20 to 0 .45 . . .	—	1	31	168	56	88	8	—
0 .45 to 0 .80 . . .	—	—	1	46	20	23	1	—
0 .80 to 2 .00 . . .	—	—	1	12	19	13	—	—
Over 2″.00	—	—	—	1	6	6	—	—
Mean P. M.	0″.02	0″.03	0″.06	0″.17	0″.18	0″.12	0″.07	0″.04
Percentage of stars with $\mu > 0''.20$ . .	0	0.2	5	25	18	10	4	0

Very striking relations are here evident. The stars of Class B have uniformly small proper motions, while large motions appear in Class A and increase in number in classes F and G. Class K shows a few very large motions and many small ones, while in Class M the large ones have practically disappeared, leaving only small ones, though not so small as for Class B. The few stars of classes O and N all have very small proper motions.

729. Determination of Radial Velocity. To find the radial velocity we must do three things: identify the stellar lines, meas-

ure their wave-lengths with precision, and compare them with measures of at least equal accuracy, made in the laboratory, for a source at rest relatively to the observer.

Enough lines (such as those of hydrogen) can almost always be identified by inspection to meet the first requirement. To measure the wave-lengths a slit spectrograph must be employed, and a comparison spectrum must be photographed on each side of the star's spectrum, by means of light from an arc, spark, or vacuum-tube mounted near the instrument. Any source that gives numerous sharp lines in the region under observation will suffice. From the measures of these lines it can be found just what wave-length corresponds to any given micrometer reading, and the wave-lengths of all the stellar lines can then be determined, whether they appear in the comparison spectrum or not. The measurements in the laboratory are made in the same way, by means of grating spectroscopes of high dispersion, and usually by employing the iron spectrum (which has been very carefully measured) as a standard.

730. Precautions Necessary for Accuracy. Special care is taken in radial-velocity work to avoid instrumental errors. The spectrograph employed is of very rigid construction, to minimize flexure; is inclosed in a thick case to protect it against changes of temperature; and, moreover, is provided with a delicate thermostat and electrical heating apparatus, which keep the temperature inside the case constant within a small fraction of a degree. The comparison spectrum is photographed in two short exposures (usually of a few seconds each) near the beginning and end of the exposure on the star; so that any minute, but steady, shift of the spectrum on the plate should affect the two alike. During these exposures the middle of the slit is covered by a shutter, so that the comparison spectrum is not confused with that of the star.

The exposures on the star are long, — sometimes several hours, — and the necessary guiding is usually done by having the outer face of the slit-plate polished, so that the reflected image of the star may be seen in a suitable eyepiece and set upon the slit.

Since the prismatic spectrum is not normal (§ 561), the reduction of the measures is rather tedious, though not difficult. In the "later" spectra (F to M) complications arise from the fact that many of the stellar lines are "blends" of components usually arising from different elements and too close to be separated by the relatively low dispersion available. The mean wave-length of such a blend depends on the relative intensities of the components; this usually varies with the spectral class, and allowance has to be made for it.

731. Accuracy of the Results. For the brighter stars, spectrographs with three prisms are used, giving considerable dispersion; for the fainter ones, a single prism. Stars of about the sixth photographic magnitude may thus be reached by the 36-inch Lick refractor, with exposures of a couple of hours. With the 100-inch reflector at Mt. Wilson, and a spectrograph of small dispersion, the tenth, or even the eleventh, magnitude may be reached.

For stars with numerous good lines, and for spectra obtained with three prisms, the probable error of the radial velocity derived from a single plate is not far from ± 0.5 km./sec. This corresponds to a determination of wave-length with a probable error of one part in 600,000 — really remarkable accuracy. When the spectral lines are few and diffuse, as in many stars of classes A and B, the probable errors are much greater and sometimes rise to ± 5 or even to ± 10 km./sec.

732. Reduction to the Sun; Determination of the Solar Parallax. Spectroscopic observations, of course, give the velocity of the star relative to the terrestrial observer, and must be corrected for the effects of the earth's rotation and orbital motion before they can be compared with one another. The former correction is small, but the latter is important. Application of these gives the radial velocity of the stars *relative to the sun.*

Observations of the radial velocity of the stars may be employed (reversing this process) to determine the earth's orbital velocity and hence the solar parallax (§ 218). A careful determination of the solar parallax has been made at the Cape of Good Hope in this way, using a spectrograph of high dispersion. The mean probable error of the velocity derived from one plate, for the seven bright stars that were observed, was only ± 0.4 km./sec. The resulting value of the solar parallax, $8''.802 \pm 0''.004$, is one of the best determinations.

733. Present State of the Observations. The number of stars whose radial velocities have so far been determined is about three thousand. Stations have been established in the southern hemisphere, — notably by the Lick Observatory, — and the survey has been carried from pole to pole. Much work remains to be done on the fainter stars.

Velocities from 10 to 30 kilometers a second (comparable to those of the planets in their orbits) are very common. Those over 100 km./sec. are rather rare. The greatest value so far observed is 385 km./sec. for the variable star VX Herculis.

MOTIONS OF THE STARS IN SPACE

734. Radial, Tangential, and Space Velocities. The motions of the stars in space may now be considered. Let O (Fig. 221) be the observer's position (which on the scale of stellar distances is imperceptibly removed from the sun), S that of a star at a given date, and S' the same star's position a year later. The star's true motion (referred to the sun as a standard) is represented by the vector SS' and is called the *space velocity*, denoted here by v. This vector may, as the figure shows, be resolved into components, — SR, directed away from the sun, which is the *radial velocity* V,

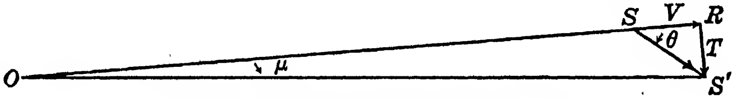


FIG. 221. Radial and Tangential Velocities

and RS', at right angles to the former, termed the *tangential velocity* T. The angle ROS', by which the direction of the star from the sun changes, is evidently the observed annual proper motion μ.

To find RS' we must know the distance OS' or the parallax p of the star. From the definition of the latter it follows that, in astronomical units, $RS' = \mu/p$, which is therefore the tangential velocity, in astronomical units per year. To reduce this to kilometers per second (the units ordinarily used) we must multiply by 4.74, which gives the equation (p. 634)

$$T = 4.74\ \mu/p \quad \text{(km./sec.)}. \tag{1}$$

The radial velocity V is derived in kilometers per second from spectroscopic observations. Finally, the space velocity v is given by

$$v^2 = V^2 + T^2. \tag{2}$$

If θ is the angle RSS', we have

$$V = v \cos\theta;\ \ T = v \sin\theta. \tag{3}$$

The space velocity, being found from the proper motion, parallax, and radial velocity, demands for its determination the combination of observations made in very different ways.

735. Track of a Star in Space relatively to the Sun. When the quantities just mentioned are known for any star, it is possible not only to calculate its space velocity but to follow out its whole past and future motion, assuming, as is legitimate in this connection, that it is moving uniformly in a straight line. The resulting equations afford so good an illustration of the fundamental formulæ of Chapter XVIII that it is worth while to work them out in full.

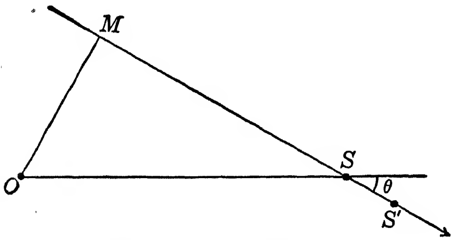


FIG. 222. Track of a Star in Space

In Fig. 222, O and S represent the sun and star, as before, and MSS' the star's rectilinear track in space. Let M be the point on this which is nearest to the sun. At what time was (or will be) the star at this point, and what were the values then of its parallax, proper motion, visual magnitude, etc.?

The angle at S is θ. Hence $MS = OS \cos\theta$; $OM = OS \sin\theta$. But, in astronomical units, $OS = 206{,}265/p$ (§ 710). If p' is the parallax corresponding to the distance OM, $OM = 206{,}265/p'$, whence

$$p' = \frac{p}{\sin\theta}. \tag{4}$$

If M is the absolute magnitude of the star (which we suppose to be unaltered), and m and m' its observed magnitudes at S and M, respectively, we have

$$M = m + 5 + 5\log p;$$
$$M = m' + 5 + 5\log p';$$

whence

$$m' = m + 5\log p - 5\log p' = m + 5\log\sin\theta. \tag{5}$$

Next, if T', V', are the tangential and radial velocities at the point M, we have evidently

$$V' = 0;\quad T' = v = 4.74\,\mu'/p';$$

whence

$$\mu' = \frac{vp'}{4.74} = \frac{1}{4.74}\frac{T}{\sin\theta}\,\frac{p}{\sin\theta} = \frac{\mu}{\sin^2\theta}. \tag{6}$$

Finally, if t' is the number of years ago at which the star was at M, we have (since the velocity along MS is $v/4.74$ astronomical units per year) $t' = 4.74\,MS/v$. But $MS = (206{,}265\cos\theta)/p$, and $v = \dfrac{T}{\sin\theta} = \dfrac{4.74\,\mu}{p\sin\theta}$.

Substituting these values, we find

$$t' = \frac{206{,}265}{\mu} \sin\theta \cos\theta = \frac{206{,}265}{\mu} \frac{VT}{V^2 + T^2}. \tag{7}$$

When the radial velocity V is negative, t' comes out negative, indicating that the closest approach is in the future.

736. Secular Changes in Proper Motion, Parallax, etc. It is evident from Fig. 222 that for a star that is receding from us the parallax and proper motion continually decrease, while the radial velocity increases numerically and the observed magnitude grows fainter. The opposite changes occur for a star that is approaching. The only one of these changes which is perceptible within the time covered by observation is the alteration of proper motion, for which the effects are cumulative with the time; and even this is barely sensible for a very few of the fastest-moving stars. It has been observed for Groombridge 1830 and 61 Cygni.

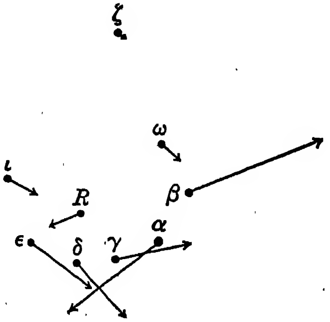


FIG. 223. Corona Borealis—Proper Motions in 108,000 Years

These are evidently at random

737. Moving Clusters. The proper motions of stars belonging to the same constellation are usually so different in amount and direction as to show at once that there can be no real connection between them (Fig. 223, Corona Borealis). But in certain cases the motions of a group of stars are strikingly similar, as in the Dipper in Ursa Major (Fig. 224), and suggest at once that the stars in the group are actually neighbors in space and moving together like a flock of birds. The radial velocities of such stars are also found to be almost the same, which puts the reality of their common motion beyond doubt. Their spectra are also generally (but not always) similar; and, when unlike, show certain characteristic relations to their brightness (§ 799).

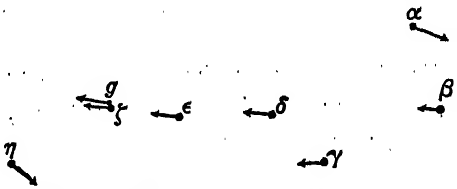


FIG. 224. Ursa Major—Proper Motions in 50,000 Years

All are nearly equal and parallel, except those of α and η

Such community of motion is almost always found among the members of the star clusters, such as the Pleiades, which occur here and there in the sky. The few stars which do not really belong to the cluster, but lie in the same line of sight, before it or behind it, can be picked out at once by their discordant motions.

Convergent points. When the cluster covers a considerable area in the heavens, it is found that the proper motions of the stars are not exactly parallel, but *converge* toward a definite point, — as was first found by Boss for the Hyades cluster in Taurus

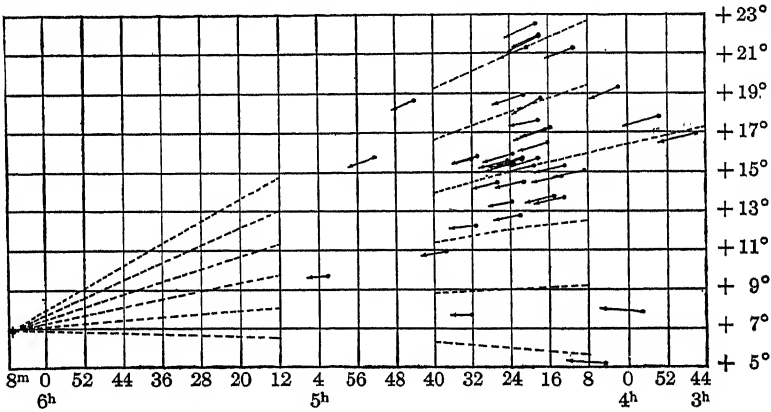


FIG. 225. Convergent of the Taurus Cluster

The arrows represent the motions of the stars in an interval of 50,000 years. The convergence of the motion, in the directions indicated by the dotted lines, and the more rapid apparent motion for the stars farther from the convergent point, are obvious. (From a diagram by Lewis Boss)

(Fig. 225), — or perhaps *diverge* from one. This is evidently an effect of perspective, like the divergence of meteor tracks from a common radiant (§ 534). When the motions converge, the stars must then be receding, and their radial velocities are actually found to be positive; when they diverge, the radial velocities are negative.

738. Distances of Moving Clusters. *The Taurus cluster.* In such cases the parallax and distance of every star in the cluster can be found with great accuracy if the radial velocities of one or more stars are known. The line from the sun to the point on the celestial sphere toward which the apparent motions converge must evidently be parallel to the direction of motion of the

cluster relative to the sun, and the angular distance in the heavens between any star and this point is equal to the angle θ which appears in the equations of § 734 (see Fig. 226).

These equations may be rewritten

$$v = \frac{V}{\cos\theta}; \tag{8}$$

$$p = \frac{4.74\,\mu}{v \sin\theta}. \tag{9}$$

When the radial velocities of several cluster stars are known, it is best to take the mean of the values of v given from these by (8). The parallax of each star in the cluster may then be found by (9). For example, for δ Tauri in the Hyades, $\mu = 0''.115$, $V = +38.6$ km./sec., and $\theta = 29°.1$, which gives $v = 44.0$ km./sec., $p = 0''.025$.

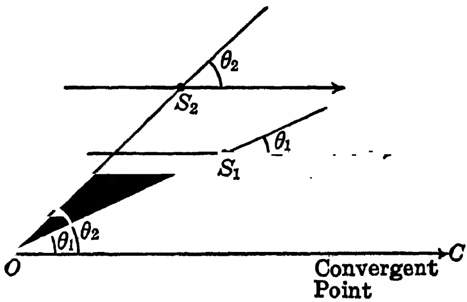


FIG. 226. Distances of Stars in a Moving Cluster Found from their Radial Velocities

The convergent point, C, is directly observed; so that the angle in the heavens, COS, between it and a given star may readily be found. The angle is the θ of equations (8) and (9)

About eighty members of this Taurus cluster are known, ranging in brightness from the fourth to the tenth magnitude. When their positions in space are worked out, it is found that they form a nearly spherical cluster, with its center at a distance of about 42 parsecs (135 light-years). Most of the stars lie within 5 parsecs from the center, but the stragglers extend twice as far.

739. Other Notable Moving Clusters. *The Ursa Major group* comprises the five Dipper stars already mentioned (β, γ, δ, ϵ, ζ Ursæ Majoris), which are from 70 to 80 light-years distant, and a number of others in different parts of the sky, including Sirius. The region occupied by this cluster (whose stars are sparsely scattered, nearly in one plane) includes the sun, so that its members appear all over the sky and some are much nearer to us than others.

The Scorpius-Centaurus group is a huge aggregation of stars, mainly of spectral class B, in the constellations named and in the Southern Cross; it was discovered by Kapteyn (Fig. 227). These stars have small proper motions, usually less than 0″.05 per year, and are very remote. The mean parallax of the bright stars of the group, in Scorpius, is about 0″.009, and of those in the Southern Cross 0″.015. Many of the brightest stars belong to this group, — for example, β Centauri, Antares, and Spica.

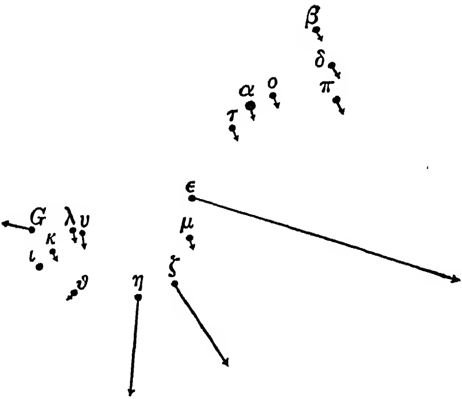


FIG. 227. Proper Motions of Stars in the Constellation Scorpius

The arrows indicate the motion in 108,000 years. Most of the stars, including Antares (α), are evidently moving together. They form a part of Kapteyn's great group

The Perseus group is a somewhat similar, but smaller, assemblage in Perseus, with a proper motion of about 0″.04 and a parallax not far from 0″.01, which is harder to find accurately than in the previous cases, since the convergent point is poorly determined.

Most of the bright stars in Orion probably form another cluster; but this group is moving straight away from us, the radial velocities being about + 20 km./sec. and the proper motions insensible, so that it is not easy to get good values of the parallaxes. Kapteyn has had good success, and finds a mean parallax of 0″.006 and a distance of about 500 light-years.

Other groups. There are many other clusters (for example, the Pleiades, the group in Coma Berenices, and the Præsepe cluster in Cancer) which are known to be moving; but these occupy so small an area in the heavens that the convergent point cannot be found with any certainty. Fortunately a good idea of the distances of these groups can be obtained by other methods (§ 863). The great majority of the stars, however, do not belong to such clusters, but are apparently moving independently.

It has sometimes been suggested that there may be a "solar cluster" of stars, moving in space at the same rate and in the same direction as the sun. If so, the stars of this cluster should have large parallaxes but no sensible proper motion or radial velocity. No reliable evidence of the existence of such a cluster has been found.

740. The Sun's Motion among the Stars; Definitions. So far we have taken the sun as our origin of reference and treated it as though at rest in space. In all probability, however, the sun is actually in motion like the other stars. Such a motion can only be defined as motion relative to something else (§ 360), and the obvious standard to choose is the *general average of the stars themselves*. Such a standard is, of course, not absolute. The average motion of one group of stars (say those visible to the naked eye), and of another group differently chosen, need not be the same, and indeed often is not; but when once the standard is chosen the problem is definite.

The point of the celestial sphere toward which the sun (referred to such a standard) is moving is called the *solar apex*; the opposite point, the *antapex*. A statement of the apex and velocity of the solar motion suffices to define it.

Each star, referred to the new standard, has a motion of its own, which is commonly called its *peculiar motion*. The observed proper motion and radial velocity result from the combination of this peculiar motion and the solar motion, and the practical problem is to disentangle the two.

741. Solar Motion from Proper Motions. How this is done in the case of proper motions can best be illustrated by an example. Suppose that we were on the fighting top of a battleship at night, on a sea where nothing was visible except the lights of other vessels, which were moving in all directions and at different speeds. If these motions were *at random*, then if our ship were at rest we should see the same number of lights moving toward the right and toward the left, no matter in what direction we looked. If our ship were moving, this would still be true if we looked dead ahead or astern, but not if we looked in other directions. Looking abeam to the starboard, the lights of all ships which were going in the opposite direction, and also of those which

were going the same way but more slowly, would appear to drift astern, that is, to the right, while only the lights which were overtaking us would move to the left. On the port side of our course the preponderance would be to the left, away from the bow in both cases, and the disparity in numbers would evidently be greatest at right angles to the course. To find out in what direction we were moving we should therefore have only to count, for each point of the compass, the number of lights that were moving to the right and to the left. As for our speed, we obviously could not in this way find it in miles per hour; but we could tell whether we were on a fast ship or a slow one, compared with the others, for on a fast ship very few vessels would overtake us, and almost all the lights abeam would be drifting toward the stern, while on a slow ship the disproportion obviously would be much less.

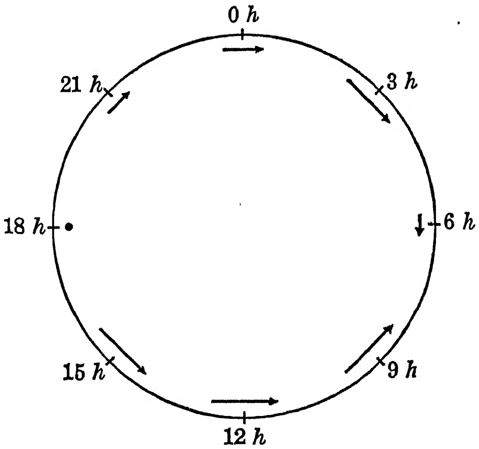


FIG. 228. Distribution of Proper Motions of Stars in Right Ascension, indicating Sun's Motion

This illustration may be extended with profit. The range in speed from destroyers to barges roughly represents that of peculiar velocity among the stars; that of brightness between searchlights and lanterns hardly equals that in stellar absolute magnitude; a group of vessels in a convoy or tow represents a moving cluster; and so on.

The actual problem of the solar motion differs only in being in three dimensions, and not in one plane. In the vicinity of the apex or antapex the stars should appear to move at random in all directions in the sky. In other regions there should be an average apparent drift toward the antapex, the majority of the motions, though not all, being in this general direction.

742. Numerical Example. A very good determination of the solar apex may easily be made by applying the principle just explained. A moment's consideration shows that motion of the sun toward or from the celestial pole can cause only an apparent southward or northward drift of the stars, — that it will affect the proper motions in declination but not those in right ascension. Only the latter can tell us the right ascension of the apex, and in studying these we may use the simple two-dimensional method just explained.

Taking Boss's great *Preliminary General Catalogue*, selecting the two hundred stars which lie nearest to the right ascensions 0^h, 3^h, etc., and simply counting the *number* for which the proper motions in right ascension are positive and negative (dividing zero values equally between the two), we find

R.A.	0^h	3^h	6^h	9^h	12^h	15^h	18^h	21^h
Positive	132	152	107	39	46	47	101	143
Negative	68	48	93	161	154	153	99	57

These results are plotted in Fig. 228, in which the arrows indicate the favored direction of motion, and their lengths the degree of preponderance. The points at which the number of positive and negative proper motions are equal appear to be about $6^h\ 20^m$ and $18^h\ 0^m$ (not exactly opposite, owing to the errors of "random sampling"). The latter is obviously the apex, and its right ascension is thus fixed at $18^h\ 10^m$ (taking the average).

To find the declination of the apex, we may now take the stars in a narrow strip of the sphere, between $17^h\ 30^m$ and $18^h\ 30^m$ R.A., and on the other side between $5^h\ 30^m$ and $6^h\ 30^m$, and consider their motions in declination only. We then find numbers

Declination	90° to 60°	60° to 30°	30° to 0°	0° to − 30°	− 30° to − 60°	− 60° to − 90°
R.A. near 6^h						
Positive	3	9	22	29	31	16
Negative	10	35	72	40	32	3
R.A. near 18^h						
Positive	10	25	22	8	6	4
Negative	7	13	38	59	46	9

Plotting these as before, we obtain Fig. 229. The declination of the apex is now indicated as about + 35°, and that of the antapex as − 40°. Taking the mean, we find + 37° for the apex.

Boss, from a thorough study of the proper motions, both in right ascension and in declination, of 5972 stars distributed all over the heavens, finds for the position of the apex $18^h\ 2^m \pm 5^m$, $+34^\circ.5 \pm 1^\circ.1$. The probable errors represent the uncertainties due to the random motions of the stars, which are not completely eliminated even in so large a sample. The value obtained by our rough process, in which only a small portion of the stars are used, agrees surprisingly well.

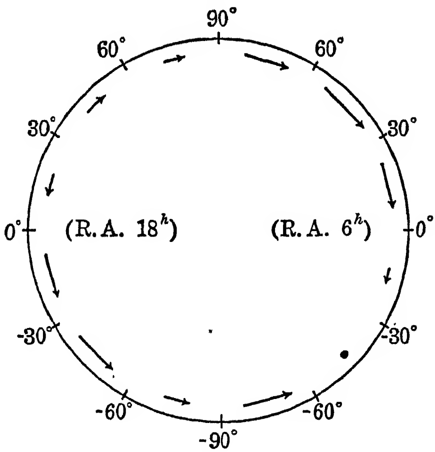


FIG. 229. Distribution of Proper Motion of Stars in Declination, indicating Sun's Motion

743. Solar Motion from Radial Velocities; Numerical Example. This is really much easier to determine. In the region of the apex the stars should, on the average, be approaching the sun; near the antapex, receding; halfway between, neither one nor the other. By taking the mean (with regard to signs) of the radial velocities in a given (not too large) region of the sky, the individual peculiar motions nearly cancel out, since they are as likely to be positive as negative, leaving the effect of the solar motion almost unaffected. This method gives not only the direction of the sun's motion but also its velocity in kilometers per second.

As a simple example of this method, Campbell's catalogue of radial velocities is found to contain 32 stars within a square 40° on a side, with center at the vernal equinox. The radial velocities for these stars range from + 45 to − 27 km./sec., but the algebraic mean is + 4.4 km. Treating regions near five other points similarly, we find

Right ascension	0^h	12^h	6^h	18^h	—	—
Declination	0°	0°	0°	0°	+ 90°	− 90°
Number of stars	32	27	33	26	24	43
Mean radial velocity	+ 4.4	+ 0.8	+ 23.7	− 15.1	− 8.2	+ 9.3

The last two columns indicate that the sun, compared with these stars, is approaching the north pole at the rate of 8.2 km./sec. and receding from the south pole at 9.3 km./sec. Taking the average, we have a motion of 8.7 km./sec. toward the north pole, and, similarly, of 19.4 km./sec. toward the point on the equator at 18^h, and 1.8 km./sec. toward one on the equator at 12^h. The combination of these three velocity components in the usual way gives a motion of 21.3 km./sec. toward R.A. $17^h\ 40^m$, Decl. $+24°$.

Campbell and Moore (1925), from a thorough solution including 2034 stars, find 19.0 km./sec. toward $17^h\ 54^m$, $+27°.2$. Thus the rough process that we have used gives again a surprisingly good result.

744. Difference of the Results. The right ascensions found by Boss and Campbell agree fairly well, but the declinations differ far more than can be attributed to errors of sampling, with such large groups. Some real influence must be at work.

It is now almost certain that errors in the proper motions are mainly to blame. If the older determinations of declination (made a century or more ago) were subject to errors which, on the average, made the stars come out too far to the north, and from which modern work is free, it is clear that a spurious southward motion of the stars will be added to the real motions in declination. Substantially the same effect would be produced by a northward motion of the sun relative to the stars, and hence this motion, when calculated from the faulty observations, would come out too great, and the computed apex would be too far north.

Various investigators have found strong evidence that Boss's proper motions in declination need correction by an average amount of $0''.004$ or thereabouts. Wilson (1925) finds that when this correction is applied, the proper motions give the declination of the apex as $+28°$, in close agreement with the radial velocities.

His latest determination, R.A. $18^h\ 0^m$, Decl. $+28°$, with Campbell's velocity of 19.0 km./sec., or 4.0 astronomical units per year, may be adopted as the best values now obtainable (1925) for the sun's motion relative to the naked-eye stars as a whole. It should be borne in mind that, with respect to groups of stars chosen in other ways, both the direction and the velocity of the solar motion may be, and sometimes actually are, very different (§ 759).

745. The peculiar motion of a star may be found by calculation when the solar motion has been determined. If V_0 is the velocity of the latter in kilometers per second, and λ the angular distance of the star from the apex, the sun's motion evidently gives to the star the radial velocity $-V_0 \cos \lambda$, and the tangential velocity $V_0 \sin \lambda$ in the direction away from the apex.

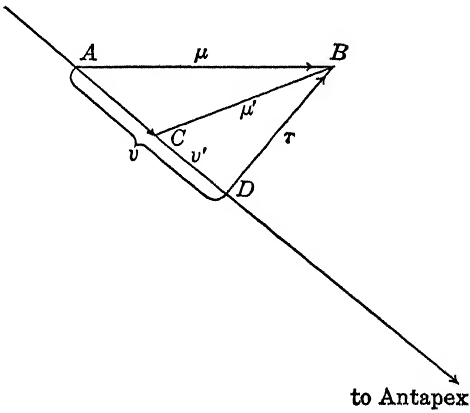


FIG. 230. Components of Proper Motion with Respect to Sun's Motion

Correction for the first is very simple; if V' is the peculiar radial velocity, we have simply

$$V' = V + V_0 \cos \lambda. \qquad (10)$$

The effects on proper motion are more complicated, for the peculiar motion of the star usually makes an angle with the drift introduced by the solar motion.

If AB (Fig. 230) is the observed proper motion, and AC the drift due to the sun's motion, the peculiar motion is represented by CB.

The simplest way of handling the matter is to resolve the observed proper motion into two components, — AD in the direction of the drift, and BD at right angles to it. The former, following Kapteyn, is called υ (the upsilon component) and is counted positive *toward* the apex, while the latter is denoted by τ (tau). The tau component of the motion is evidently unaffected by the solar motion. The upsilon component is diminished algebraically (increased numerically in the figure because it is negative) by the full effect of the solar motion, which corresponds to the tangential velocity $V_0 \sin \lambda$ and is therefore $pV_0 \sin \lambda / 4.74$. If υ' and τ' denote the components of the peculiar velocity, we have therefore

$$\tau' = \tau; \qquad (11)$$

$$\upsilon' = \upsilon + \frac{pV_0 \sin \lambda}{4.74} \qquad (12)$$

Since the parallax appears in the last equation, the peculiar motions cannot be fully computed unless this is known. This might appear to be a disadvantage, but it is in fact a great advantage, for it leads to much information concerning the distances of the stars which would otherwise be unobtainable.

746. Determination of Mean Parallax from Parallactic Motion. Equation (12) may be written

$$p = \frac{4.74(\upsilon' - \upsilon)}{V_0 \sin \lambda}.$$

Here υ, V_0, and λ are known quantities. If only we knew υ', we could find the parallax. This is impossible in individual cases; but if we take the average for a large number of stars, the peculiar motions, υ', which are just as likely to be positive as negative, will tend to average out, and we obtain a good value of the mean parallax. Denoting, as is customary, algebraic means by bars placed above the corresponding symbols, we have

$$\bar{p} = -\frac{4.74\ \bar{\upsilon}}{V_0 \sin \lambda}.$$

It may be proved that, when dealing with stars at different distances from the apex, this equation should be modified to

$$\bar{p} = -\frac{4.74\ \overline{\upsilon \sin \lambda}}{V_0\ \overline{\sin^2 \lambda}}. \tag{13}$$

This equation gives the *mean parallax* of the stars. Its accuracy depends on how well the peculiar motions, υ', cancel one another in taking the average. If they are small in comparison with the parallactic motion, fairly good values can be derived from twenty or more stars; but if they are large, a great number of stars may be needed to obtain tolerable accuracy.

It should be especially noticed that this formula cannot legitimately be applied when the group of stars has been selected on account of proper motion — directly or indirectly. For example, if we pick out stars of small proper motion, those cases are evidently favored in which AC and CD (Fig. 230) are oppositely directed, — that is, cases in which υ' is positive; and the assumption that the peculiar motions will cancel out is no longer tenable. But selection on almost any other basis — magnitude, spectral type, double or variable stars, etc. — is permissible, so that the method is of great value.

The apparent drift due to the solar motion is often called the parallactic motion. Its value at 90° from the apex is sometimes termed the "secular parallax." For a group of stars the latter quantity is $-\overline{\upsilon \sin \lambda}/\overline{\sin^2 \lambda}$. With the solar motion adopted above, the secular parallax is 4.1 times the actual mean parallax.

747. Mean Parallax from τ Components and Radial Velocities. Another quite independent method of finding the mean parallax depends upon the assumption that the average peculiar motions of the stars in the radial direction, and in the direction corresponding to the τ components of proper motion (which is always perpendicular to the first), are the same. This would be true in the case of random motion and is probably nearly true for any group of stars that are distributed fairly uniformly over the sky (that is, at random directions from the sun).

Here we must take *arithmetical means*, disregarding sign; and our formula is

$$\bar{p} = \frac{4.74\,\bar{\tau}}{\overline{V'}}. \tag{14}$$

The accuracy of this equation depends upon the degree to which the quantities τ and V' may be trusted to "average up" to the same value in successive groups. This depends on the number in the group, but not on the size of the peculiar velocities (so long as these are considerably larger than the errors of observation). Fifty stars should give a probable error of about 10 per cent. It can be shown that if the average peculiar motion V' is less than $0.7\,V_0$, the method of section 746 is likely to be more accurate than the present method. Otherwise the situation is reversed.

748. Mean Parallax and Mean Distance. These two methods, with the limitations stated, give good mean parallaxes for sufficiently large groups of stars. It must be remembered, however, that the distance corresponding to this parallax will always be less than the mean of the actual distances of the stars. Taking, as an example, two stars with parallaxes 0″.1 and 0″.01 and distances of 10 and 100 parsecs, the mean parallax is 0″.055, corresponding to a distance of 18 parsecs, while the mean of the distances is 55 parsecs. The magnitude of this difference decreases with the dispersion of the parallaxes, that is, with the average difference of individual values from the mean.

Similarly, the mean absolute magnitude of a group of stars is brighter than that computed from the mean apparent magnitude and the mean parallax. (The geometrical mean of the parallaxes would give the correct result.)

749. Mean Parallaxes of Stars of Different Magnitudes. Information of very great importance can be obtained by the methods just described. For example, the mean parallaxes of the stars of

different visual magnitudes may be determined. The most recent results (due to Seares), which have been "smoothed" to remove the remaining accidental errors, are as follows:

TABLE XVIII. MEAN PARALLAXES OF STARS OF DIFFERENT MAGNITUDES

MAGNITUDE	MEAN PARALLAX	m	$\bar{p}$	m	$\bar{p}$
1	0″.083	6	0″.0120	11	0″.0018
2	0 .056	7	0 .0082	12	0 .0013
3	0 .038	8	0 .0056	13	0 .0009
4	0 .026	9	0 .0039		
5	0 .018	10	0 .0027		

These are average values for the whole sky. For stars in the Milky Way the mean parallaxes are about 12 per cent smaller; for those near the galactic pole, about 35 per cent greater. The dispersion of the individual parallaxes about their mean is very large, so that the mean distance of the stars of a given magnitude is much greater than that corresponding to the mean parallax.

One important use of this table is in the reduction of differential observations of stellar parallax (§ 711) to absolute values. The comparison stars are usually of the ninth magnitude or fainter, so that even if the average parallax of the few which were used should be 20 per cent greater or less than the mean for all the stars of the same magnitude, the error would be negligible.

750. Mean Parallax of Stars of Different Spectral Classes. If the naked-eye stars in Boss's *Catalogue* are divided according to their spectra, noteworthy differences in their proper motions and parallaxes appear. For the stars in Boss's *Catalogue* (brighter than $6^m.5$ and averaging about $5^m.7$) which have proper motions less than $0''.17$ per year (after approximate correction for the effect of the sun's motion in space) he finds the mean proper motions and parallaxes given in Table XIX.

The second and third columns give the mean values of the total proper motions and of the τ components, and the fourth those of the peculiar radial velocities V' as found by Campbell for about 1200 stars. The mean parallax calculated from the υ components of proper motion by equation (13), taking the solar motion as 19.5 km./sec., is given in the fifth column, and that found from the τ components by equation (14), in the sixth. The two independent determinations agree closely.

TABLE XIX. MEAN PARALLAXES OF THE REMOTER NAKED-EYE STARS

Sp.	Mean μ	Mean τ	Mean V' Km./Sec.	Mean Parallax		Number of Stars	
				From v	From τ	Included	Rejected
B	0″.024	0″.0086	6.2	0″.0066	0″.0066	490	1
A	0 .046	0 .0217	10.5	0 .0100	0 .0094	1647	47
F	0 .077	0 .0403	14.4	0 .0121	0 .0133	656	259
G	0 .052	0 .0289	15.9	0 .0076	0 .0086	444	113
K	0 .057	0 .0304	16.8	0 .0098	0 .0086	1227	183
M	0 .050	0 .0282	17.1	0 .0080	0 .0078	222	15

From these data the important conclusion follows that the large majority of the stars visible to the naked eye are very remote, lying at distances of 100 parsecs or more.

It must be remembered, however, that all the stars with proper motions greater than 0″.17 per year were excluded from the groups under consideration. The numbers of stars of each spectral class which were included and excluded are given in the last two columns of Table XIX. The excluded stars, of large proper motion, have a much greater mean parallax than those included in the table. For 559 such stars with proper motions between 0″.17 and 0″.80 Boss finds a mean parallax of 0″.053; and that of the stars with proper motions exceeding 0″.8 must be greater still. If *all* the stars were included in the mean for each spectral class, the mean parallax for Class B would be practically unaltered. Those for classes A and M would be increased slightly, and for F, G, and K very considerably, and the general mean would be brought into agreement with Table XVIII.

751. Absolute Magnitudes of the Brighter Stars. A very important deduction from this is that the great majority of the stars visible to the naked eye are very much brighter than the sun.

The mean visual magnitude of the stars in Table XIX is 5.5. If this is adopted for each spectral class separately, which is doubtless roughly correct, the corresponding absolute magnitudes come out $-0^m.4$ for Class B, $+0.4$ for Class A, $+1.0$ for Class F, $+0.0$ for Class G, $+0.3$ for Class K, and 0.0 for Class M. These values represent typical stars having visual magnitudes and parallaxes equal to the mean value for the respective group.

The mean of the individual absolute magnitudes, if these could be separately determined, would be considerably brighter.

The stars of spectrum B0 — B2, in particular, are much more luminous, their average absolute magnitude (found by the same method) being about -3. The stars of the less common spectral classes all have small proper motions and are of high luminosity. For 29 stars of Class R, Sanford finds a mean peculiar radial velocity of 73 km./sec. (or 21 km./sec. after excluding three very large velocities). The mean parallax computed by combining these data with the proper motions determined by Wilson for 18 stars is $0''.0011$, which, since the mean apparent magnitude is 8.4, leads to a mean absolute magnitude about -1.5.

For Class N, Moore finds a mean peculiar radial velocity of 18 km./sec. from 25 stars, and with Wilson's proper motions a mean parallax of $0''.0031$ for 19 stars of mean apparent magnitude 6.1, giving again a mean absolute magnitude of about -1.5. These stars showing carbon bands therefore average some 300 times as bright as the sun.

The stars of Class O are still brighter. For 42 stars Plaskett finds peculiar radial velocities averaging 25 km./sec., but the proper motions are very small, those of 26 stars brighter than the seventh magnitude averaging only $0''.012$ per year. The resulting mean parallax is only $0''.0011$, and the mean absolute magnitude -4.3, corresponding to a real brightness 4500 times that of the sun and far greater than that of any other spectral class.

The stars of Class S are almost all variable in brightness. Merrill and Strömberg find a mean peculiar radial velocity of 22 km./sec. and a mean absolute magnitude at maximum of $+0.4$.

752. Mean Parallax of Stars of Given Magnitude and Proper Motion. Kapteyn, supplementing this method of study by others, has found that the mean parallaxes of stars of given magnitude and proper motion can be closely represented by the formula

$$\log \bar{p} = -0.690 - 0.0713\, m + 0.645 \log \mu. \tag{15}$$

Comparison with directly measured parallaxes shows that the "probable error of prediction" by this formula is about 40 per cent. For individual stars the formula is often seriously wrong, but it gives a good average value for a group, unless the group has been selected in some such way as to favor the nearer or remoter stars, which is only too often the case. For typical groups the mean distance is 40 per cent greater than that corresponding to the mean parallax (§ 748).

This equation is something more than a convenient empirical formula. As Schwarzschild showed, it can be derived from simple and plausible assumptions regarding the distribution of the stars in space, and the relative numbers which have various absolute magnitudes and velocities.

If a quantity H is introduced, defined by the equation $H = m + 5 \log \mu$, and M is the absolute magnitude corresponding to the parallax computed by Kapteyn's formula for any stars, then $M = 0.645\, H + 1.55$, as is easily shown.

From the definition it follows that

$$H = M - 5 + 5 \log \left(\frac{\mu}{p}\right) = M + 5 \log T - 8.38, \tag{16}$$

where T is the tangential velocity in kilometers per second.

A grouping of stars according to the value of H therefore puts bright, slow-moving stars at one end, and faint, fast-moving ones at the other. Such groupings are valuable in various statistical studies.

More complicated formulæ and tables for the mean parallax of stars of different magnitudes, proper motions, spectral classes, and distances from the Milky Way have been given by van Rhijn (Groningen Publications, No. 34, 1923).

753. Peculiar Motion and Absolute Magnitude. The peculiar motions of the fainter stars are more rapid than those of the brighter. On the average the mean peculiar radial velocity increases by about 7 per cent for stars a magnitude fainter in absolute brightness, and is approximately doubled for a change in M of ten magnitudes. The exact amount of this increase is hard to determine, as it is influenced in a peculiarly intricate manner by the effects of selection. The differences in peculiar velocity between the various spectral classes remain of almost the same size when this is allowed for.

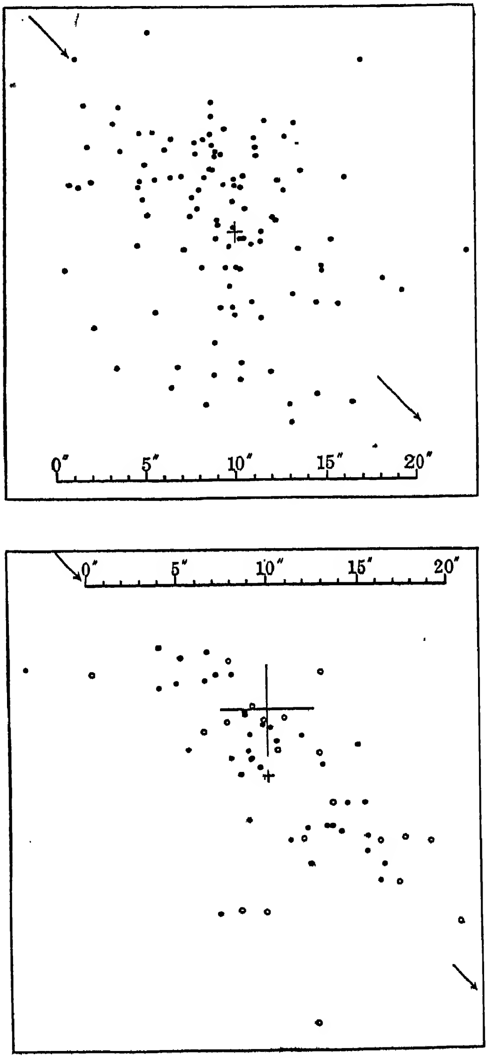
754. Apparent Recession of the Stars: the K Term. The mean peculiar radial velocity of the stars (taking account of sign) does not vanish (as it ought to do if the motions were at random) but is definitely positive. This fact was discovered by Campbell, and the resulting mean velocity is commonly called the K term, as he denoted it by this letter in his equations.

The K term varies with the spectral class, being large (about + 4 km./sec.) for Class B, small for classes A, F, and G, + 2 km./sec. for Class K, and + 4 km./sec. for Class M, according to Campbell. It might be interpreted as due to a general motion of recession of the stars, indicating a tendency of the stellar system to expand; but this is highly improbable. Small errors in the laboratory wave-lengths are more probably the cause. Albrecht has shown that when better values for these wave-lengths are used, the K term for the B stars is much diminished. The question is, however, not fully cleared up.

755. The Preferential Motions of the Stars; Star Streaming. Kapteyn, in 1905, was the first to discover that the peculiar motions of the stars were not at random. If they were so, then, in a large sample, as many would be moving in any one direction as in any other, and with the same average velocity. What actually

happens can best be shown by the use of diagrams of a type first published by Boss.

Suppose that all the stars in a given region of the sky could be collected at one point of that region and then allowed to move off over the heavens, each in the direction and at the rate corresponding to its own proper motion. After a century they would have spread out into a cluster, as seen in the sky, with the faster-moving stars at the edges and the slow-moving ones near the middle. Though this experiment cannot be performed, we can make a diagram showing just what such a cluster would look like, by choosing a convenient origin and laying off for each star a distance and direction equal to its centennial proper motion.



Figs. 231 and 232. Star Streaming

The arrows point toward the vertex (§ 756). (From diagrams by Benjamin Boss)

The points so obtained will represent, not the positions of the stars, but their motions, and may therefore be called velocity points. Such diagrams are shown in Fig. 231 and Fig. 232 (from Boss's papers). The origin of coördinates cor

responds to a star with no proper motion (that is, to one moving in space at the same rate and direction as the sun), and may therefore be taken as the sun's "velocity point."

If the peculiar motions of the stars were at random, the velocity points for the stars of given parallax would form a round cluster, — denser near the middle, because small velocities would be more common than large ones, and with its center at a distance from the origin equal to the parallactic motion of the group. For stars of different parallax the point clusters would also be round, but with their centers at different distances from the origin. Combining all, we should get a roundish diagram, somewhat elongated in the direction of the parallactic motion.

In some regions this is actually found (Fig. 231), but in others (Fig. 232) the cluster of velocity points is strongly elongated along a line quite different from that of the parallactic motion, which indicates that the peculiar velocities of those stars which are moving nearly parallel to this line are greater, on the average, than those of stars which are moving nearly perpendicular to it.

756. Nature of the Preferential Motion. This *preferential motion* should be carefully distinguished from the common proper motion of the stars of a moving cluster. In the latter case the proper motions of the cluster stars are almost identical in amount and direction, and their velocity points fall close together, forming a conspicuous close-packed group on the general diagram. In the former the motions of the individual stars are still independent and may be in any direction, but the average component of motion in either direction parallel to a certain line is greater than that at right angles to it. There are more stars whose directions of motion are within 10° of the preferential line than within 10° of the line perpendicular to it, and the average proper motion is greater in the first case than in the second; but the numbers moving in the two opposite directions along the preferential line are nearly equal, as are the mean proper motions.

When these preferential lines are determined for different regions in the sky and plotted on a celestial globe, it is found that they all pass close to two antipodal points of the celestial sphere, which are called the *vertices* of preferential motion. According to concordant determinations by several investigators they are in

$6^h\,15^m$, $+13°$ (in Orion) and $18^h\,15^m$, $-13°$ (in Scutum), both close to the central line of the Milky Way.

This evidently means that the real direction of preferential motion is always the same in space, — along the line joining the vertices. Velocities along this line average greater than in any of the directions at right angles to it, while the velocity differences along the latter directions are small. If this is true, the proper-motion velocity diagrams should be nearly round for regions of the sky near the vertices (where the preferential motion is in the radial direction), and increasingly elongated with increasing distance from the vertices, to a maximum at 90°; and this is actually the case. The average peculiar radial velocities, however, should be greatest near the vertices and lowest 90° away from them, which is also found to be true. The position of the vertices may be determined from radial velocities and agrees closely with that found from proper motions.

The ellipsoidal and two-stream hypotheses. The *fact* of preferential motion is therefore that the velocity-point diagram for the stars, if made in three dimensions, using the space velocities, in kilometers per second, is not globular, as it should be if the stars were moving at random, but *elongated* in the direction of the vertices. The mathematical *description* of this fact may be made in different ways. An elongated point-cluster much like the actual one may be constructed by taking a globular cluster and stretching it in the direction of the vertices by increasing in a fixed ratio the distances of all its points from the equatorial plane perpendicular to this direction, thus transforming the cluster into an ellipsoidal form.

Or we may suppose two globular clusters to be superposed, with their centers separated by but a fraction of their diameters, and the points of one intermingling freely with those of the other, and thus build up an elongated cluster very similar to the first. The latter scheme corresponds to the "two-stream" hypothesis of Kapteyn, the former to the "ellipsoidal" hypothesis of Schwarzschild. Both are primarily mathematical devices for the convenient description of the facts. From this standpoint each has its advantages: the two-stream method makes it possible to construct a cluster which is denser at one end than at the other; and the ellipsoidal, one whose cross section at right angles to the line of vertices is flattened and not round.

The latter property appears to be very often shown by the actual velocity distribution, and the ellipsoidal form of description is now widely adopted. The average velocity is greatest in the direction of the vertices, which lie in the plane of the Milky Way, smaller in the perpendicular direction in this plane and usually smaller still in the direction of the galactic poles.

757. Preferential Motion for Different Groups of Stars. Boss found that the preferential motion is almost absent in the stars of Class B, strong in those of Class A, and rather less conspicuous in the later spectral classes, where the peculiar motions are greater in all directions. The direction of the observed vertices is nearly the same in all cases.

The latest study, by Strömberg (1925), indicates that this difference is probably related to absolute magnitude. The fainter and faster-moving stars (in the sense of real, not apparent, brightness and motion) show a very strong preferential motion, the average velocity in the direction of the vertices being nearly twice the mean of the velocities at right angles to this direction. For the brighter and slower-moving stars the difference diminishes, till, for the B stars, it is actually reversed, the motions in the directions of the vertices being slightly less.

758. Explanation of the Preferential Motion. The question what forces could produce this preferential motion appeared at first very puzzling, but it was found that a sufficient and very probable explanation could be found in the gravitational attraction of the whole assemblage of the stars.

The stars visible with small telescopes form, as we shall see later, a huge flattened cluster many thousands of light-years in diameter. It is natural to think that this great cluster is in what is called a "steady state," that is, that the motions of the stars, under the attraction of the whole mass, are such as to preserve its general size and shape, individual stars moving freely from one part of the swarm to another, and other stars coming in to take their places. It can be shown that in such a case the stars would exhibit a preferential motion of very much the observed type, with the directions of greater average velocity coinciding with the two opposite directions in which a star following a circular orbit about the center of the cluster would move.

Kapteyn has shown that this explanation is qualitatively as well as quantitatively satisfactory. Determining the size and shape of the huge cluster, or "universe," of stars from independent evidence, he found that both the calculated average peculiar motion of the stars and the computed amount and direction of the preferential motion were in good agreement with observations.

Other solutions appear to be mathematically possible, and it is of course by no means certain that our "universe" of stars is in a steady state. In fact, the differences in motion found for stars of different absolute magnitude or spectral type can best be accounted for by supposing that some of these belong (in part at least) to subordinate systems within the greater one. But the possibility of a dynamical explanation of "star streaming" in terms of motion under gravitational attraction is practically beyond doubt.

759. The Asymmetry of Stellar Motions. Further investigation has revealed a still more remarkable peculiarity in the motions of the stars. This was first recognized by Boss among stars of high velocity (larger than 100 km./sec.). With reference to the general average of the naked-eye stars these stars do not move in equal numbers in opposite directions but are nearly all moving toward points in one half of the celestial sphere. If the velocity points for these stars are plotted, they form a diagram showing the familiar elongation in the direction of the vertices; but the center of this point cluster differs widely from that corresponding to the slower-moving stars. That is, these high-velocity stars are moving, as a group, relative to the slow-moving stars, while, with reference to their own average motion, their velocities are distributed ellipsoidally.

Strömberg, from a study of all the available radial velocities (1925), finds that this is but one instance of a general relation. Objects with high velocities can be selected in various ways; certain star clusters, nebulæ, variable stars, etc. are found to possess them; and among ordinary stars those of high velocity can be picked out by means of the large values of H (§ 752). However such a group is selected, it is found that the group as a whole is moving relatively to the general average of the naked-eye stars, and always in substantially the same direction (toward $20^h\ 40^m$ R. A. and $+57°$ declination, or galactic longitude $61°$ and galactic latitude $+9°$, in the constellation Cepheus). The greater the dispersion of velocities within a group, about their own mean, the greater is the systematic motion of the group as a whole.

This is illustrated in Fig. 233, in which the ellipses represent, by their major and minor axes, the average dispersion of velocities

within each group in the direction of the vertices and at right angles to it, while their centers represent the average motions of the groups. The groups plotted in the diagram represent

(*a*) Stars of types F–M, with small H.
(*b*) Stars of Class O.
(*c*) Variable stars of long period.
(*d*) Stars of Class F, with very large H.
(*e*) Variable stars with periods between 150 and 210 days.
(*f*) Globular star clusters.

Many more might be added, but they would confuse the figure.

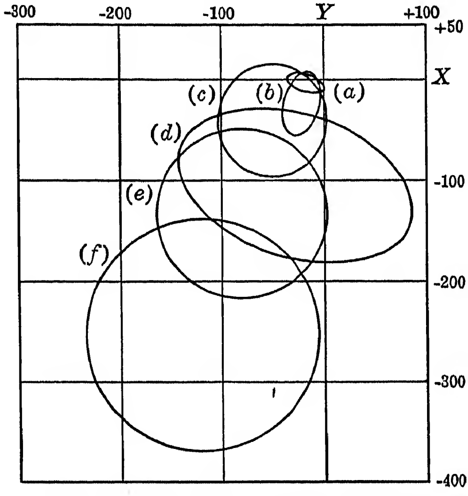


Fig. 233. Asymmetry of Stellar Motions
From diagram by G. Strömberg

The intersection of the lines X and Y represents the sun's velocity point. It is evident that the sun's motion relative to a group of stars (which is represented in the figure by the line joining this intersection to the center of one of the ellipses) differs enormously, according to the group of stars that we are dealing with. For the stars with smallest velocity dispersion it is about 15 km./sec. toward $17^h\ 43^m, +22°$; for the globular star clusters, which show the very large dispersion of 117 km./sec. relatively to their own mean, the solar motion is 286 km./sec. toward $20^h\ 24^m, +62°$. The last is a very exceptional case, but there are groups of variable stars with respect to which the solar motion exceeds 100 km./sec.

Conclusions regarding the mean parallax of stars, based on the solar motion, should therefore be drawn with caution, unless enough radial velocities are available to give an idea of the solar motion, and the internal dispersion of velocities, for the group concerned. Such data may be had in most of the important cases.

Perhaps the most remarkable fact of all is that the mean motion of translation of any group, referred to a suitably chosen origin of velocities, is always nearly in a certain direction and is very closely proportional to the square of the dispersion of velocity within the group in this same direction. Strömberg's data put this relation beyond doubt. It is still unexplained, but it shows clearly that a law of some sort is at work, which operates even at the great distances of the globular clusters.

It is probable that this too may be explained by motions under the gravitational attraction of some vast system, — far larger than the "universe" of stars whose attraction produces the ordinary "streaming."

Oort maintains that the asymmetry sets in abruptly, at a velocity of about 60 km./sec., which he regards as the velocity of escape from the limited stellar "universe."

The B-stars and some of the most luminous A-stars are discordant, having a considerable group velocity and small dispersion. It is probable that these stars belong to a subsidiary cluster within our "universe" (§ 879).

EXERCISES

1. Find the date of closest approach of α Centauri to the sun, and its parallax, proper motion, and visual magnitude at that time.

Ans. Given, $V = -22$ km./sec., $T = 23$ km./sec. (Table XIV), we find (§ 735) $\tan\theta = T/V = -1.04$, whence $\theta = 134°$, $\sin\theta = +0.719$, $\cos\theta = -0.695$.
Hence $p' = 0.758/0.719 = 1''.05$;
$\mu = 3.68/(0.719)^2 = 7''.11$.
$m' = m + 5 \log(0.72) = m - 0.75$;
so that $m' = -0.4$ and $+1.0$ for the two components.
Finally, $t' = (206{,}265/3.68) \sin\theta\cos\theta = (56{,}300)(-0.50) = -28{,}150$. Since t' is negative, the closest approach is yet to come, about A.D. 30,050.

2. Find the same quantities for Barnard's star, taking the data of Table XV.

Ans. $\theta = 142°.4$, $p' = 0''.80$, $\mu' = 22''.5$, $m' = 8.9$, $t' = -9800$, date is A.D. 11,700.

3. What will be the proper motion and parallax of Barnard's star 2000 years hence?

Ans. It is easy to see from Fig. 222 that cot θ, which equals MS/MO, changes uniformly with the time. At present cot $\theta = V/T = -1.30$. In 9800 years it will be zero. Hence, 2000 years hence it will be -1.03, whence $\theta = 135°.9$, sin $\theta = 0.716$. Reversing the equations of § 735, we find $p = p' \sin \theta = 0''.574$, $\mu = \mu' \sin^2 \theta = 11''.56$.

4. If the sun had a velocity in space greater than that of Groombridge 1830, how would this assist the astronomer in finding the distances of the stars?

5. Prove that the radial velocities of all the stars in the sky are slowly but steadily increasing algebraically. (Consider the way in which cos θ in equation (3), § 734, varies with the time.) What does this result mean physically?

6. Is there any way in which the radial velocities of the stars could, in the course of time, be determined by purely geometrical methods?

Ans. Yes; by means of the slow increase in parallax and proper motion for a star which was approaching us.

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CHAPTER XX

DOUBLE STARS

OPTICAL AND PHYSICAL PAIRS · VISUAL, SPECTROSCOPIC, AND ECLIPSING BINARIES: NUMBER, MEASUREMENT, ORBITAL ELEMENTS · DATA FOR INDIVIDUAL PAIRS · STATISTICS: MASS, MASS-RATIO, MASS-LUMINOSITY CURVE, DYNAMICAL PARALLAX, DENSITY, DISTRIBUTION WITH RESPECT TO SPECTRAL TYPE · MULTIPLE STARS

760. Definitions. The telescope shows numerous cases in which two stars lie so near each other that they can be separated only by a high magnifying power. These are *double stars*, and at present more than twenty thousand such couples are known. There is also a considerable number of triple stars, and a few are quadruple.

In a few cases the two components of a double star are at very different distances from us, and appear close together in the heavens only because they lie nearly in the same line of sight. These are called *optical pairs*. The great majority of double stars, however, are *physical pairs*, the components being really near each other and doubtless in slow orbital motion. When this orbital motion is clearly perceptible, the pair is called a *visual binary*. A few physical pairs are so widely separated that they may be resolved with the naked eye or with a field-glass, but most binaries are close, and many are at the very limit of resolution of the greatest telescopes.

Still closer pairs may be detected by the variations in radial velocity, due to the orbital motion, and observed with the spectroscope. These are known as *spectroscopic binaries*.

Again, if the orbit of such a close pair happens to lie nearly edgewise toward the sun, the stars may eclipse one another at every revolution, causing periodic variations in brightness, which reveal its duplicity, even though the pair may be far too close for telescopic resolution and too faint for spectroscopic study. Such systems are called *eclipsing binaries*.

VISUAL DOUBLE STARS

761. Discovery. A few conspicuous bright pairs were noted asually in the seventeenth century (the earliest being ζ Ursæ Iajoris, by Riccioli, in 1650), but no serious attention was paid ɔ them till about 1775. Shortly after this, Sir William Herschel egan the first systematic search for such objects. His catalogues 1782–1784) contain about 700 double stars, including many nportant binary systems. The next great step was taken by Vilhelm Struve, whose catalogue (1827) contains 3110 pairs. ince his time, observers have been numerous and enthusiastic.

The discovery of double stars, unlike most astronomical disɔveries, is made by simple telescopic examination. The prinipal requisites are a trained eye, a telescope of excellent optical uality, and good atmospheric conditions. The latest extensive earch, by Aitken and Hussey, with the Lick 36-inch refractor, inolved the examination of all stars brighter than the ninth ıagnitude between $+90°$ and $-14°$ declination ($-23°$ from 3^h to 1^h R.A.) and resulted in the discovery of more than 4300 ew pairs. Further discoveries are likely to be made mainly in ıe southern skies, upon which work is in progress.

762. Nomenclature. A double star is designated by the initial of s discoverer and the number which it bears in his catalogue. 'he principal observers are W. Struve (Σ), Otto Struve ($O\Sigma$), urnham (β), Hussey (Hu), and Aitken (A). Thus, Σ 2398 deotes the double star bearing that number in Struve's catalogue. Iany bright stars thus obtain an additional alias; for example, Ursæ Majoris $= \beta$ 1077.

The brighter star of the pair is usually denoted by A, and the ıinter by B; for example, β 648 A, 70 Ophiuchi B. In a mulple star the components are called A, B, C, D, ... in order of rightness.

Burnham's *General Catalogue* (1906) includes 13,665 pairs (all ut a few north of declination $-31°$), and Innes's *Reference 'atalogue* covers the southern sky. A new catalogue is in preparation by Aitken at the Lick Observatory, where a complete ard catalogue of all observations is kept, and made available to ıose interested.

763. The limiting separation, beyond which a pair should not be called a double star, is more or less a matter of convention; but it should evidently be greater for the brighter stars, which, on the average, are nearer. Aitken suggests as a "working definition" a limit of 40″ for stars brighter than the second magnitude, diminishing to 20″ at the fourth magnitude, 10″ at the sixth, 5″ at the ninth, 3″ at the eleventh, and 1″ for still fainter stars. For stars of large proper motion or parallax these limits may be extended. They include practically all objects of physical interest, but exclude the faint wide pairs which encumber the older catalogues.

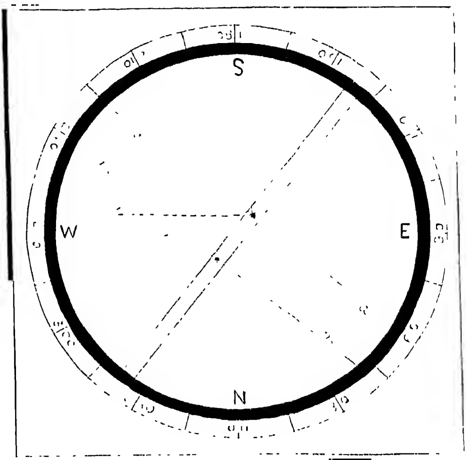


FIG. 234. Measurement of Distance and Position Angle of a Double Star

The fixed and movable wires (a and m) are set on the component stars, to measure the distance, while the two close parallel ones indicate the position angle (here about 320°). If the driving-clock is stopped, the star appears to move to the west (as indicated by the arrow). (The telescopic field of view is *inverted.*) In this way the circle-reading corresponding to position angle 270° is found, determining the "zero point" of the circular scale

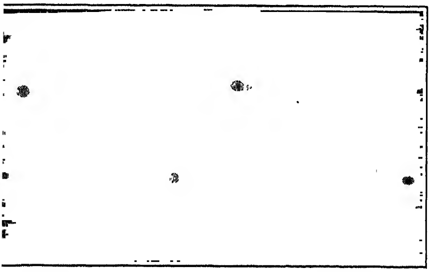
There are 5400 pairs, north of the celestial equator and brighter than the ninth magnitude (on the B.D. scale), which come within these limits. This is nearly one eighteenth of all the stars brighter than the ninth magnitude in this hemisphere. Of these pairs 64 per cent have separations less than 2″; 41 per cent, less than 1″; and 23 per cent, less than 0″.5. Among the stars brighter than $6^m.5$, *one in nine* is a visual double. Duplicity is therefore a very common phenomenon among the stars.

764. Measurement. The discovery that a star is double is of little value until the relative position of the components has been accurately measured. The measures are usually made with the filar micrometer, and determine the *distance d* in seconds of arc, and the *position angle p*, which is the angle made at the principal

between a line drawn to the smaller star and one to the north ;tial pole. It is reckoned *counterclockwise*, from the north ugh the east completely around the circle, as is illustrated in 234. When the two stars are equally bright, either one of ı may be taken as the principal component, and the position es recorded by different observers may be 180° apart.

he separating power of a good telescope, for pairs of stars of ıl brightness, is about equal to the theoretical resolving power 3), being 0″.11 for a 36-inch telescope. Somewhat closer pairs may be seen "elongated," even though the diffraction disks overlap (Fig. 13). For unequal pairs the limit is much greater. The companion of Sirius, though of the eighth magnitude, was invisible even in the greatest refractors from 1892 to 1896, while within 4″ of its primary.

South



1908 1913

FIG. 235. The Triple Star Krüger 60

are two photographs (left and right) of the system, five years apart. The wide pair is evidently an al one, and the change in it arises from proper mo- The close pair is binary, since the motion is orbital. n photograph by E. E. Barnard, Yerkes Observatory)

Close pairs, and faint companions of bright stars, can only be obed visually, and demand an excellent optical system and d seeing. Wider pairs — beyond 2″ or thereabouts — may be ısured accurately by photography. A few important pairs, too e to be resolved by any telescope, have been separated by the rferometer (§ 820).

65. Discrimination between Optical and Physical Pairs. The ıciples of probability show that the great majority of double s must be physically associated, optical pairs being comparaly rare. Consider, for example, the chance that two naked- stars, which have no physical connection, will be within 10″ ach other.

raw a circle 1° in diameter about a naked-eye star. The area of this e is 1/52,500 of the area of the whole sky. There are about 6500 naked-

eye stars. The chance that another will fall within this circle, that is, within 30′ of the one selected, is 6500 in 52,500, or about 1 in 8.

The chance that a naked-eye star will have another within 3′ of it is 1/100 as great, or 1 in 800. There should be about 6500/800 = 8 such cases. But each case has been counted twice, so 4 such pairs are to be expected.

If the separation is reduced to 10″, the probability becomes $(18)^2$, or 324 times smaller, and the chance that there would not be a single such pair in the sky is 80 to 1. Actually there are dozens.

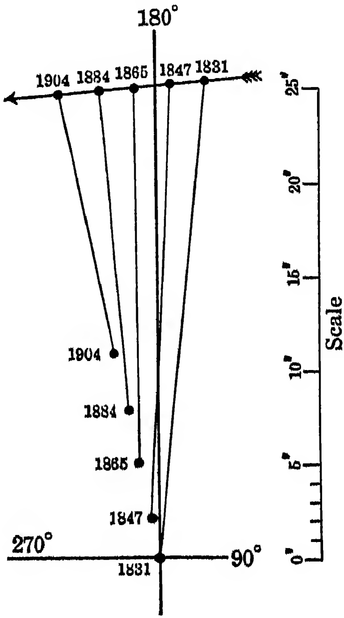


FIG. 236. Motions of δ Herculis, Σ 3127

Marked difference in proper motion identifies this star as an *optical* pair. The component shown above is moving at right angles to the lower one. (From Burnham's *General Catalogue of Double Stars*, by courtesy of The Carnegie Institution of Washington)

Definite evidence that a pair is physically connected is afforded by (1) *orbital motion*, the relative path being curved, and concave toward the principal star, and (2) *common proper motion*, the two stars moving together in the heavens at nearly the same direction and rate (usually with a small difference due to slow orbital motion). In some cases physical connection is certain from the time of discovery, — for example, when a star of considerable proper motion is found to have a close companion, which, if it had not been moving with the other, would have been so far away from it twenty or thirty years before as to form a wide pair, which earlier observers could not have missed.

We can be practically sure that a pair is optical when the proper motions of the components differ entirely in magnitude and direction, as in δ Herculis (Fig. 236). The relative motion is usually much more rapid in this case than in orbital pairs of the same angular separation. When the proper motion is very small or unknown, discrimination is difficult. It is probable, however, that almost all the "fixed" pairs, which have shown no relative motion since discovery, are physically connected.

766. Visual binaries were first recognized in 1803 by Sir William Herschel, who had measured many pairs (supposing them to be optical) in the hope of detecting an annual parallactic displacement of one of the stars relative to the other. He found instead a true orbital motion. At the present time it has been possible to compute orbits for more than 100 binaries, and the number of other pairs in which orbital motion (often very slow) is known to be present exceeds 1500.

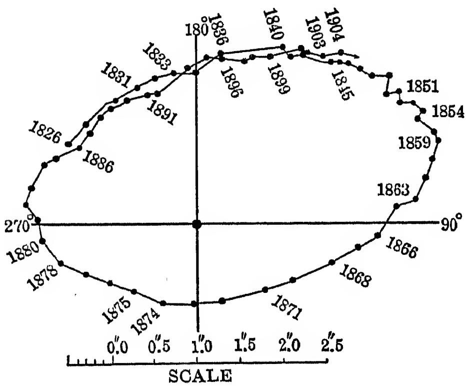


FIG. 237. Orbit of ξ Ursæ Majoris, Σ 1523

The pair has made more than one complete revolution from 1826 to 1904. The period is 59.8 years. (From Burnham's *General Catalogue of Double Stars*, by courtesy of The Carnegie Institution of Washington)

The *apparent orbit* of the companion about the primary may be found by plotting the observed position angles and distances (Fig. 237). It is always an ellipse, but the principal star is not usually in the focus. This is because the *true orbit* is almost always seen more or less obliquely and is distorted by foreshortening. If the true orbit lies nearly edgewise to the line of sight, the apparent ellipse is long and narrow; indeed, in one case (42 Comæ) it is exactly edgewise, and the companion appears to oscillate in a straight line from one side of the primary to the other.

The law of areas still holds good in the apparent motion, the radius vector sweeping out equal areas in equal times. This follows at once geometrically from the fact that the apparent orbit is the projection of the true orbit upon a plane perpendicular to the line of sight, often called the "plane of the sky." (The very small portion of the celestial sphere which is involved can be treated as if it were flat.)

767. The calculation of a double-star orbit therefore consists of two stages: (1) to represent the plotted observations, as well as may be, by an ellipse in which the law of areas is obeyed; (2) to pass from this apparent orbit to the true orbit. The latter prob-

lem is one of pure geometry, and many solutions are known, some of much elegance; the former often involves a good deal of judgment in the interpretation or rejection of poor observations, and demands considerable skill, especially when the components are of equal brightness and when some of the observed angles may need correction by 180°.

If the observed arc is short, a variety of ellipses, of different sizes, may be made to fit it, within the error of the observations (Fig. 238); and it is only when the observations cover at least half the circumference of the orbit that reliable elements can be obtained.

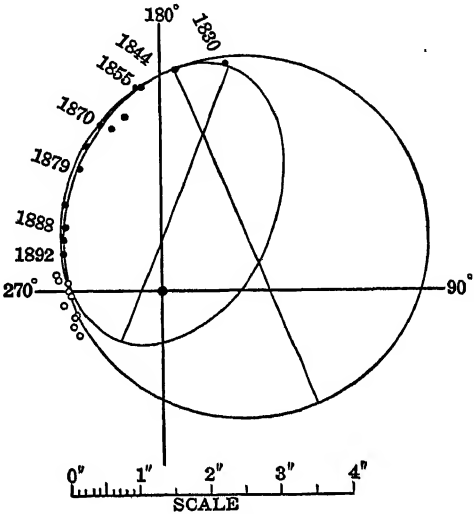


FIG. 238. Orbit of Σ 1785

The uncertainty of this orbit is illustrated by the two ellipses, which give periods, respectively, of 130 and 300 years, and which satisfy equally well the observations from 1830 to 1892. Observations since 1892 (white circles) indicate that the true orbit is at least as big as the larger ellipse; but the future positions of the companion are still unpredictable with any accuracy. (From Burnham's *General Catalogue of Double Stars*, by courtesy of The Carnegie Institution of Washington)

The *elements of a double-star orbit* are very similar to those of a planet (§ 280). They are

- P, the period of revolution, in years,
- T, the time of periastron[1] passage,
- e, the eccentricity,
- a, the semi-major axis of the true orbit, in seconds of arc,
- Ω, the position angle of the node of the orbit plane on the "plane of the sky" (this is always taken less than 180°),
- ω, the angle, in the plane of the orbit, from the node, in the direction of motion, to the periastron,
- i, the inclination of the orbit plane to the "plane of the sky,"
- μ, the "mean motion" in degrees per year ($= 360°/P$).

[1] Periastron, the point of the orbit at which the stars are nearest. Compare with the terms "perihelion" and "perigee."

It should be stated whether the position angles increase or decrease with the time.

The inclination i is to be regarded as positive when the companion is moving away from the observer at the node of position angle Ω, and negative if it is approaching. Which of these is the case can be determined only by observations of radial velocity; in their absence, i must be given with the double sign ($\pm$).

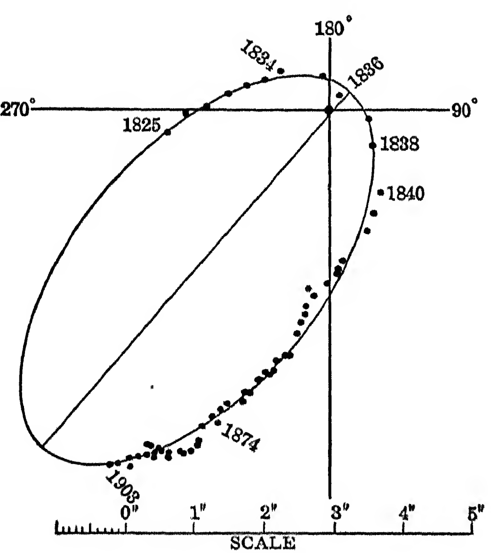


FIG. 239. Orbit of γ Virginis, Σ 1670

From, Burnham's *General Catalogue of Double Stars*, by courtesy of The Carnegie Institution of Washington

768. Characteristics of the Orbits. Fairly good orbits are now available for rather more than a hundred visual binaries. The periods range from 5.7 years (δ Equulei) and 6.9 years (13 Ceti) upward. About a dozen other pairs have periods less than 25 years, and some fifty more lie between 25 and 100 years. Both the upper and the lower limits are fixed entirely by the conditions of observation. Systems of short period (of which the spectroscope reveals great numbers) are too close to be resolved by the telescope, while those of long period move so slowly that the observed arc of the orbit (which represents at most a century's observations, and often much less) is too short to determine the shape of the rest. For the same reason computed orbits with periods longer than 150 or 200 years are usually not very reliable.

The positions of the orbit planes appear to lie at random in space. The eccentricities are usually large, being notably different in this respect from those of the planetary orbits. The average value for the orbits so far computed is 0.50. The eccen-

tricity is less than 0.2 for only 10 per cent of the orbits and exceeds 0.85 for an equal number. For ξ Boötis and OΣ 341 it is 0.96 (equal to that of Halley's comet), and for γ Virginis (Fig. 239) it is 0.89.

769. Real and Apparent Separations. The apparent separations (mean distances) range from $0''.16$ (close to the limit of telescopic resolution) to $17''$ (for α Centauri). The mean distance is less than $2''$ for 83 per cent of the whole, less than $1''$ for 62 per cent, and less than $0''.5$ for 29 per cent.

The real separation can be determined only if the parallax, p, is known. If A is the mean distance in astronomical units, and a the same distance in seconds of arc, then evidently

$$A = a/p. \tag{1}$$

The resulting values for those binary systems which have well-determined parallaxes are given in Table XX. The stars are arranged, in this table, in order of increasing period. Except for α Aurigæ (which can be resolved only with the interferometer) and for a few pairs at the end, the orbits are all comparable in size with those of the outer planets, from Jupiter to Neptune.

There is a conspicuous correlation between the period and the true separation, the two increasing together. This has a very important physical significance, namely, that the various systems are comparable in mass (§ 770).

For the wider pairs the spectra of the two components can be separately observed, and are given in the table, while the difference of magnitude is based on photometric measures. For the closer pairs the spectrum represents the combined light and, unless the magnitudes are nearly equal, must be that of the brighter star. The spectral classes given for α Aurigæ depend upon observations of the star as a spectroscopic binary.

The periods, eccentricities, and other orbital elements are usually well determined for the systems of shorter period, which have completed one or more revolutions.

Some of the stars of Table XX are triple or quadruple, having distant companions. Data concerning these are given in Table XXI.

TABLE XX. VISUAL BINARY STARS

	MAG.	SP.	μ	P	e	i	a	p	p. e.	A	$m_1 + m_2$	m_1 m_2	ABS. MAG.
α Aur . . .	0.8 1.1	G0 F5	0″.44	0ʸ.285	0.01	±40°	0″.054	0″.063	±0.001	0.85	7.5	4.2 3.3	−0.2 0.1
δ Equ . . .	5.2 5.7	F5	0 .31	5 .70	0.39	±81	0 .27	0 .060	± .006	4.5	2.8	—	4.1 4.6
κ Peg	4.8 5.5	F5	0 .03	11 .35	0.49	±77	0 .29	0 .026	± .005	11.2	11.1	—	1.9 2.6
ϵ Hya . . .	3.7 5.2	F8	0 .20	15 .3	0.65	+50	0 .23	0 .020	± .005	11.5	6.5	3.5 3.0	0.2 1.7
42 Com . . .	5.6 5.6	F5	0 .45	25 .9	0.52	±90	0 .66	0 .063	± .008	10.6	1.8	—	4.6 4.6
85 Peg . . .	5.8 11.3	G0	1 .29	26 .3	0.46	±53	0 .82	0 .092	± .006	8.9	1.0	0.35: 0.65:	5.7 11.2
Σ 3121 . . .	7.9 8.2	K0	0 .51	34 .3	0.33	±75	0 .67	0 .056	± .008	12.0	1.5	—	6.7 7.0
ζ Her	3.0 6.5	G0	0 .60	34 .5	0.46	−48	1 .35	0 .111	± .005	12.2	1.6	1.1 0.5	3.2 6.7
α CMi . . .	0.5 13 :	F5	1 .24	39 .0	0.32	±14	4 .05	0 .312	± .006	13.0	1.5	1.1 0.4	3.0 15.5:
β 416	6.1 8.1	K2	1 .20	42 .2	0.55	±50	1 .83	0 .169	± .016	10.9	0.7	0.4 0.3	7.2 9.2
μ Her BC . .	10.0 11.0	M2	0 .82	43 .2	0.20	±63	1 .30	0 .111	± .006	11.7	0.9	0.5 0.4	10.2 11.2
Krüger 60 . .	9.7 11.3	M3	0 .89	44 .3	0.38	±26	2 .46	0 .257	± .004	9.6	0.45	0.27 0.18	11.8 13.4
ξ Sco	4.8 5.1	F8	0 .07	44 .7	0.75	±29	0 .72	0 .040	± .005	18	2.9	—	2.8 3.1
Σ 2173 . . .	5.9 6.2	G5	0 .21	46 .0	0.18	±81	1 .06	0 .052	± .006	20	3.9	—	4.5 4.8

τ Cyg . . .	3.8 8.0	F0	0 .47	47 .0	0.22	± 43	0 .91	0 .050	± .006	18	2.7	—	2.3 6.5
α CMa . . .	− 1.6 8.4	A0 F0	1 .30	50 .0	0.60	+ 43	7 .57	0 .371	± .004	20.4	3.4	2.44 0.96	1.3 11.3
β 648	5.2 8.7	G0	0 .24	56 .6	0.23	± 66	1 .25	0 .064	± .005	19	2.3	—	4.2 7.7
ξ UMa . . .	4.4 4.9	F9 G2	0 .73	59 .8	0.41	± 53	2 .51	0 .146	± .006	17	1.4	0.7 0.7	5.2 5.7
99 Her . . .	5.2 10.5	F8	0 .11	63 .0	0.76	± 25	1 .00	0 .042	± .006	24	3.4	—	3.3 8.6
α Cen . . .	0.3 1.7	G0 K5	3 .68	78 .8	0.51	+ 79	17 .65	0 .758	± .010	23.3	2.04	1.10 0.94	4.7 6.1
70 Oph . . .	4.3 6.0	K0 K6	1 .13	87 .7	0.50	− 59	4 .50	0 .192	± .005	23.5	1.7	0.9 0.8	5.7 7.4
γ CrB . . .	4.0 7.0	A0	0 .11	87 .8	0.42	± 84	0 .73	0 .022	± .006	33	4.7	—	0.7 3.7
OΣ 79 . . .	7.1 7.9	G0	0 .10	88 .9	0.63	± 56	0 .57	0 .027	± .004	21	1.2	—	4.3 5.1
ξ Boo . . .	4.8 6.7	G6 K4	0 .17	152 .8	0.51	± 38	4 .83	0 .168	± .007	29	1.0	0.53 0.47	5.9 7.8
Σ 2052 . . .	7.8 7.8	K0	0 .53	153 .2	0.92	± 51	1 .62	0 .055	± .006	29	1.1	—	6.5 6.5
o_2 Eri BC . .	9.7 11.4	A0 M6	4 .08	248	0.40	± 72	6 .89	0 .203	± .008	34	0.64	0.44 0.20	11.4 12.9
α Gem . . .	2.0 2.8	A0 A0	0 .20	306	0.56	+ 67	6 .06	0 .076	± .004	80	5.5	—	1.4 2.2
η Cas	3.7 7.4	F8 K0	1 .25	346	0.33	± 39	10 .1	0 .182	± .005	55	1.4	0.8 : 0.6 :	5.0 8.7
σ CrB . . .	5.8 6.7	G0	0 .30	500 900 1680	0.72 0.78 0.85	± 22 ± 26 ± 29	4 .18 6 .00 9 .02	0 .048	± .007	87 125 188	2.6 2.4 2.4	1.7 : 0.8 : —	4.2 5.1 —

TABLE XXI. DISTANT COMPANIONS OF VISUAL BINARIES

	MAG.	SP.	DIST.	RELATIVE MOTION	
ϵ Hya C	6.8	F5	3″.4	0″.031	Physical
ϵ Hya D	12.5	—	19 .8	0 .02 :	Physical
85 Peg C	9.8	—	60	1 .29	Optical
β 416 C	10.5	—	30 .7	0 .078	Physical
μ Her A	3.48	G5	31 .1	0 .038	Physical
Krüger 60 C . .	9.2	—	60	0 .89	Optical
ξ Sco C.	7.2	—	7 .2	0 .025	Physical
o_2 Eri A	4.48	G5	83 .2	0 .060	Physical
α Gem C	9.03	M0	72 .7	0 .032	Physical

770. Masses of Binary Systems. By Kepler's harmonic law (§ 304)

$$\frac{m_1 + m_2}{\odot + E} = \frac{A^3}{P^2},$$

where m_1 and m_2 are the masses of the two stars, $\odot$ the sun's mass, E the earth's mass, A the mean distance of the stars in astronomical units, and P the period in years. Using equation (1) and neglecting the earth's mass in comparison with the sun's, we have

$$\frac{m_1 + m_2}{\odot} = \frac{a^3}{p^3 P^2}. \tag{2}$$

The *combined* mass of the pair, determined by this equation, is given for each star in Table XX. In every case the masses are of the same order as the sun's and are remarkably similar to one another. The range of luminosity, from α Aurigæ to Krüger 60, is fully twelve magnitudes, or in the ratio of 60,000 to 1, while the range of masses is only 25 : 1. This is all the more remarkable since the *cube* of the parallax appears in the denominator in (2), and since a very small error in the parallax may therefore cause a large one in the computed mass. If, for example, the parallax of κ Pegasi is increased by its probable error, p becomes 0″.031, A comes out 9.3 astronomical units, and $m_1 + m_2$ becomes 6.3 instead of 11.1.

It may be noticed that for σ Coronæ the period is very uncertain, orbits with periods ranging from 500 to 1680 years representing the observations about equally well; but the masses computed from these three orbits are

almost the same. The poor orbits give good masses. This is usually the case, the reason being that the observations, though they do not suffice to fix the size of the unobserved portion of the ellipse, are sufficient to determine the curvature of the apparent orbit due to the attraction between the stars at a given distance. When the inclination is high, however, the rule sometimes fails.

771. Mass-Ratio of the Components. In most cases the relative orbit of one component with respect to the other is all that can be determined from the observations, and in this case only the sum of the masses can be found. But if the positions of one or both of the components can be measured with respect to some external point of reference (by observations with the meridian circle or by micrometric or photographic measures from neighboring stars), the motions of the two stars relative to their center of gravity can be separately found and the ratio of their masses determined.

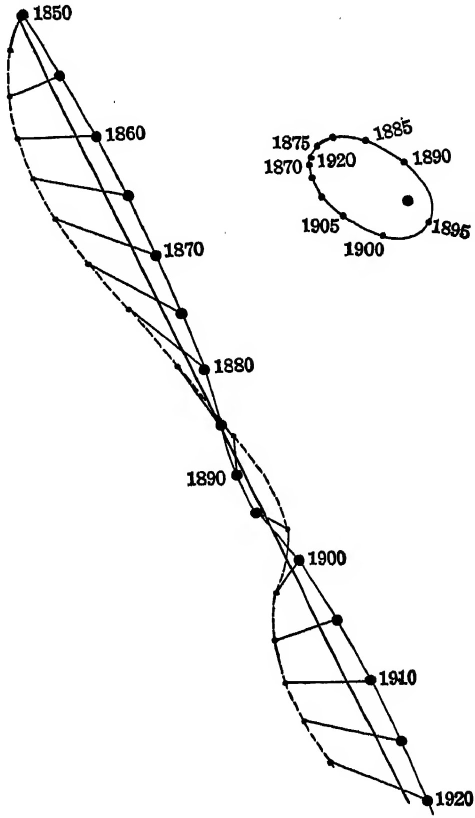


FIG. 240. Relative and Absolute Positions of Sirius and Companion, 1850–1920

The case of Sirius is in point. Before 1850 Bessel found, from the meridian observations, that the bright star was not moving uniformly in the heavens, but in a wavy line (Fig. 240), and concluded that it must be revolving about the center of gravity of itself and an invisible companion in an elliptic orbit, with a period of about fifty years. In 1862 Clark found near it a minute companion which explained everything. Since that date this has

been followed through a whole revolution, and its period turns out to be just the fifty years predicted by Bessel.

The bright star and its companion are always on opposite sides of the center of gravity (which moves uniformly), and their distances from it are in the inverse ratio of their masses. It is thus found that the companion of Sirius, though but 1/10,000 as bright as its primary, has a mass two fifths as great. Procyon shows a similar but smaller oscillation, which is due to the attraction of a very faint companion that was not discovered till 1896.

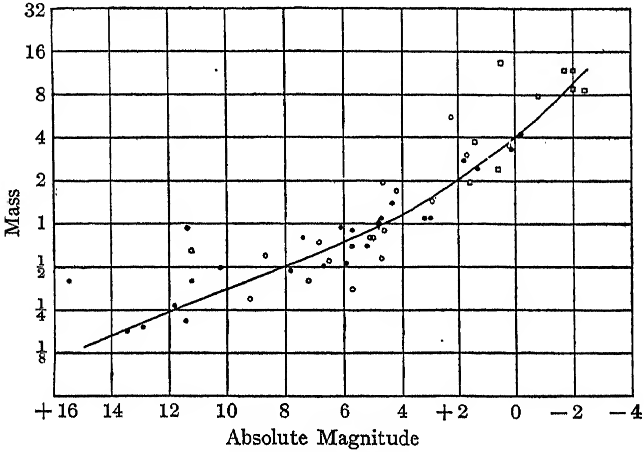


FIG. 241. The Mass-Luminosity Curve

The vertical scale is logarithmic, the masses being given in terms of the sun's mass

Almost all the pairs for which the mass-ratio is known are included in Table XX, and the individual masses are given for these. A few uncertain cases are marked by colons. When the components are nearly equal in brightness, they are also nearly equal in mass. When they are unequal, the brighter star is always the more massive, except in the case of 85 Pegasi, where the observations, though not quite conclusive, indicate a disparity in the other direction.

772. Mass and Absolute Magnitude. Though the masses of the stars differ much less than most of their other characteristics, they nevertheless show very definite differences between stars of different sorts. These differences were once supposed to de-

pend upon the spectral class, but it is now generally recognized that they depend almost entirely upon the real brightness, or absolute magnitude, of the star. How nearly this is the case is illustrated by Fig. 241, in which the masses of the stars in Table XX (and also of a few spectroscopic and eclipsing binaries, to be discussed later) are plotted against their absolute magnitudes. The data for the individual components are employed when the relative masses are known. In other cases, when the difference of magnitude is small, the mean of the masses, $\frac{1}{2}(m_1+m_2)$, and of the absolute magnitudes of the two components has been taken. When the components differ in brightness by more than a magnitude, this process does not give a safe average, and the star has been omitted from the diagram. The more accurately determined data (for which the probable error of the parallax is less than 10 per cent and the mass-ratio is well determined) are plotted as black dots, the rest as open circles. Several spectroscopic and eclipsing binaries, for which the masses and absolute magnitudes can be determined (§ 785), are also plotted. The four-pointed star (at mass 1, absolute magnitude 4.8) represents the sun.

Almost all the plotted points lie close to a smooth curve. From this curve the *masses corresponding to given absolute magnitudes* may be read (Table XXII.)

TABLE XXII. THE RELATION OF MASS AND ABSOLUTE MAGNITUDE

Abs. mag.	15	12.5	10	7.5	5	2.5	0	− 2.5
Log mass	− 0.86	− 0.66	− 0.46	− 0.26	− 0.04	0.24	0.60	1.08
Mass	0.14	0.22	0.34	0.55	0.91	1.72	4.0	12
Ratio of increase	1.6	1.6	1.6	1.7	1.9	2.3	3.0	

The ratio of increase of light from each tabular entry to the next is 10, but that for the mass (which is given in the lowest line of the table) is usually less than 2. For all but two of the more accurately observed stars the observed masses differ from those read from the curve by only 20 per cent, on the average; and much of this difference must arise from the small errors in the observed parallaxes. The less reliable observations give more discordant results, as might have been expected. The faint com-

panions of Sirius and Procyon are exceptional, being between three and four times as massive as their brightness would indicate.

This very remarkable relation between the mass and the luminosity of a star has been explained theoretically by Eddington (Chapter XXV).

773. Dynamical Parallaxes of Binary Stars. Accepting this relation for the present simply as an observed fact, it becomes possible to find the parallax of any double star whose orbit is known. Equation (2) may be written in the form

$$p = \frac{a}{\sqrt[3]{P^2(m_1 + m_2)}} \tag{3}$$

(if $m_1 + m_2$ is expressed in terms of the sun's mass). The absolute magnitudes of the components are given by the equation

$$M = m + 5 + 5 \log p, \tag{4}$$

where m is the observed visual magnitude in each case. If we begin by guessing at $m_1 + m_2$ (twice the sun's mass would be a good general average), equation (3) gives the corresponding value of p, and (4) the absolute magnitudes of the components. The corresponding masses may then be taken from the mass-luminosity curve (Fig. 241), and an estimate obtained which is much better than the first guess. One or two repetitions of this process lead to the final value.

Take, for example, κ Pegasi, for which $a = 0''.29$, $P = 11.35$ years, and the magnitudes of the components are 4.8 and 5.5. Assuming $m_1 + m_2 = 2$, we find $p = 0''.29/\sqrt[3]{257} = 0''.046$. This parallax gives absolute magnitudes $M_1 = 3.1$, $M_2 = 3.8$, whence, from the curve, $m_1 = 1.48$, $m_2 = 1.23$. This gives the corrected value $m_1 + m_2 = 2.71$, whence $p = 0''.042$, $M_1 = 2.9$, $M_2 = 3.6$; then $m_1 = 1.55$, $m_2 = 1.29$, $m_1 + m_2 = 2.84$. This gives $p = 0''.040$, $M_1 = 2.8$, $M_2 = 3.5$, $m_1 = 1.58$, $m_2 = 1.32$, $m_1 + m_2 = 2.90$, whence again $p = 0''.040$, showing that the approximation is complete. The whole computation (with a slide-rule) takes but a few minutes. In this particular case the observed parallax is $0''.026 \pm 0.005$. This differs from the computed dynamical parallax by 2.8 times the probable error, an amount inadmissible in the average case but quite tolerable when, as in the present instance, we have picked out the worst discordance among 29 stars (cf. § 124).

If the masses of the stars of the same absolute magnitude all had the values given in the table, these dynamical parallaxes

would be perfectly accurate. This is, of course, not true; but the differences in mass for individual stars are usually so small that the dynamical parallaxes are likely to be within a small percentage of the truth. Only the larger parallaxes (greater than 0″.10) can be determined as accurately as this by direct observation.

774. Masses and Dynamical Parallaxes of Slow-Moving Pairs. There are hundreds of double stars which are known to be binary systems, but in which the observed motion covers far too small an arc to permit the calculation of the elements of the orbit. The masses of such systems may be found (if the parallaxes are known) by a statistical process (invented independently by Russell and by Hertzsprung) which gives good average values, though the results are rough in the individual case.

To understand it, imagine first a pair moving in a circular orbit, in the "plane of the sky." The observed distance a will be constant. If v is the rate of relative motion of the components in seconds of arc per year, the period, P, will be $2\pi\, a/v$.

The mass is then given by

$$m_1 + m_2 = \frac{a^3}{P^2 p^3} = \frac{av^2}{4\,\pi^2 p^3} = \frac{av^2}{39.7\, p^3}$$

and could be determined without waiting for the whole orbit to be described.

In an actual binary pair the apparent distance s will usually be less than the true distance r, owing to foreshortening; and the latter, again, may be greater or less than the mean distance a. Similarly, the apparent motion w will usually be less than the true motion, and this again may be greater or less than the value v in a circular orbit. It is impossible to take account of these differences in individual cases unless the orbit is known; but the average effect in a large number of cases distributed at random can be accurately computed by the theory of probability. It is thus found that, on the average, sw^2 will be 43 per cent of av^2, so that the formula

$$m_1 + m_2 = \frac{sw^2}{17\, p^3} \tag{5}$$

will give results which, on the average, will be correct. The "probable error" of such a determination, arising from the dif-

ference of the ratio sw^2/av^2, in the individual case and in the average, may be found from theory and is about 49 per cent.

By reversing the process, an equation for the parallax may be obtained in the form

$$p = 0.42 \sqrt[3]{\frac{sw^2}{m_1 + m_2}}, \tag{6}$$

which gives the parallax if the mass is known or can be estimated. The "statistical probable error" of the determination is $\pm$ 20 per cent of the value given by the formula.

Equation (6) does not agree numerically with the result obtained by solving equation (5) for p, because the average value of $\sqrt[3]{sw^2}$ is not equal to the cube root of the average value of sw^2 itself.

With the aid of equation (5) the masses of about four hundred double stars of known parallax can be found. Preliminary results show that for these too the mass depends primarily on the absolute magnitude, and give values agreeing closely with those read from the mass-luminosity curve. The number of stars for which dynamical parallaxes can be calculated by equation (6) exceeds fifteen hundred. The parallaxes thus found (especially for the fainter stars) are considerably greater than the mean values for all stars of the same magnitude. The reason is simple: the resolving power of telescopes is limited, and the nearer pairs have a much better chance of being detected.

775. Eccentricity of Wide Pairs. Some information regarding the average shape of the orbits of these wide pairs may be obtained statistically. It is not hard to see that if all the orbits were circular, so that the real orbital motion was always perpendicular to the radius vector, there would be an excess of apparent motions perpendicular to the line joining the stars, in spite of the effects of foreshortening. The greater the orbital eccentricity, the less marked will be this effect. In this way Russell has found that the average eccentricity is 0.61 for more than 500 stars, of average period estimated roughly as 2000 years, and 0.76 for nearly 800 more, with average periods of perhaps 5000 years.

776. Multiple Stars. A considerable number of visual "binaries" — 4 or 5 per cent of the whole, according to Aitken — are really triple or quadruple systems. In many cases a close pair has

a distant attendant which shares the proper motion. Several such instances are listed in Table XXI (p. 688). Often one component of a star long known to be double is visually resolved into a close pair (missed by earlier observers either because their telescopes were less powerful or because the close pair was then still closer), and in others its duplicity is revealed spectroscopically by variable radial velocity. In a few cases the spectroscopic observations alone show that a close pair is in orbital motion about the center of gravity of itself and a third body (as in β Persei, § 785).

The distance in the close pair is almost always small compared with the distance between the pair and the third body, so that the orbital periods are very different in the two cases. When the close pair is visually separable, the motion in the wide pair is too slow to permit the calculation of an orbit from a century's observations.

Zeta Cancri consists of a close pair, with a period of 60 years, eccentricity 0.34, and mean distance $0''.85$, attended by a third star $5''.4$ away, which is moving in the same direction at the rate of half a degree a year, but with regular oscillations in position angle and distance, which indicate that it is revolving about the center of gravity of itself and an invisible companion, in a nearly circular orbit of radius $0''.22$, in a period of 17.6 years.

Castor (α Geminorum) is a visual binary with period exceeding three centuries, accompanied by a faint attendant $73''$ distant, which shares the proper motion and must be revolving about the bright pair with a period probably exceeding ten thousand years. All three components are spectroscopic binaries of short period, so that the system consists of six stars.

The system of ϵ Lyræ consists of two pairs, of separation $2''.5$ and $3''.2$ (both in slow retrograde motion) and $207''$ apart, with a common proper motion of $0''.06$. The periods of the closer pairs must be many centuries in length, and that of the wide pair (which is just resolvable by a keen eye without optical aid) is probably several hundred thousand years. The brightest of the four stars is a spectroscopic binary.

Most of the few cases in which the components of a triple star are all at about the same apparent distance can be accounted for by foreshortening. In a few instances, however, several stars are associated in a group at not very unequal distances, — for example, the six components of θ Orionis. These systems may be regarded as small star-clusters. They all have very small proper motions, and the real separations of the components, in astronomical units, are much wider than for ordinary binary systems.

SPECTROSCOPIC BINARIES

777. Discovery. One of the many interesting results of spectroscopic work is the discovery, dating from 1889, of numerous pairs of double stars, so close that no telescope can separate them, but proved to be double by the behavior of the lines in their spectra.

After correction has been made for the observer's motion, the radial velocity is found to be variable; the spectral lines shift back and forth across their normal position. The star, therefore, is undergoing acceleration. In almost all cases the variations in radial velocity are found to be periodic, — repeating themselves exactly, at intervals which range (as observed) from two and one-fourth hours (γ Ursæ Minoris) to fifteen years (ϵ Hydræ). There is therefore reason to believe that these stars are double, — "spectroscopic binaries."

In some cases the lines periodically become double (the two sets shifting in opposite directions), showing that there are two bright components, the one receding when the other is approaching. The spectrum of the fainter star can seldom be seen if this star is less than one third as bright as the other, that is, if there is a difference of more than about one magnitude. The fact that but one set of lines is visible therefore cannot be regarded as evidence that the companion gives out no light at all.

The number of these systems is great. The *Third Catalogue of Spectroscopic Binaries* (Lick Observatory, July, 1924) lists 1054 of them. About one in six shows double lines.

778. The Velocity Curve. The period of variation of the velocity may be found by comparing the observed times at which it had the same value and was changing in the same direction. The velocity at any two dates separated by an exact number of periods is the same; hence each observation can be "carried forward" or back by an integral number of periods, so that the "reduced times" all fall close together as if the observations had all been made in a single revolution. The observed velocities are plotted against these "reduced" times, and the *velocity curve* is drawn to represent them.

Fig. 242 represents a few typical cases. In β Capricorni only one spectrum is visible; in p Velorum, two spectra, which show

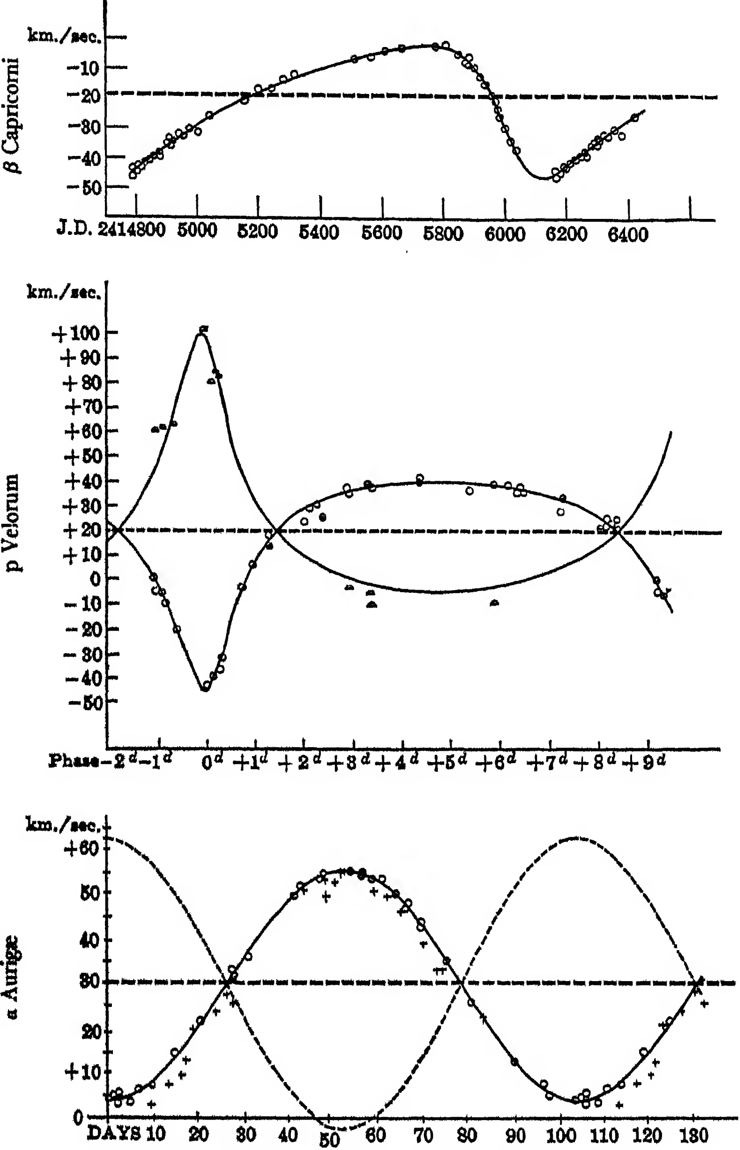


FIG. 242. Velocity Curves of Spectroscopic Binaries

By courtesy of the Lick Observatory

velocity changes in opposite directions, represented by curves of the same form but inverted and with different ranges. In α Aurigæ the second spectrum is present and the velocity curve of the companion (though not the individual observations) is plotted in the figure.

779. Stationary Lines. In many spectroscopic binaries with spectra of Class B3, or earlier, the (H) and (K) lines of ionized

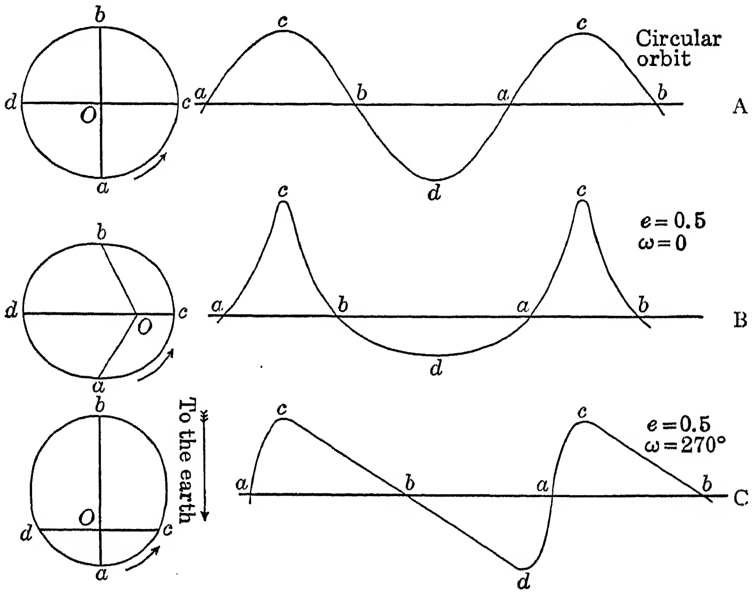


FIG. 243. Velocity Curves

calcium are narrow and sharp (quite unlike the rest, which are broad and diffuse), and do not share in the periodic displacements of the other lines. The velocity indicated by these lines is sometimes nearly the same as that of the center of mass of the system, but not always so. The (D) lines of sodium behave in the same way. It is evident that these lines do not originate in the atmospheres of the components, but must be produced in an outer envelope of some sort, too far away to be much influenced by the rotation of the close pair.

Whether the gas which produces this absorption is fairly near the star or diffused in space is not yet certain (§ 946).

780. Determination of the Orbit. The relation between the velocity curve and the real ellipse which the star describes about the center of gravity of the system is illustrated in Fig. 243, which shows three orbits and the corresponding velocity curves. The horizontal line represents the velocity of the center of gravity. The points *a*, *b*, where the curve intersects this line, correspond to the points *a*, *b*, on the orbit, where the star is moving at right angles to the line of sight. The points *c*, *d*, of maximum and minimum velocity, correspond to the points *c*, *d*, of the orbit, at which the line joining the two stars is perpendicular to the line of sight (that is, the nodes). (Between *a* and *c* the attraction of the companion obviously pulls the star away from the earth; between *c* and *b*, toward the earth. The maximum velocity of recession is therefore reached at *c*.)

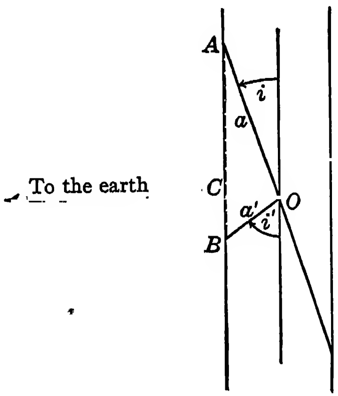


FIG. 244. Effect of the Inclination of the Orbit of a Spectroscopic Binary

The inclination cannot be determined from the velocity curve

If the orbit is circular (A), the time-intervals between the phases *a*, *b*, *c*, *d*, are all equal and the velocity curve is a simple sine-curve. Otherwise these intervals (which are proportional to the areas of the sectors *Oac*, *Ocb*, etc. and are represented by the horizontal distances *ac*, *cb*, between the points on the velocity curves) are unequal. The maximum velocities of approach and recession (relative to the center of gravity) will also be unequal (unless the line of apsides points toward the earth, as in C).

In practice the horizontal line must be drawn so as to make the area between it and the curve on the upper side equal to that on the lower. The height of this line above or below the line of zero velocity represents the velocity of the system.

The elements of the orbit may then be found from the shape of the curve by any one of several methods which have been devised.

The eccentricity of the orbit, *e*, the angle of periastron, ω (measured from the node), and the time of periastron passage,

T, may thus be found. The position angle of the node, Ω, and the inclination, i, cannot possibly be found from measures of radial velocity. The first statement is obvious; the second depends on the fact that the radial velocity depends only on the *changes of distance from the observer*, and these may be exactly the same for a small orbit with a high inclination and a large orbit with a low inclination, as is illustrated in Fig. 244, which represents two circular orbits, of different radii and inclination, seen edgewise (so that they reduce to straight lines).

The actual distances OA and OB are very different, but the corresponding changes OC in the distance from the earth are exactly equal, being $a \sin i$ in both cases. It is this quantity $a \sin i$, and not the actual distance of the star from the center of gravity, which can be found from the spectroscopic observations. It is usually given in kilometers. The period, P, and the radial velocity of the system, V_0, are also counted among the elements.

The half-range of velocity, K, is likewise commonly tabulated. This is given by the equation

$$K = \frac{2\pi a \sin i}{P\sqrt{1-e^2}}$$

(as is easily proved from the laws of elliptic motion). If K is desired in kilometers per second, a should be expressed in kilometers and P in seconds. If the latter is given in days,

$$K = \frac{a \sin i}{13{,}750\, P\sqrt{1-e^2}}.$$

781. The Mass Function. If m_1 and m_2 are the masses of the components, a_1 and a_2 their mean distances from the center of gravity, and a the mean distance of the stars, then

$$\frac{a_1}{a} = \frac{m_2}{m_1+m_2}, \qquad \frac{a_2}{a} = \frac{m_1}{m_1+m_2}.$$

Also, by Kepler's law,

$$m_1 + m_2 = \frac{a^3}{P^2},$$

provided a is expressed in astronomical units and P in years. If we know only the orbit of one component, m_1, about the center of gravity, the best that we can do is to compute $(a_1 \sin i)^3/P^2$. From the foregoing equations we have

$$\frac{(a_1 \sin i)^3}{P^2} = (m_1+m_2)\frac{m_2{}^3 \sin^3 i}{(m_1+m_2)^3} = \frac{m_2{}^3 \sin^3 i}{(m_1+m_2)^2}. \tag{7}$$

This expression is called the *mass function* and represents all that can be found out about the mass of the system *when but one spectrum is visible.*

If both spectra are visible, we know $a_2 \sin i$ and can compute

$$\frac{(a_1 + a_2)^3 \sin^3 i}{P^2} = (m_1 + m_2) \sin^3 i. \tag{8}$$

Since $m_2/m_1 = a_1 \sin i / a_2 \sin i = K_1/K_2$, the values of $m_1 \sin^3 i$ and $m_2 \sin^3 i$ may in this case be found separately.

If $a \sin i$ is given in *millions of kilometers,* and P in *days,* equation (8) becomes

$$(m_1 + m_2) \sin^3 i = \frac{(a_1 + a_2)^3 \sin^3 i}{25.0\, P^2}. \tag{9}$$

The proof of this is a good exercise for the student. The actual value of $\sin i$ can be found only when the star is also observable as a visual binary (with the telescope or interferometer) or as an eclipsing binary (§ 793). Except in these cases all that we can say for an individual system is that the mass is at least as great as the value given by (9).

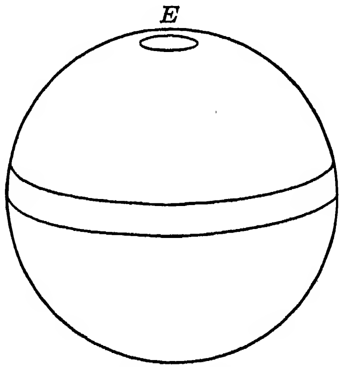


FIG. 245. The Mean Inclination

There is a much larger probability that the inclination lies between 80° and 90° than that it is less than 10°

For the average of a number of cases, however, the mean value of $\sin^3 i$ may be found on principles of geometrical probability. It might be supposed at first that all values of i were equally probable; but this is not so. The orbital inclination i is equal to the angle between the perpendicular to the orbit plane and the line of sight toward the earth. Consider the celestial sphere as centered upon the star (Fig. 245). If i is between 0° and 10°, this perpendicular, carried out to the celestial sphere, must meet it in a point P within 10° of the apparent position E of the earth; if i is between 80° and 90°, in a zone lying between 80° and 90° from E. The latter evidently covers a much larger portion of the sphere than the former. Here the chance

that P will be within it is much the greater of the two. In this way (by simple applications of the calculus) it may be shown that half the values of i will be between 60° and 90°; that the mean value of $\cos i$ will be $\frac{1}{2}$, and that of $\sin i$, $\pi/4$; while the mean value of $\sin^2 i$ is 2/3 and that of $\sin^3 i$ is $3\pi/16$, or 0.59.

For spectroscopic binaries with double lines, however, there will be fewer cases of small inclination than this theory demands; for if the inclination is small, K is small, the lines are not widely separated, and in many cases they will not be seen double. The average value of $\sin^3 i$ for such systems should therefore be taken a little larger, — probably about 2/3. If therefore the observed values of $m \sin^3 i$ are increased by 50 per cent, a good approximation to the true masses should, on the average, be obtained.

When but one component is visible, the mass function (which for the moment we may call f) may be expressed in the form

$$f = (m_1 + m_2) \sin^3 i \left(\frac{m_2}{m_1 + m_2}\right)^3.$$

In almost all binaries, spectroscopic and visual, for which the mass-ratio is known, m_2 is less than m_1, so that the mass function is less than 1/8 of the mass of the system. If, as in Procyon, $m_2 = \frac{1}{3} m_1$, this fraction reduces to 1/64. It is not surprising, therefore, that the mass function is very small for some spectroscopic binaries.

782. Other Possible Causes of Variable Velocity. For a number of stars, however, the computed mass function is less than 0.0001, — too small to explain in this fashion unless the companion is of very small mass compared with the primary. Moreover, this peculiarity is associated with others, being displayed, for example, by all the variable stars of the so-called Cepheid type (Chapter XXII).

There are good reasons to believe that in many such cases the stars are not really double at all, but are *changing in diameter*, alternately expanding and contracting. Such oscillations, or "pulsations," are theoretically quite possible under the combined action of gravitation and the elasticity of the gases of which the stars are composed. The observed changes of radial velocity would in such a case arise from the approach of the star's surface

toward us as it expanded, and the recession as it contracted. The period (depending on the density of the star) should be constant, but the range of velocity should be much smaller than in a true binary system of the same period (§ 843).

Otto Struve (the younger) has recently shown that among spectroscopic binaries the mean range K increases steadily for groups of stars of diminishing period (as it ought to do if the average mass is about the same) down to a certain limiting period, below which the range falls abruptly to much smaller values. This limiting period is a little more than a day for stars of spectra B and A, and about a week for classes K and M. The stars of shorter period than this, and some of those with longer period and small range, may be "pulsating" bodies and not really double. Polaris (Sp. F8, $P = 3^d.97$, $K = 2.7$ km./sec., $f = 0.000008$) and β Canis Majoris (Sp. B1, $P = 0^d.257$, $K = 9.1$ km./sec., $f = 0.00002$) are good examples. In most cases the variations of velocity are very regular; in a few, such as β Canis Majoris, the amplitude varies greatly.

A very few stars, such as ϵ Aurigæ, show apparently irregular changes of radial velocity, which are as yet unexplained.

783. Statistics of Spectroscopic Binary Stars. Orbits have been computed for 248 of the 1054 spectroscopic binaries in the *Third Catalogue*. Omitting 19 variable stars, which are probably not binary systems, it is found that 56 per cent of the rest have *periods* less than 10 days, and 26 per cent have periods between 10 and 100 days, leaving only 18 per cent with longer periods. The shortest period for a star which is certainly a true binary is 8 hours (for the eclipsing variable W Ursæ Majoris), and the longest period which has so far been followed through is 15.3 years (for ϵ Hydræ). The shortest observed periods probably represent about the shortest periods possible in binary systems for stars revolving practically in contact (§ 796), and the longer ones grade imperceptibly into those of visual binaries; indeed, several systems — δ Equulei, ϵ Hydræ, etc. — can be observed in both ways.

There is a significant relation between *period* and *spectral class*, the percentage with periods less than 10 days being 71 for classes O and B, 64 for Class A, 52 for F and G, and 16 for K and

M, while the percentage with periods longer than 100 days is represented by 12, 6, 18, and 61, so that the earlier-type stars tend to have shorter periods.

The *range in velocity* of the brighter stars extends from about 5 km./sec. (the smallest change readily detectable) up to 450 km./sec. The greatest relative orbital velocity of the two components, when double lines are visible, is 604 km./sec. for the eclipsing variable V Puppis. The average range diminishes steadily with increasing period, being roughly proportional to the cube root of the latter,— as it should be according to Kepler's harmonic law, for systems of the same average mass.

The *orbital eccentricities* are much smaller than for visual binaries, the general average being only 0.17. There is a significant increase with the period. For periods less than six days the mean eccentricity is only 0.06, and no individual values exceeding 0.25 appear, except for stars which may not be really binary. Large eccentricities appear rather suddenly at periods of 8 or 10 days, so that the mean eccentricity increases to 0.20 for periods between 6 and 20 days, and 0.32 for periods longer than 20 days, beyond which it varies but little.

The mean eccentricity of visual binaries is 0.50 for an average period of about 120 years; of physical pairs of long period, 0.61 and 0.76, respectively, for periods of about 2000 and 5000 years. There is evidently a steady increase in the mean eccentricity with the mean period over this long range.

784. Masses of Spectroscopic Binaries. When both spectra are visible, an inferior limit to the actual values of the masses of the components is given by the quantities $m \sin^3 i$. These are in general comparable with the sun's mass, but are sometimes a good deal greater. The most notable instance is $+ 6° 1309$, a sixth-magnitude star in the constellation Monoceros, for which Plaskett finds $m_1 \sin^3 i = 75$, $m_2 \sin^3 i = 63$. This is much the most massive star known, though there are several cases in which the computed masses exceed ten times that of the sun. Taking means for all the cases listed in the *Third Catalogue*, we find the data of Table XXIII. Here the stars are grouped according to spectral class, and the number in each group, except the first, is great enough to give a significant average.

TABLE XXIII. MASSES OF SPECTROSCOPIC BINARIES

SPECTRUM	NUMBER OF STARS	MEAN $m_1 \sin^3 i$	MEAN $m_2 \sin^3 i$	RATIO OF MASSES
O8	1	75	63	0.84
B0 to B2	8	12.2	9.4	0.77
B3 to B5	8	4.9	3.5	0.71
B8 to A3	22	2.3	2.0	0.87
A5 to F3	12	1.4	1.1	0.79
F5 to G5	12	1.2	1.1	0.89

The great increase in mass for the hottest stars is very striking. It is, however, only a particular instance of the general relation between mass and absolute magnitude, for stars of these spectral classes are known to be of exceptionally great luminosity (§ 751). A good approximation to the actual mean masses may be obtained by increasing the tabular numbers by 50 per cent.

The *ratio of the masses* of the components ranges in different systems from 1.00 to 0.28. When the spectra are nearly equal in intensity, the masses are nearly equal; and when the fainter star has a small mass, its lines are barely perceptible. The general mean of m_2/m_1 is about 0.8, with a slight tendency to be greater for the later spectral classes. It exceeds 0.8 for 61 per cent of the systems, and 0.5 for 93 per cent; but it would be quite unjustifiable to suppose that this was typical of spectroscopic binaries in general, for the pairs in which both spectra are visible form a selected group in which the components are nearly alike in magnitude. The average mass-ratio 0.8 corresponds (according to the curve of Fig. 241) to a difference in magnitude of about 0.6,— which is a reasonable value. For the systems in which only one spectrum is visible, the difference in magnitude, and the disparity in mass, are probably considerably greater. There is good evidence for this. According to Struve the average orbital velocity (K) for the bright component corresponds to a mass function $\dfrac{m_2^3 \sin^3 i}{(m_1 + m_2)^2}$ of 0.45 for Class B, 0.16 for A, 0.12 for F, and 0.08 for G. If the values of $(m_1 + m_2) \sin^3 i$ average the same as those given above, the values of $m_2/(m_1 + m_2)$ must be 0.31, 0.33, 0.37, and 0.32 in the four cases, — that is, m_2/m_1 must average about 0.50.

TABLE XXIV. SPECTROSCOPIC BINARIES

NAME	MAG.	SP.	PERIOD	e	ORBITAL VELOCITY K	$a \sin i$ MILLIONS OF KM.	$m_1 \sin^3 i$ / $m_2 \sin^3 i$	$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	PARALLAX	ABS. MAG.
+6° 1309	6.36	O8	14d.41	0.04	206	40.9	76.	13.2		
					247	48.9	63.			
η Ori . .	3.44 *v* *	B1	7 .99	0.02	145	15.9	11.2	2.51	0″.0057	−2.0
					152	16.8	10.6			−2.0
β Sco . .	2.90 *v*	B1	6 .83	0.27	126	11.4	13.0	1.26	0 .0085	−1.7
					197	17.8	8.3		± .0014	−1.7
ψ Ori . .	4.66	B2	2 .53	0.07	144	5.0	5.5	0.78	0 .0057	−0.8
					190	6.6	4.2			−0.8
α Vir . .	1.21 *v*	B2	4 .01	0.10	126	6.9	9.6	0.83	0 .014	−2.6
					208	4.4	5.8		± .003 †	−2.2
ζ Cen . .	3.06	B2	8 .02	0.50	312 ‡	29.8 ‡	16.4		0 .0217	+0.5
									± .0014	+0.5
μ Sco . .	3.09 *v*	B3	1 .45	0.05	480 ‡	9.5 ‡	16.5		0 .0074	−2.2
									± .0017	−1.8
β Aur . .	2.07 *v*	A0	3 .96	0.00	109	5.93	2.21	0.55	0 .034	+0.6
					111	6.04	2.17		± .006	+0.6
ζ UMa A §	2.40	A2	20 .54	0.54	69.2	16.4	1.67	0.41	0 .045	1.4
					68.8	16.4	1.66		± .002	1.4
o Leo. . .	3.76	F5	14 .50	0.0	54.0	10.8	1.30	0.24	0 .026	1.6
					63.1	12.6	1.12		± .007	1.6
α Aur . .	0.21	G0	104 .02	0.01	25.8	36.8	1.19	0.18		
					32.5	46.4	0.94			
29 CMa .	4.90	Oe	4 .39	0.16	218	13.0		4.58	0 .0071	−0.8
ι Ori . . .	2.87	Oe5	29 .14	0.75	110	28.9		1.14	0 .0051	−3.6
β Per . .	2.1 *v*	B8	2 .87	0.04	44.1	1.73		0.025	0 .027	−0.7
			688	0.13	10.0	93.0		0.070	± .010	
β Ari . .	2.72	A5	107 .0	0.88	32.6	22.9		0.042	0 .064	1.7
									± .006	
β Cap . .	3.25	G0	1375 .3	0.44	22.2	377.		1.13		
κ Peg A .	4.8	F5	5 .97	0.03	40.5	3.36		0.040		
ξ UMa A .	4.41	G0	665	0.41	7.0	58.3		0.018		
α Gem A .	1.99	A0	9 .22	0.50	13.6	1.49		0.0015		
α Gem B .	2.85	A0	2 .93	0.01	31.8	1.28		0.0097		

* The letter *v* denotes that the star is an eclipsing variable, which means that the inclination is near 90°.

† Parallax, if the star is a member of Kapteyn's Scorpius-Centaurus group.

‡ Relative orbital velocity and mean distance of the two stars, from measures on objective-prism plates.

§ Interferometer measures give $i = 50°$, $m_1 = m_2 = 3.7$.

785. Data for individual stars of interest are given in Table XXIV. The stars for which only one spectrum is observable are chosen to illustrate various characteristics,—large mass function, high eccentricity, long period, etc.; β Persei is interesting as a spectroscopic triple, the close eclipsing pair, with a period less than three days, revolving about the center of gravity of itself and a much remoter star in a period of a little less than two years.

The last four stars are components of visual binaries listed in Table XX — ξ Ursæ Majoris is of special interest, for Nörlund has detected a periodic variation in the relative motion of the visual components due to the motion of the brighter one in its short-period orbit. Hertzsprung finds that the apparent radius of the smaller orbit is $0''.052$, and its inclination 90°. The value computed from the observed parallax and the spectroscopic $a \sin i$ is $0''.056$. If, as the meridian observations indicate, the mass of the close pair is half that of the wider system, we should have $\frac{a'^3}{P'^3} = \frac{1}{2}\frac{a^3}{P^2}$. Introducing $a = 2''.51$, $P = 60^{y}.0$, $P' = 1^{y}.83$, we find $a' = 0''.195$ for the mean distance of the spectroscopic companion. It follows that in the close pair the mass of the companion is 0.38 times that of the visible star. The other component of the visual binary is also a spectroscopic binary of unknown, but probably short, period.

The average ratio of the mass function to the mass of the brighter component, when both are known, is 0.18.

The parallaxes of several of the stars are known, and these, with the absolute magnitudes, are given in the last two columns. The stars for which the data are fairly good are plotted in Fig. 241, taking the mean absolute magnitude of the components and assuming the mean mass to be 10 per cent greater than the mean $m \sin^3 i$ in the case of eclipsing variables, and 60 per cent greater in other cases, except where the inclination is known. The resulting points serve to extend the curve connecting mass with absolute magnitude. The extension of this curve indicates for Plaskett's star an absolute magnitude of about -6 for each component (or about 20,000 times the sun's light), and hence a parallax of $0''.00025$ and a distance of 4000 parsecs, or 13,000 light-years.

ECLIPSING BINARIES

This third class of binary stars was the first to be detected by observation. As long ago as 1670 Montanari noted that the second-magnitude star β Persei (Algol) was sometimes fainter than usual. Goodricke, in 1782, discovered that these variations were *periodic* and occurred at regular intervals of $2^{d}\ 20^{h}\ 49^{m}$. For about $2^{d}\ 11^{h}$ the star remains of substantially constant bright-

ness. During the next five hours it loses two thirds of its light and returns to its original brightness in the five hours following. Goodricke realized that this variation in brightness might be caused by the partial eclipse of the star by a large body revolving round it; but no other binary stars were known at that time, and his explanation was almost forgotten until revived by Pickering a century later.

About 200 stars are now definitely known to vary in this fashion, and a hundred or so more are suspected and await confirmation.

786. The Light Curve. Like their prototype Algol, these stars usually remain at a nearly constant magnitude for some time, following which their brightness decreases rapidly to a minimum.

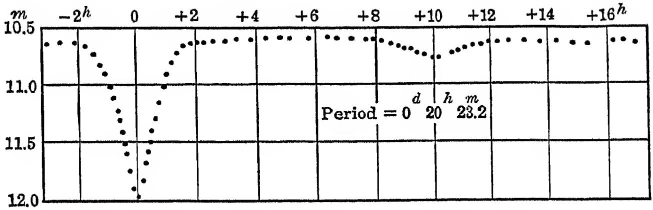


FIG. 246. Mean Light Curve of RT Persei

From observations by R. S. Dugan, Princeton University Observatory

The light may remain constant at the minimum for some time or only for a moment, but in either case the increase to normal is as rapid as the decrease. After remaining practically stationary for some time the brightness falls again, usually much less than before and sometimes almost imperceptibly, then rises again to normal and remains stationary for about the same time as before, after which the whole cycle is repeated with very exact periodicity. The deeper minimum is called the *primary minimum*; the other, the *secondary*.

The observations are typical of those of any variable star; the difference in brightness between the variable and a neighboring comparison star of constant brightness is repeatedly measured with a photometer and the time of the observation noted. Since the variation is very strictly periodic, observations may be made at any convenient time and all combined to form a mean light curve in a manner similar to that already described in the case of

a spectroscopic binary. Many hundreds of accurate observations, well distributed throughout the period, are necessary to precise definition of the mean curve.

Fig. 246 is a plot of the observations of the star RT Persei, period $20^h\ 23^m.2$. The dots, each of which represents about two hundred photometric readings, suffice to define the variation of light without drawing any curve. The primary minimum has a depth of $1^m.33$, and the secondary, $0^m.16$. Each minimum lasts about four hours, and the secondary comes almost exactly in the middle of the interval between successive primary minima.

787. Cause of the Light Variation. This characteristic change in brightness is readily explained on the assumption that the variable star is a binary pair with components usually differing in size and brightness, and that the orbital inclination is nearly 90°, so that the components eclipse one another during every revolution. When the fainter star begins to pass in front of the brighter, the light from the system begins to decrease.

If the inclination differs considerably from 90°, the eclipses will be partial; the light will be a minimum at the middle of eclipse and will begin immediately to increase (Fig. 247, β Aurigæ). If the orbit is very nearly edgewise toward us, one eclipse will be total and the other annular. There will be a stationary phase (Fig. 247, U Cephei) during the total eclipse while one star is hidden behind the other, and also one during the annular eclipse (if the star-disks are of uniform brightness).

If the orbit is circular, the eclipsed area is the same at the two minima (Fig. 249, β Aurigæ). The principal minimum therefore corresponds to the eclipse of the star which has the greater surface brightness.

788. The proofs of the eclipse theory are extremely convincing.

(1) Every light curve which has been accurately observed is found to be such as can be produced by the mutual eclipses of a binary pair.

(2) Every eclipsing variable which has been observed spectroscopically has proved to be a spectroscopic binary. The period is equal to that of the light variation, and the spectroscopic data place the bright star in superior conjunction at the time of pri-

mary minimum. When the light curve indicates that the two components are of comparable brightness, two spectra are visible.

(3) Many spectroscopic binaries, upon precise photometric observation, have been found to be eclipsing variables of small

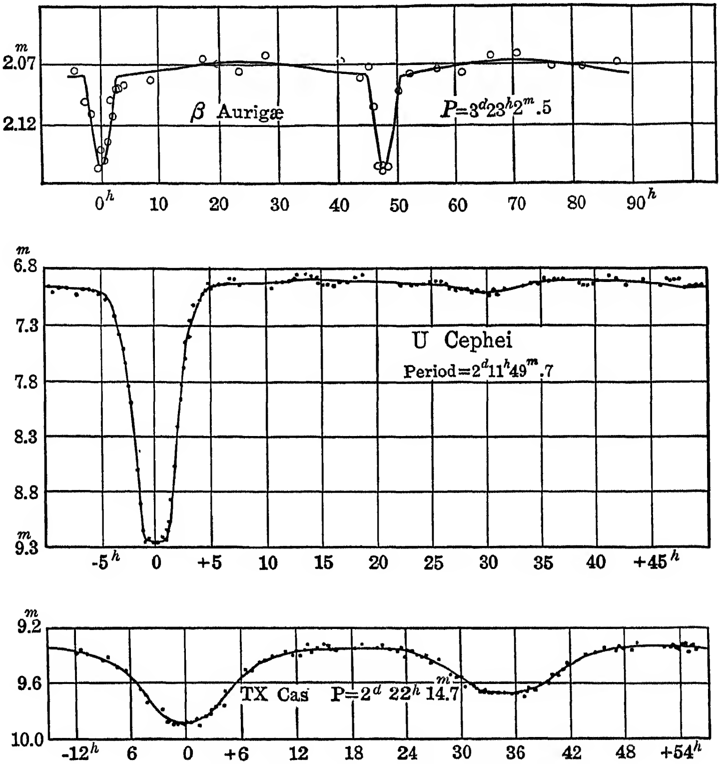


FIG. 247. Light Curves of Eclipsing Binaries

The observations of β Aurigæ were made by J. Stebbins, at the University of Illinois; of U Cephei by R. S. Dugan; of TX Cassiopeiæ by R. J. McDiarmid, at Princeton

range. There is again complete agreement between the spectroscopic and photometric results. Other spectroscopic binaries show no variation, as might be expected, since in most cases the orbits will not be so inclined as to make eclipses possible.

789. Characteristics of the Light Curve. The depths and shapes of the two minima depend primarily on the relative brightness of

the two stars, on the ratio of their radii, and on the inclination. The primary minimum corresponds to the eclipse of the star which has the greater surface brightness. The difference in surface brightness is often large (sometimes 20 to 1 or even 50 to 1), but there is almost always direct evidence that the companion gives some light.

If the two stars could be seen separately, it is doubtful whether their difference in brightness would ever exceed five magnitudes. This is confirmed by the fact that the observed loss of light during a total eclipse never exceeds four magnitudes.

The apparent orbit of one star about the other is usually a narrow ellipse. When the inclination is 90°, this becomes a straight line, and the transit during eclipse is central. The whole duration of eclipse, as a fraction of the period, depends mainly on the ratio of the sum of the radii of the stars to the radius of the orbit. The total phase is longest in proportion to the whole eclipse when the stars are very unequal in size and the transit central. Partial eclipses should theoretically be much more numerous than total ones; the shallow ones, however, stand a poor chance of being detected.

790. The light elements of an eclipsing binary are the period P, the time of the middle of some specified primary minimum, T_0 (which may be called the initial epoch), the magnitudes at maximum and minimum, and the duration of eclipse and that of the stationary phase, if present. The time of any other minimum is then given by the equation $T = T_0 + PE$, where E is the number of revolutions since the initial epoch.

The *phase* of an observation is the difference in time between the observation and the next preceding minimum, or it may be counted from the following minimum and taken as negative. By plotting the magnitude against the phase the *light curve* is obtained.

791. Example of the Determination of the Period. The star RV Ophiuchi was observed at minimum brightness at the following Julian dates (the decimal shows the fraction of a day since Greenwich mean noon). These give the moments of mid-eclipse derived from a plot of numerous photometric observations on each night, and should be correct within a few thousandths of a day. Successive differences appear in the third column.

NUMBER	DATE	INTERVAL	E
1	$2416604^d.701$		0
2	6641 .572	$36^d.871$	10
3	7334 .753	693 .181	198
4	8112 .739	777 .986	409
5	8138 .548	25 .809	416
6	8477 .764	339 .216	508

Each interval, barring the uncertainties of observation, must be an exact multiple of the period, and so also the difference of the two short ones, $11^d.062$. It appears on inspection that one third of this is $3^d.687$, and one seventh of $25^d.809$ is also $3^d.687$. The other intervals are approximately 10, 188, 211, and 92 times this quantity. Dividing them by these numbers, we obtain 3.6871, 3.68713, 3.68714, 3.68713.

The values derived from the three long intervals, which should be the most accurate, agree almost perfectly. The formula

$$Min = 2416604.701 + 3.68713\,E$$

therefore represents all these observations. It agrees also with all the other observed minima. The star was faint at all other times of observation which fell near the moments predicted by this formula, and at no times besides; hence this may be accepted as the correct period. A still more accurate value may be obtained by discussing all the observed times of minima by the method of least squares. In this case the resulting value was

$$P = 3^d.6871267.$$

792. Variations in Period. Almost all the eclipsing variables which have been long and well observed show small but unquestionably real alterations in their periods; the observed minima, after an interval of several years, may be as much as an hour or two from the predicted. These variations appear to be fluctuating rather than progressive, but it has not been possible to obtain formulæ which represent them well enough to allow of safe prediction. Some of these changes may arise from a revolution of the line of apsides resulting from the ellipticity of the stars, but the actual changes are complicated and very imperfectly understood.

793. The orbital elements of an eclipsing binary include, in the simplest case, the radii r_b and r_f of the bright and faint stars (in terms of the radius of the relative orbit as unit), the orbital inclination i, and the light of the two stars L_b and L_f (in terms of their combined light). Methods of computation by the aid of

tables have been developed, by means of which the elements may be expeditiously derived from the light curve.

From a precise light curve additional information can usually be derived. If the orbit is eccentric, the intervals from primary to secondary eclipse and from secondary to primary will, in general, be unequal, as also will the durations of the two eclipses. The eccentricity and longitude of periastron can be found from these differences. Most of the orbits are very nearly circular.

The light curve often shows that the brightness continues to increase after the eclipse is over, and reaches a maximum halfway between eclipses (Fig. 247, TX Cassiopeiæ). This indicates that the stars are not spherical, but ellipsoidal, in form. During the minima the presentation is endwise and the stellar disks have their smallest area; at quadrature they show the largest possible area and appear, therefore, the brightest. The ellipsoidal form is taken on because of mutual tidal action, which distorts the stars so that their longest axes point toward one another. The polar axis is the shortest, and the other axis in the equatorial plane, intermediate. This effect of ellipticity is found in nearly all the light curves that have been precisely observed. As might be expected, it is greatest when the stars are least separated. This effect is very conspicuous in β Lyræ, where the components are almost in contact, and stars which show it are often said to be of the β Lyræ type.

Often the light just outside the secondary minimum is greater than that just outside the primary. This indicates that the companion keeps the same face toward the brighter star and that this side is hotter and brighter than the other.

Finally there is evidence in some cases that the star-disks, like that of the sun, are darker at the limb than at the center. This usually reveals itself only by slight modifications in the form of the light curve; but when one eclipse is total and the other annular, the light is constant during the total eclipse, but during the annular phase is less at the middle, when the brightest part of the larger star is hidden, than at the second and third contacts. This has been observed in TX Cassiopeiæ (Fig. 247); the primary eclipse is annular with no stationary phase, and the secondary eclipse is total with constant light for 4.7 hours.

794. Visual and photographic observations have both been made upon a number of eclipsing variables. The visual and photographic light curves can be represented by exactly the same geometrical elements, but the relative brightness of the two stars is often different. In stars of large range the primary eclipse is always total or nearly so, and the star appears considerably redder at minimum than at maximum, so that the photographic range exceeds the visual, sometimes by almost a magnitude. It follows that the fainter component, which in these cases is the larger and of much smaller surface brightness, is of considerably lower temperature than the primary. This is confirmed by spectroscopic observation of U Cephei, which shows a spectrum of Class A0 at maximum and of Class K0 during the total eclipse.

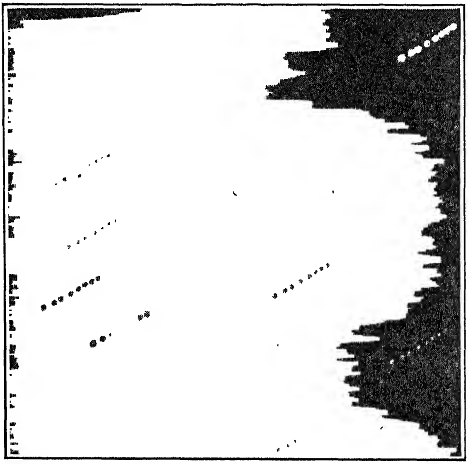


FIG. 248. A "Multiple-Exposure" Photograph of U Cephei Passing through Primary Minimum

Photographic light curves of eclipsing variables are commonly found from such a series of star-images obtained by successive exposures of equal length. The telescope or the plate must be moved slightly between exposures. Comparison is made between the images of the variable and those of close-by stars of constant brightness. (From photograph at Harvard College Observatory)

The typical eclipsing binary of large range consists, therefore, of a small, bright component of early spectral type and a large, faint companion of late spectral type. It would be quite wrong, however, to assume that the typical spectroscopic binary was of similar constitution, since it is only when the companion is large and faint that a conspicuous total eclipse can occur. If the star of low surface brightness is small, the range of variation will necessarily be small, and the variation not likely to be discovered. Stebbins has found a case of this kind in the spectroscopic binary 1 H Cassiopeiæ (Table XXVI, p. 720), and others are likely to be found in future among spectroscopic binaries.

When the components are equal in surface brightness, the two minima are equally deep (Fig. 247, β Aurigæ); the stars are similar in color, and the visual and photographic ranges are equal.

When the stars are nearly equal in surface brightness, the two minima are of nearly equal depth, never exceeding 0.75

β Aur

Sun

U Cep

RT Per

TX Cas

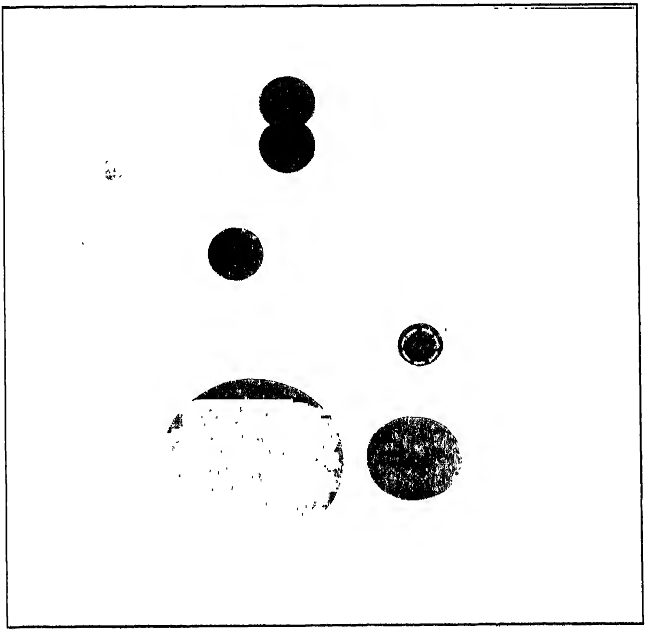


FIG. 249. Eclipsing Binary Systems

These drawings are proportioned on the assumption that the combined masses of the two components are equal in each case. The relative size of the sun is determined from the observations of β Aurigæ. The attempt has been made to indicate approximately the relative brightness in each system. No attempt, however, has been made to portray the darkening of the limb, of which the light curves of the three lower systems gave indication. It is this darkening which causes the curve of annular eclipse of RT Persei to look like a partial eclipse (Fig. 246)

magnitude, — corresponding to a loss of half the light. The components are similar in color, and the visual and photographic ranges are equal.

795. Real Dimensions. The photometric observations suffice to determine the relative dimensions in the system, and enable us to make a correctly proportioned drawing of it. To put the scale of miles on this drawing, spectroscopic observations are necessary,

and both spectra must be observable. In this case $(a_1 + a_2) \sin i$ can be found from the velocity curves, and i from the light curve, so that the actual distance between the components, $a_1 + a_2$, can be found, and then the diameters of the individual stars. This was the first way in which the diameter of a star was ever measured, and it is still the only direct method for the hotter stars (cf. § 825). Data are now available for about a dozen pairs, as summarized in Table XXV, p. 719. The diameters of most of these stars are between two and five times that of the sun. When only one spectrum is visible, only a_1 is known. The mass-ratio of the components, and hence the value of a_2, may be estimated from their known difference in brightness by the aid of the mass-luminosity curve (Fig. 241), and the diameters of the stars then found. The resulting data, however, will be considerably uncertain.

When the components are unequal and the bright star is halfway into eclipse, the observed radial velocity departs from the smooth velocity curve. The star is rotating with the orbital period, and at this time we get light only from the half which is turning away from us. An opposite effect occurs as the star comes out. The diameter of the star can be estimated from this effect, but only roughly, since the degree of darkening at the limb is unknown.

796. Densities. When the masses and dimensions of the components are known, their densities may be immediately calculated. Even without knowledge of the true dimensions of the stars a close approximation to the densities may be found as follows. For the mass of the system we have

$$m_b + m_f = \frac{(a_b + a_f)^3}{25.0\, P^2},$$

if a is measured in millions of kilometers, and P in days. The radius of the brighter component is $(a_1 + a_2)r_b$, where r_b is known from the photometric data. The volume V_b of this star, in terms of the sun's volume as unity, is $2.98(a_1 + a_2)^3 r_b^3$, since the radius of the sun is 695,000 kilometers. Hence the density, in terms of the sun's density as unity, is

$$\rho_b = \frac{m_b}{V_b} = \frac{1}{74.4\, P^2 r_b^3} \cdot \frac{m_b}{m_b + m_f}, \tag{10}$$

the unknown value $(a_b + a_f)^3$ canceling out. In the second member everything is known except the ratio $\frac{m_b}{m_b + m_f}$, and this may be estimated with considerable certainty from the mass-luminosity curve or from the relation

$$\frac{m_b}{m_f} = \left(\frac{L_b}{L_f}\right)^{\frac{2}{9}},$$

which agrees closely with the course of this curve for stars of the brighter absolute magnitudes.

It should be noted that systems of low density cannot have short periods. If, for example, the stars are of equal radius, r_b and r_f cannot exceed 0.5. But equation (10) may be written

$$P^2 = \frac{1}{74.4\,\rho r^3}.$$

Hence $$P^2 > \frac{1}{9.2\rho}, \quad \text{or} \quad P > \frac{1}{3\sqrt{\rho}}.$$

For a density 1/100 the sun's, the period must be longer than three days.

The information which can be obtained in this way regarding the densities of the stars is of great importance. Almost all eclipsing binaries are less dense than the sun. There is a conspicuous correlation between density and spectral class. Stars of Class B are of rather low density, the mean density for B0 to B3 being about 0.04,[1] and for B5 to B8, 0.08. Those of Class A (B9 to A3) are considerably denser, averaging about 0.15, and show a marked similarity in density. More than half of them have densities between one half and twice the mean value, and only 2 out of 48 have densities outside the limits which would be obtained by changing a star having the mean density to half or twice its diameter. For A5 to F2 the mean density is 0.4. There are few eclipsing binaries with spectra of Class F5 or later, but these fall definitely into two groups, one with mean density 0.9, and the other with densities lower than the average for stars of any other spectral type, ranging from 1/50 to 1/500,000 that of the sun. The significance of these very striking differences

[1] In cases like this the geometrical mean gives a better typical value than the arithmetical, and it is here adopted.

will be discussed in Chapter XXI. The highest known density is 2.6 times the sun's, for the faint companion of Castor. W Ursæ Majoris, which has the next highest density — 1.8 times the sun's, taking the average for the two components — and W Crucis, for which the mean density is a million times smaller, have well-observed light curves of the β Lyræ type, whose course can be represented accurately by the eclipse theory. It may seem incredible that a mass of gas, 1/500 as dense as ordinary air, could eclipse anything, but we have good evidence that gases of less than this density in the solar photosphere may be opaque in a thickness of a hundred miles or less.

797. Tables of Elements of Eclipsing Binaries. Table XXV comprises the eclipsing binaries whose real dimensions have been reliably determined; that is, the velocity curves of both components, and the light curve, have been well observed. Table XXVI contains a selection of eclipsing binaries for which there are good photometric data and which represent various types of these variables. For a few of them, spectroscopic observations of the brighter component have been made.

In both tables, column 2 contains the period in days; column 3, the spectral type; column 4, magnitude at maximum; column 5, above, the observed range in magnitude from maximum to the bottom of primary minimum, and, below, the corresponding amount for the secondary minimum. The symbols following the ranges have the following significance: T, total eclipse; An, annular; P, partial, large star in front; p, partial, small star in front; pq, partial eclipse of equal stars; G, grazing eclipse, large star in front; g, grazing, small star in front. Column 6 contains the inclination, i, of the orbit; column 7, the ellipticity, or ratio of the shorter to the longer equatorial axis of each star; column 8, the light, L_b, of the brighter star, and L_f, of the fainter star, in terms of the combined light as unit. From column 9 on, the two tables differ. In Table XXV, column 9 contains the radii of the two stars; column 10, their densities; and column 11, their masses, all in terms of the sun as unit. Column 12 gives the distance between the centers of the two stars — the radius of the relative orbit — in millions of kilometers.

TABLE XXV. ELEMENTS OF ECLIPSING VARIABLES OF KNOWN DIMENSIONS

NAME (1)	P (2)	SP. (3)	m (4)	RANGE (5)	i (6)	ELLIPT. (7)	L (8)	r $\odot = 1$ (9)	ρ $\odot = 1$ (10)	MASS $\odot = 1$ (11)	$a_1 + a_2$ (12)
											10^6 km.
V Pup .	$1^d.45$	B1 p	4.1	$0^m.64$ p	74°	0.88	0.60	8.45	0.044	19.4	8.83
		B3		0 .53			0.40	7.70	0.058	19.4	
Y Cyg*	2 .996	B2	7.1	0 .60 pq	85	—	0.58	4.6	0.17	16.6	19.25
				0 .40			0.42	4.6	0.16	15.3	
u Her .	2 .05	B3	4.6	0 .71 P	74	0.93	0.71	4.56	0.095	7.66	10.29
				0 .24			0.29	5.35	0.022	2.93	
σ Aql †	1 .95	B3	5.2	0 .18 pq	72	0.96	0.57	3.56	0.15	6.19	10.22
				0 .15			0.43	3.56	0.12	5.14	
Z Vul .	2 .45	B3	7.3	1 .65 G	89	0.92	0.75	4.23	0.085	5.24	10.49
				0 .34			0.25	4.46	0.033	2.36	
U Oph .	1 .68	B8	6.0	0 .69 pq	84	0.94	0.54	3.23	0.18	5.36	8.90
				0 .59			0.46	3.23	0.16	4.71	
TV Cas	1 .81	B9	7.4	1 .05 P	74	0.97	0.84	2.50	0.12	1.83	6.19
				0 .09			0.16	2.83	0.048	1.01	
RS Vul	4 .48	B8	6.9	0 .96 P	79	0.98	0.80	4.27	0.060	4.59	14.46
				0 .08			0.20	5.58	0.009	1.44	
β Aur†	3 .96	A0 p	2.1	0 .09 pq	77	0.99	0.50	2.81	0.11	2.40	12.31
				0 .09			0.50	2.81	0.11	2.36	
TX Her	2 .060	A2	8.1	0 .70 p	87	—	0.64	1.58	0.52	2.06	7.41
				0 .34			0.36	1.58	0.45	1.77	
S Ant	0 .648	F0	6.3	0 .44 p	62	0.75	0.67	1.66	0.31	0.75	2.30
				0 .40			0.33	1.29	0.38	0.42	
Z Her	3 .99	F5 p	7.1	0 .81 P	82	—	0.57	1.77	0.29	1.6	10.49
				0 .12			0.43	3.29	0.04	1.3	
W UMa	0 .334	G0	7.9	0 .60 pq	76	0.85	0.50	0.78	2.1	0.69	1.53
				0 .60			0.50	0.78	1.5	0.49	
αGemC†	0 .814	M	9.0	0 .54 pq	86	—	0.50	0.58	2.60	0.52	2.58
				0. 54			0.50	0.58	2.60	0.52	

* Photometric elements somewhat uncertain.
† Discovered first as spectroscopic binaries.

In column 9 of Table XXVI the radii of each pair of stars in the eclipsing systems tabulated are expressed in terms of the radius of the relative orbit as unit; and the densities, in column 10, are derived with the aid of the mass-luminosity law. Column 11 contains the radius of the orbit of the brighter star, in millions of kilometers, for the three cases where spectroscopic orbits have been determined.

The ratio of surface brightness, J_b/J_f, of the two stars may be found by dividing their light-ratio, L_b/L_f, by the square of the ratio of their radii. Thus, for β Persei,

$$\frac{J_b}{J_f} = \frac{0.93}{0.07} \div \left(\frac{21}{24}\right)^2 = 17.$$

The photometric elements, for most of the stars, have been derived on the assumption that the star-disks are darkened at the limb.

TABLE XXVI. ECLIPSING VARIABLES WITH WELL-OBSERVED LIGHT CURVES

NAME (1)	P (2)	SP. (3)	m (4)	RANGE (5)	i (6)	ELLIPT. (7)	L (8)	r (9)	ρ (10)	a_1 (11)
										10^6 km.
1 H Cas *	6d.07	B3	4.9	0m.13 An	85°	1.00	0.97	0.19	0.04	4.94
				0 .032			0.03	0.066	0.25	
TX Cas .	2 .93	B3	9.3	0 .55 An	88	0.93	0.83	0.57	0.0063	
		B5		0 .32			0.17	0.30	0.023	
β Per . .	2 .87	B8	2.3	1 .23 P	82	0.99	0.93	0.21	0.13	1.76
				0 .05			0.07	0.24	0.03	
U Cep . .	2 .49	A0	6.9	2 .28 T	86	0.96	0.84	0.20	0.14	
				0 .09			0.16	0.32	0.034	
RW Tau .	2 .77	A0	7.9	3 .47 T	90	—	0.96	0.19	0.21	
				[0 .02] †			0.04	0.25	0.02	
S Cnc . .	9 .48	A0	8.2	2 .12 T	85	1.0	0.86	0.10	0.10	
				0 .05			0.14	0.18	0.009	
RZ Cas .	1 .20	A0	6.4	1 .59 P	82	0.99	0.92	0.28	0.33	1.15
				0 .06			0.08	0.29	0.11	
RT Per .	0 .85	F ?	10.6	1 .33 An	89	0.99	0.88	0.32	0.40	
				0 .16			0.12	0.26	0.34	
W Cru . .	198 .5	G0p	8.7	0 .62 g	76	0.91	0.90	0.61	1.3×10^{-6}	
				0 .28			0.10	0.34	3.1×10^{-6}	
SW Lac .	0 .32	G2p	8.6	0 .78 P	73	0.78	0.54	0.42	1.62	
				0 .68			0.46	0.46	1.16	
RZ Oph .	262 .0	cG0	9.7	0 .83 T	87	—	0.53	0.046	0.001	
				[0 .07] †			0.47	0.15	3×10^{-5}	

* Eccentricity of orbit, 0.25.
† Secondary not observed; depth computed from elements.

798. The distribution of binary stars with respect to spectral class is exhibited in Table XXVII, which gives the total numbers of double stars of various kinds, and the percentages of these which fall into various spectral divisions; thus, of the 1054 spectroscopic binaries, 27 per cent are of Class B, 30 per cent of Class A, etc.

The upper part of the table deals with the stars brighter than $6^m.5$ (including practically all the known spectroscopic binaries); the lower part, with all stars for which the spectra have been observed (the average limiting magnitude being about 8.75).

TABLE XXVII. PERCENTAGE OF STARS OF VARIOUS SPECTRAL CLASSES

	NUMBER	O–B8	B9–A3	A5–F2	F5–G2	G5–K2	K5–M
Brighter than $6^m.5$							
All stars	8,773	9	32	12	11	29	7
Visual pairs	460	13	31	14	18	20	4
Spectroscopic binaries. .	1,054	27	30	11	14	15	3
Brighter than $8^m.75$							
All stars	98,675	5	24	11	19	35	8
Visual pairs	3,939	4	32	14	28	21	1
Eclipsing pairs	132	18	58	12	8	3	1
"Visual orbits"	110	1	16	15	47	17	4

The visual pairs include all double stars which satisfy Aitken's criterion (§ 763). A very few of them may be optical pairs, but not enough to affect the percentages at all. Among the brighter stars 17 per cent are binaries of some sort. No less than 42 per cent of the stars of Class B are known to be double, and the actual percentage is probably at least 50, as spectroscopic studies are not complete to $6^m.5$. The percentage of known binaries is 17 for Class A and also for Class F, 25 for Class G, 10 for Class K, and 8 for Class M.

The spectroscopic binaries (that is, the systems of short period) show a marked preference for the earlier spectral types, and especially for Class B. This cannot be an effect of observational selection, which works the other way, since small variations of radial velocity are easier to detect in the later types, when the lines are better to measure.

Among the fainter stars the percentage of visual doubles falls off, as might be expected, since these stars are, on the average, more distant, and so stand less chance of being telescopically resolved. The deficiency of double stars in classes K and M is pronounced. The eclipsing variables show a very strong concentration in classes B and A, like the spectroscopic binaries.

The visual binaries for which orbits have been computed are listed in the last line of the table. They show a strong concentration in Class G. This is known to be an effect of observational selection (§ 802).

EXERCISES

1. What is the limit of resolution of a 10-inch telescope for double stars in which the components are equally bright?

2. Is it larger or smaller for unequal pairs, and why?

3. What did Herschel expect to learn from the measurement of unequal pairs?

4. What are considered to be proofs of binary character?

5. If the ratio of the masses of the two components of a double star is 4/3, the orbits circular, and the inclination 60°, draw the orbits of the two components and the relative orbit in correct proportion.

6. When is it possible to find the value of m_2/m_1? the values of a_1 and a_2 (§ 781)?

7. Did the position angle of Krüger 60 increase or decrease from 1908 to 1913 (Fig. 235)?

8. What are the velocities of the centers of gravity of the spectroscopic binaries β Capricorni, p Velorum, α Aurigæ (Fig. 242)?

9. Give approximate values from Fig. 243 for the angle of periastron, ω, in these three systems.

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CHAPTER XXI

THE LUMINOSITIES, TEMPERATURES, AND DIAMETERS OF THE STARS

RELATION BETWEEN ABSOLUTE MAGNITUDE AND SPECTRAL CLASS · GIANT AND DWARF STARS · SPECTROSCOPIC PARALLAXES · C-STARS · STARS AS PERFECT RADIATORS · RELATION BETWEEN ABSOLUTE MAGNITUDE AND TEMPERATURE · TEMPERATURES FROM COLOR-INDICES, HEAT-INDICES, AND SPECTRAL ENERGY CURVES · RELATION BETWEEN ABSOLUTE MAGNITUDE, DIAMETER, AND TEMPERATURE · COMPUTED DIAMETERS AND DENSITIES · THE STELLAR INTERFEROMETER · RESULTS WITH DOUBLE STARS · MEASUREMENTS OF STELLAR DIAMETERS · CONFIRMATION OF THEORY · THE WHITE DWARFS AND THE RELATIVISTIC SHIFT OF SPECTRAL LINES · TABLE OF STELLAR TEMPERATURES

At the beginning of the present century little was known about the real brightness of the stars, and next to nothing concerning their actual dimensions and surface temperatures. Advance in these fields has been rapid, and ignorance has been replaced by extensive knowledge.

ABSOLUTE MAGNITUDES AND SPECTRA

799. The relation between the absolute magnitudes and spectra of the stars has proved of fundamental importance. The number of good determinations of parallax is now great enough to provide reliable information on this subject. The results are best exhibited graphically, by plotting the absolute magnitudes, M, against the spectral classes, as in Fig. 250. (Such a diagram was first made by Russell in 1913.) Bright stars are represented by points near the top of the diagram, faint stars by points near the bottom, red stars at the right, and white ones at the left.

The points are by no means distributed at random. The great majority (more than five sixths of the whole) fall in a fairly narrow belt extending diagonally downward to the right, from the bright white stars of classes B and A to the faint red ones of Class M. The stars in Orion's belt, Sirius, Procyon, the Sun,

61 Cygni, and Barnard's star all belong to this group, which Eddington calls the *main sequence*. Along this sequence the absolute magnitude falls steadily and rapidly, from about − 2.5

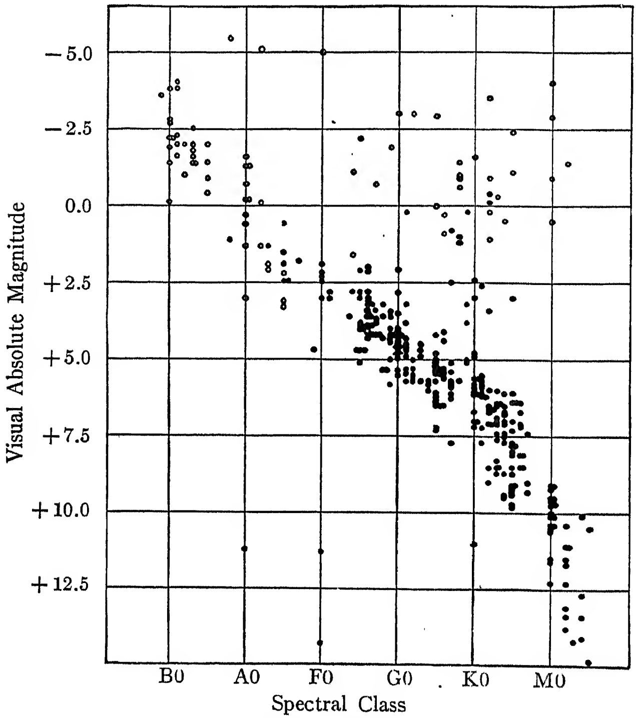


FIG. 250. The Relation of Absolute Magnitude to Spectral Class

Each dot in this figure represents a star; the ordinate is the visual absolute magnitude M, and the abscissa the spectral class. The sun, for which M is 4.8, and the spectrum is G0, is represented by the small x near the middle. The solid black dots represent those stars nearer than 20 parsecs for which fairly reliable parallaxes are available; the open circles, all stars of known parallax which are brighter than the third magnitude and more distant than 20 parsecs. These latter are included to give more data regarding the stars of high luminosity. All good determinations of parallax — trigonometric, spectroscopic, and from moving clusters — have been included. The vertical gaps in the diagram mean only that the classification of the spectra is a little rough. Thus stars which are really of classes K8 and K9 have been recorded as K7 or M0, leaving a blank space

for B0 to + 12.5 for M4, the intermediate values being approximately A0, 0.0; F0, 2.5; G0, 4.5; K0, 6.5; M0, 10.0. Nearly two thirds of the individual stars of the sequence are within one magnitude of the brightness given by this mean curve; and if

the errors of observation were eliminated, the agreement would doubtless be even closer.

Outside the main sequence there are a good many bright yellow and red stars and a few faint white ones. It is certain in both cases that the separation from the main sequence is real and not due to errors of observation.

800. Giant and Dwarf Stars. The stars of high luminosity are often called *giant stars*, and those of low luminosity *dwarf stars*. These felicitous names (proposed by Hertzsprung) were intended to describe only the relative *brightness* of the stars, though, as we shall soon see, they often (although not always) give a correct idea of their relative size.

The dwarfs are fainter and fainter the redder they are, as Fig. 250 shows, while all giants are of about the same brightness. Thus the distinction between giants and dwarfs is well marked only in the spectral classes later (redder) than F. Stars intermediate between the giants and dwarfs occur, except in Class M, where the difference in brightness is greatest; and even there such stars may exist, though none have yet been detected. The sun belongs to the main sequence, and ranks as a rather bright dwarf star.

The term "giants" is now often used in a slightly different sense, to denote those stars which are decidedly brighter than those of the same spectral class in the main sequence.

Certain very bright stars, much more brilliant than the ordinary giants, are sometimes called *super-giants*. Examples are β Orionis (spectral class B8), α Cygni (A2), α Carinæ (F0), and α Scorpii (M0), which are represented by the four points nearest to the top of Fig. 250.

The faint *white dwarfs* form a very distinctive group on the lower side of the main sequence and widely separated from it, so far as can be judged from the few such stars now known.

Detailed study and comparison of the characteristics of giant and dwarf stars is of great importance. The recognition of the distinction between giants and dwarfs is of great aid in interpreting many observed phenomena. As an illustration of the manner in which it brings order out of apparent chaos, we may take the spectra of the components of double stars.

801. The Spectra of the Components of Double Stars. These may be observed separately if the pair is not too close. The most thorough study has been made by F. C. Leonard. He finds that when the stars are equal in brightness, or nearly so, the spectra (with a very few exceptions) are very nearly alike. When the difference in magnitude is considerable, the spectra are usually different, and the difference in magnitude is roughly proportional to that in spectral type. In the majority of cases the fainter star is the redder, and of later type, by about half a spectral class for each magnitude of difference in brightness; for example, η Cassiopeiæ (3.7, F8 and 7.4, K5). In a considerable number, however, the fainter star is of earlier type, and the average difference in spectrum is about 1.2 classes for each magnitude of difference in brightness; for example, ϵ Boötis (2.7, G8 and 5.1, A1). There are a few stars, too, for which the spectra are of almost the same class, though the brightness is very different; for example, ϵ Hydræ AC (3.5, F9 and 6.8, F5). When the absolute magnitudes are studied, it is found that in pairs of the first sort the principal star is usually fainter than the absolute magnitude + 2, except in a few cases where it is of Class A or Class B. Both stars belong to the main sequence. In pairs of the other two sorts the principal star is always brighter than absolute magnitude + 2; that is, it is a giant. In pairs like ϵ Boötis the companion is also a giant, — whiter and somewhat fainter; in those like ϵ Hydræ it is a dwarf.

802. Relative Numbers of Giants and Dwarfs. Observational Selection. *In any given region of space* the dwarf stars far outnumber the giants (as is illustrated in Fig. 250, where the solid black dots represent the stars which are known to be within 20 parsecs of the sun), and the fainter dwarfs are the more numerous.

For the stars *visible to the naked eye* the reverse is the case, and the giants are in a large majority. The reason for this is that the method of selection is very different. In a list of stars above the fifth magnitude, for example, a super-giant of absolute magnitude − 5 will be included if it is within 1000 parsecs; a giant for which $M = 0$, if it is within 100 parsecs; a bright dwarf ($M = 5$), if nearer than 10 parsecs; and a fainter dwarf ($M = 10$), not unless it is nearer than 1 parsec. The region of space within which we are selecting stars for our catalogue is 1000 times

greater in volume in each case than for the following one, and this gives an enormous preference to the more luminous stars. Three of the four nearest stars are faint red dwarfs, but not one of them is visible to the naked eye. On the other hand, all four of the super-giants named in § 800 are first-magnitude stars, although more than 100 parsecs distant. Within this distance there must be more than 100,000 dwarf stars, telescopically visible, but for the most part indistinguishable at present from the millions of more distant stars of the same apparent magnitude.

Similar effects of *observational selection* appear in all star-counts, and great care must be taken to investigate its influence, and allow for it as far as possible, when attempting to interpret the observations.

The distribution of the proper motions among the naked-eye stars of different spectral classes (§ 728) now finds its explanation. The B-stars are all very bright and are sown very thinly in space, so that even the brightest are distant and have small proper motions. Exactly the same is true of Class N. For the A-stars the case is similar but less pronounced, and the two nearest stars (Sirius and Altair) have large proper motions. At the other end of the list the dwarfs of Class M are so faint that not one of them is visible to the naked eye; so, although they are very numerous in space and have very large proper motions, they are absent from our list, which contains only giants, distant and of small proper motion. A few dwarfs of Class K get into the list, and supply almost all the cases where the proper motion is large.

The proportion of dwarfs is greatest in classes F and G. The mean proper motion and the percentage of stars with considerable proper motion are much larger for these classes than for the others.

803. Parallaxes, Proper Motions, Colors, and Masses. Among stars of the same apparent magnitude the dwarfs, on account of such observational selection, are much nearer than the giants, and so have much larger parallaxes.

Their proper motions also are larger. This is explained mainly by their proximity, but partly by a greater average value of the velocities of the dwarfs in space. It would not be safe to conclude from this difference in proper motions that the average tangential

velocity of the dwarfs in our lists is high, since most of these dwarfs have been picked out for observation on account of their large proper motions. The radial velocities, however, which should be little influenced by this observational selection, are decidedly larger for the dwarfs.

There is also a decided difference in color between giants and dwarfs, the former being the redder. For stars of spectral class earlier than F5 this difference is very small, but it increases rapidly for the later types, and for classes K0 and K5 the difference of color-index exceeds half a magnitude, diminishing again

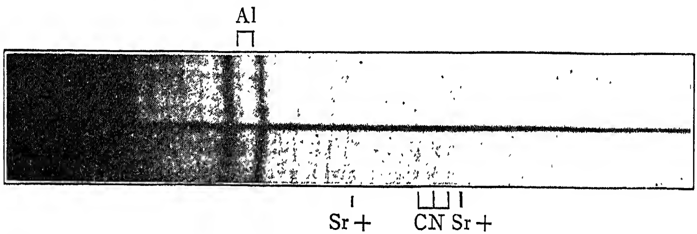


FIG. 251. Spectra of a Giant Star and of a Dwarf Star, showing Characteristic Differences

The upper spectrum is that of a dwarf, τ Ceti; the lower, of a giant, β Sagittæ. Both are of Class K0; but in the dwarf star the lines of neutral aluminum, $\lambda\lambda$ 3944, 3961, are more prominent; in the giant those of ionized strontium, $\lambda\lambda$ 4077, 4215, and the cyanogen bands between them, are strengthened. The spectrum of the dwarf extends farther into the ultraviolet, showing that it is not so red as the giant. (Harvard objective-prism spectra)

for Class M. Another important difference is that the masses of giant stars are considerably greater than those of dwarfs (§ 772).

804. Spectra of Giants and Dwarfs. The most important of the differences between giant and dwarf stars is found in their spectra. At first glance these seem to be just alike (so long as stars of the same spectral class are compared), but on closer inspection various minor but highly significant differences appear. Some lines are stronger in the giants, others in the dwarfs (Fig. 251); and the same differences recur from star to star and, in large measure, from one spectral class to another. What is more, these differences are greater, the greater the difference of absolute magnitude between the stars compared, so that, by examination of the spectrum, a skilled observer not only can tell whether the star is a giant or a dwarf but can estimate how bright it *actually* is.

Photometric measurements have already determined how bright it *looks*, and a comparison gives the star's parallax and distance.

Adams and Kohlschütter, in 1914, were the first to investigate thoroughly these differences in the spectra of giants and dwarfs, and to appreciate their full importance.

805. Spectroscopic Parallaxes. If a pair of lines is selected, one of which is strong in giants and the other in dwarfs, and the relative strength of these lines is estimated in the spectra of a number of stars of the same class and of known absolute magnitude, and if these estimates are plotted against the respective absolute magnitudes, a smooth curve is obtained. From this "calibration curve" the absolute magnitude corresponding to any given relative strength of the lines can be read off.

If now the spectrum of another star of this class, of unknown parallax and absolute magnitude, is studied, each line-pair of the sort just described gives a determination of its absolute magnitude M, the results from different pairs usually agreeing very well. When the absolute magnitude is known, the parallax p may immediately be found by the equation

$$5 \log_{10} p = M - m - 5. \tag{1}$$

This equation follows at once from the definition of absolute magnitude (§ 716). (The apparent magnitude m is nearly always known with sufficient accuracy.)

806. Their Accuracy and Limitations. This method of finding the distance of a star is one of the most extraordinary developments of modern astronomy. It seems at first acquaintance almost incredible, but thorough investigations by several workers have confirmed its usefulness and accuracy.

The best proof of these is found by comparing spectroscopic parallaxes with those directly determined, for the same stars, either by the trigonometric method (§ 711) or from moving clusters (§ 738). The agreement of the two sets of values is very striking, even in the case of stars not used in determining the calibration curves mentioned in the last section. After allowing for the errors of measurement in the trigonometric parallaxes, it is found that the probable error of a determination of the absolute magnitude of a star from its spectrum is only $\pm 0^{m}.4$. This

corresponds to a probable error in the deduced parallaxes of ± 19 per cent, — the same fraction of the parallax itself, whether the latter is large or small.

Whatever the size of the parallax, the trigonometric method gives, on the average, the same probable error in thousandths of a second of arc. For a single good modern determination this averages about $\pm 0''.008$. Thus, when the parallax exceeds $0''.05$, the spectroscopic method falls behind the trigonometric in accuracy, but for stars with smaller parallaxes (which form the vast majority) it surpasses the latter.

The labor involved in getting a spectroscopic parallax is relatively small, especially since the spectrograms on which it is based are valuable also for other purposes, such as the measurement of radial velocity or the study of other details of the spectrum. The number of published spectroscopic parallaxes exceeds two thousand and is greater than that of the determinations by the trigonometric method.

The principal limitation of the spectroscopic method is that, in its original form, it is applicable only to stars of Class F or later, since the earlier spectra are poor in lines and no suitable pairs have yet been found. Promising attempts have been made to extend it to the earlier spectral types, but the results are not yet as accurate as for the later classes.

Moreover, rather high dispersion is required to reveal clearly the fine details on which the method rests. Objective-prism spectra of sufficient dispersion are satisfactory (Fig. 251). Means of utilizing the low-dispersion spectra of faint stars may yet be found.

In a very few cases, there is a real disagreement between the spectroscopic and actual parallaxes. For example, the mean trigonometric parallax of Arcturus, $0''.080 \pm 0.005$, seems quite trustworthy; but the spectroscopic parallax has been determined at Mt. Wilson as $0''.16$, and at Harvard as $0''.21$. In such instances the strength of the line must be different from that which prevails in most stars of the same absolute magnitude and spectral class, owing doubtless to exceptional physical conditions. Such cases are rare, and are theoretically explicable (§ 941).

807. The "c-Stars." In the early days of the Harvard classification of spectra (1897) it was noticed (by Miss Maury) that in certain spectra the lines were exceptionally sharp, while some lines were unusually strong. Such spectra are now denoted by prefixing the letter "c" to the ordinary designation. Thus, β Orionis is described as cB8. Hertzsprung, in 1905, showed that these

"c-stars" had very small proper motions and must be of great luminosity. It is now clear that the "c" characteristics are those of giant stars in an exaggerated degree, — in fact, those of "super-giants" (§ 800). These very bright stars can be seen at enormous distances, and though they are very rare per unit volume of space, there are five of them among the twenty brightest stars in the sky.

The spectra of ordinary giant stars are similarly denoted by "g," and dwarfs by "d"; thus, Arcturus is gK0; the sun, dG0; and Procyon, dF5. The distinction between "g" and "d" is not practicable for classes earlier than F0, while the "c" characteristics are recognizable all the way from B0 to M.

These important spectroscopic differences, as explained later (§ 938), depend mainly on differences in density in the atmospheres of the stars.

STELLAR TEMPERATURES

808. Nature of the Problem. From the laws of thermal radiation it is possible to find, with considerable accuracy, the superficial temperatures of the stars, as well as of the sun. The final formulæ of the method are of remarkable simplicity. They lead to conclusions of great astrophysical importance and can be understood by the student who takes their derivation "on trust."

The proofs of these formulæ (unlike those of celestial mechanics) are not readily accessible in other textbooks, and are therefore included here (in small type); they may be omitted by the elementary student.

As in the case of the sun, the real, and very complicated, star may be represented by a simple idealized model (a black body, or perfect radiator (§ 606), of the same size), and the surface temperature determined at which the radiation of this body would resemble that of the star.

The comparison between the radiation of the actual star and that of the model may be made in several ways, each of which gives a determination of the temperature. Thus, section 812 gives the results when the match is made with respect to *color*; section 814, when *heat radiation* is the criterion; section 815 gives temperatures derived from a comparison of the actual and

the ideal spectral energy distributions; section 826, temperatures derived from the *surface brightness*. The agreement between all these sets of values is very good, considering the observational difficulties.

809. The absolute magnitude of a star which is a perfect radiator may be expressed by the following formulæ (derived below):

$$M_v = \frac{29{,}500}{T} - 5 \log R - 0.08; \tag{2}$$

$$M_p = \frac{36{,}700}{T} - 5 \log R - 0.72. \tag{3}$$

Here T is the absolute temperature; R, the radius of the star in terms of the sun's radius as unity; M_v and M_p, the absolute visual and photographic magnitudes, respectively, on the Harvard scale; and the logarithms are to the base 10.

The formulæ are not exact; but they almost always give the absolute magnitude more accurately than it can be observed for an object at stellar distances.

810. Derivation of these Formulæ. These equations, which are not so well known as they should be, may easily be derived from Planck's formula (§ 609), which may be written in the form

$$J_\lambda = C\lambda^{-5}/\left(10^{K/\lambda T} - 1\right), \tag{4}$$

where J_λ is the *surface brightness* (the amount of radiation between wave-lengths λ and $\lambda + d\lambda$ emitted per unit area per second), and C and K are constants. The whole surface of the star is $4\,\pi R^2$, and the whole amount of light of this sort which it emits is

$$L_\lambda = 4\,\pi R^2 J_\lambda. \tag{5}$$

If M_λ is the absolute magnitude, measured with light of this sort,

$$M_\lambda = 2.5\,(\log L_0 - \log L_\lambda), \tag{6}$$

where L_0 is a constant (§ 690).

From (5) and (6) we have

$$M_\lambda = 2.5 \log (L_0/4\pi) - 5 \log R - 2.5 \log J_\lambda;$$

while, from (4), setting $10^{K/\lambda T} - 1 = 10^{K/\lambda T}\,(1 - 10^{-K/\lambda T})$,

$$\log J_\lambda = \log (C\lambda^{-5}) - K/\lambda T - \log (1 - 10^{-K/\lambda T}).$$

Substituting, we find

$$M_\lambda = 2.5 \log (L_0\lambda^5/4\pi C) - 5 \log R + 2.5\, K/\lambda T + 2.5 \log (1 - 10^{-K\lambda/T}).$$

The first term is a constant depending on λ. Calling it C_λ, introducing the numerical value $K = 0.6240$, which is appropriate when λ is measured in centimeters, and calling the last term x, we have

$$M_\lambda = C_\lambda - 5 \log R + 1.560/\lambda T + x. \tag{7}$$

The term x is small and depends only on the value of the preceding term, as follows:

$\frac{1.560}{\lambda T}$	5.0	4.0	3.0	2.0	1.0
x	-0.01	-0.03	-0.07	-0.19	-0.55

Equation (7) holds good for monochromatic light, while both the eye and the photographic plate are sensitive over a range of wave-lengths. In either case, however, the net effect is very similar to that of light of a certain "effective" wave-length. According to a recent careful investigation by Brill, this is 5.29×10^{-5} cm. (λ 5290) for the Harvard visual observations, and 4.25×10^{-5} cm. for the Harvard photographic measures.

Substituting the former value in (7), visual magnitudes are given by

$$M_v = C_v - 5 \log R + \frac{29{,}500}{T} + x.$$

The constant C_v may be found from the fact (Chapter XVI) that a black body of the size and temperature of the sun ($R = 1$) would have about the sun's absolute magnitude (4.83). Then $\log R = 0$, $T = 6000°$, $x = -0.01$ and $M = 4.83$; whence

$$4.83 = C_v + 4.92 - 0.01; \quad \text{or} \quad C_v = -0.08.$$

For photographic magnitudes,

$$M_p = C_p - 5 \log R + \frac{36{,}700}{T} + x_p.$$

Taking the sun's absolute photographic magnitude as 5.40, we have

$$C_p = -0.72.$$

These equations are not strictly accurate, for the effective wave-length is less for very hot bodies, which appear white, than for cooler ones, which look red; and, moreover, the sun's "black-body" temperature is not exactly 6000°. In view of these approximations, and of the fact that the correction x is almost always smaller than the uncertainty of the determination of the actual absolute magnitudes of the stars, it is neglected in what follows. (Comparison with precise calculations shows, indeed, that this tends to compensate for the other approximations.) Thus the final equations are those given in section 809.

811. Color-Index and Temperature. Subtracting equation (2) from equation (3), the color-index (§ 696) of the ideal star is given by

$$I = M_p - M_v = \frac{7200}{T} - 0.64. \tag{8}$$

It thus depends only on the temperature. For low temperatures it is positive and may be very large; for high temperatures it becomes negative but remains numerically small and approaches the limit -0.64. There is therefore no theoretical limit to the "redness" of a star, but a very definite one to its "blueness." This conclusion is in accordance with observation. Some stars are very red, with color-indices as great as $+4$, but stars bluer than the theoretical limit have never been found.

According to the approximate formulæ of section 809 the absolute magnitude should also approach a limit as the temperature rises; but here the approximation fails, and the exact theory shows that the brightness should slowly increase without limit, varying proportionally to the temperature when this is very high. Up to a temperature of 25,000°, however, the approximation is fairly good.

812. Color Temperatures of the Stars. Equation (8) for color-index, when solved for T, gives directly the temperature at which a black body would give off light of the same color as any star:

$$T = \frac{7200}{I + 0.64}. \tag{9}$$

Introducing the observed values of I for the various spectral classes, the temperatures come out as follows:

Spectrum	I	T	Spectrum	I	T	Spectrum	I	T
B0	-0.33	23,000°	gG0	0.67	5500°	dG0	0.57	6000°
B5	-0.18	15,000	gG5	0.92	4700	dG5	0.65	5600
A0	0.00	11,200	gK0	1.12	4100	dK0	0.78	5100
A5	0.20	8,600	gK5	1.57	3300	dK5	0.98	4400
F0	0.33	7,400	gM0	1.73	3050	dM0	1.45	3400
F5	0.47	6,500	N	2.6	2200			

The color-indices from B0 to gM0 are those determined by King at Harvard. The value for Class N is taken from Parkhurst, and the differences between the dwarfs and giants from Seares (reduced approximately to the Harvard scale).

For all classes from B to M the color-indices of individual stars differ but little from these mean values (so long as giants and dwarfs are kept separate), and the same is probably true of their temperatures. In Class N the dispersion is much greater.

It will be noticed that the stellar temperature scale begins not far from where the scale of temperatures readily attainable in the laboratory leaves off. The hottest part of the crater of a carbon arc is at about 3800° K, corresponding to Class K2. Even the temperature of an average N-star is well above the melting point of platinum, and would ordinarily be called an intense white heat.

Whether or not these computed temperatures represent fair values of the real average temperatures of the surface layers of the stars depends upon whether or not the stars radiate like black bodies. There is good evidence, as we shall soon see, that in general this is very nearly true.

The stars of Class O (which, from the character of the lines that appear in their spectra, must be the hottest of all) are slightly yellower than those of Class B; and classes M5 and M8, though showing far heavier bands of titanium oxide, are no redder than Class M0. In the latter case the bands themselves, which lie in the green, yellow, and red, and cut off much of the visual light, may suffice to explain the discrepancy. The former case is unexplained. Fortunately other methods enable us to get fairly good values of the temperature in both cases.

More precise determinations of color temperature can be made by measuring photometrically the intensity of the spectra of the stars for different wave-lengths. These observations are difficult, but several investigators have obtained results which show that the values obtained by our approximate formula are nearly correct.

813. Radiometric Magnitude and Heat-Index. With a suitable thermoelectric device the heat radiation of the stars may now easily be measured and the results expressed as "*radiometric magnitudes*," m_r; when the parallax is known, these can be converted into absolute radiometric magnitudes, M_r. The difference, $M_v - M_r$, between the visual and radiometric magnitudes is called the *heat-index*. Positive values denote that the star gives out much heat in proportion to its light, or little light in proportion to its heat. The scales are so adjusted that the heat-index is zero for stars of Class A0.

The actual amount of heat received from the stars is very small. For α Orionis, which sends us more heat than any other, Pettit finds, for the total radiation, when the star is in the zenith, 5.4×10^{-5} ergs/cm.^{2}sec., or 7.7×10^{-11} calories/cm.^{2}min.

Even when concentrated by the 100-inch mirror, this amounts to only two calories per year. Sirius, though ten times as bright to the eye, sends us only 75 per cent as much heat as α Orionis.

814. Temperatures from Heat-Indices. By Stefan's law the total radiation from unit area of a black surface is given by the equation (§ 607)

$$E = \sigma T^4.$$

Proceeding as in section 810, it appears that

$$M_r = C_r - 5 \log R - 10 \log T.$$

The situation is greatly complicated, however, by the fact that the earth's atmosphere is completely opaque to radiation of wave-length less than about λ 3000, and has heavy absorption bands in the infra-red. The effect of the latter is not great for radiation corresponding to temperatures above 2000°, though important for cool bodies like the planets; but the former opacity is very serious for hot bodies, when the maximum of the energy curve (§ 608) moves out into the ultra-violet.

If it were not for this the heat-index would be a minimum for a temperature of about 7000°, and would increase both for low temperatures, as the bulk of the energy radiated by the body moved into the infra-red, and for high temperatures, as it moved into the ultra-violet. But actually, at very high temperatures, almost all the ultra-violet radiation is cut off by the atmosphere, and the part of the heat radiation which gets through is less, in proportion to the visible light, than at lower temperatures. The observed heat-index is therefore nearly constant for the hotter stars, and increases rapidly for the cooler ones.

For temperatures less than about 6000° this effect is not serious, and the equation for M_r given above can be used. Taking, as before, the sun as a standard, for which $M_r = 4.2$ (since the heat-index is about $+0.6$) and $T = 6000°$, the value found for C_r is 42.0, so that

$$M_r = 42.0 - 5 \log R - 10 \log T, \tag{10}$$

and the heat-index is

$$M_v - M_r = 10 \log T + \frac{29{,}500}{T} - 42.1.$$

The observed values of the heat-indices, according to Pettit and Nicholson, with the temperatures computed by solving this equation, are as follows:

Spectrum	B0	A0	F0	G0	K0	M0	M5	N
$M_v - M_r$	− 0.1	0.0	0.3	0.7	1.2	2.3	3.0	3.1
T	——	——	——	5500°	4200°	3100°	2750°	2700°

No temperatures are given for the whiter stars, since the simple formula does not apply to them. Those derived for the redder stars are in good agreement with the values found in section 812 except for Class N. There are heavy bands in the violet part of the Class N spectrum, which may well make the color-index too great and the temperature computed from it too low.

Much greater heat-indices are found among the variable stars of long period (§ 847) which have spectra of Class Me. For a number of these stars the heat-index averages 4.3 at maximum brightness and 7.8 at minimum, corresponding respectively to temperatures of 2300° and 1650°.

815. Spectral Energy Curves of Stars. A remarkable advance was made by Abbot in 1923, when, with the great Mt. Wilson telescope, a specially designed spectroscope, and a very delicate radiometer, he succeeded in measuring the energy in different parts of the spectra of a number of bright stars, including the infra-red region. In this way the spectral energy curves for these stars were directly observed. Fairly trustworthy values of the temperatures can be found by fitting a Planckian curve (§ 609) to such observations. Such determinations, for several bright stars, are as follows:

Star	Spectrum	T	Star	Spectrum	T
β Orionis	B8	16,000°	α Tauri	K5	3000°
α Lyræ	A0	14,000	α Orionis	M0	2600
α Can. Majoris	A0	11,000	β Pegasi	M5	2850
α Can. Minoris	F5	8,000	α Herculis	M6	2500
α Aurigæ	G0	5,800			

These make the white stars hotter, and the red stars cooler, than the values previously given, but the differences are hardly beyond the uncertainties of the measures.

STELLAR DIAMETERS

816. Relation between the Absolute Magnitude, Diameter, and Temperature of a Star. A curious and important consequence of the above equations is that they may be used to calculate the diameter of the model star if its absolute magnitude and effective temperature (or color-index) are known. In view of the evidence

that stars actually radiate much like black bodies, the estimates of diameter thus obtained should be fairly trustworthy. Furthermore, these are confirmed by two independent observational methods of measurement, discussed below, — direct measurement of stellar diameters with the interferometer, and their calculation from the light curves of eclipsing variables.

The first equation of section 809 is equivalent to

$$\log R = \frac{5900}{T} - 0.20\, M_v - 0.02\,; \tag{11}$$

whence, eliminating T by the equation (8),

$$\log R = 0.82\, I - 0.20\, M_v + 0.51. \tag{12}$$

These equations may be used to calculate the radius of a star (in terms of the sun's radius as unity) if its absolute magnitude and effective temperature (or color-index) are known.

The linear diameter of a star may thus be found if its real brightness is known. Even if the parallax is unknown, the apparent (angular) diameter can be found from the apparent magnitude. If p is the parallax and d'' the diameter, both in seconds of arc,

$$d'' = \frac{1}{107}\, pR \tag{13}$$

(since the sun's diameter is 1/107 of an astronomical unit) and

$$\log d'' = \log p + \log R - 2.030.$$

Making use also of equation (11), and of that defining absolute magnitude (p. 729),

$$\log d'' = \frac{5900}{T} - 0.20\, m_v - 3.05\,; \tag{14}$$

or, eliminating T as before,

$$\log d'' = 0.82\, I - 0.20\, m_v - 2.52. \tag{15}$$

It is clear that the largest diameters will be found when m is small and I large (that is, for bright red stars), while faint white stars will give the smallest values.

817. Diameters and Densities of Typical Stars. As an example of the use of these equations they may be applied to Sirius. For this star m_v is -1.6, p is $0''.37$, M_v is $+1.3$. The spectrum is A0, and the corresponding color-index is 0.00. Equation (12) gives

$\log R = 0.0 - 0.26 + 0.51 = 0.25$, and equation (15) gives $\log d'' = 0.0 + 0.32 - 2.52 = -2.20$, whence $R = 1.8$ times the sun's radius, $d = 0''.006$. It is of no use to give another place of decimals, for an error of $0^m.05$ in the color-index (which is not at all improbable) changes the computed diameter by 10 per cent.

The results of similar calculations for a number of typical stars are given in Table XXVIII (p. 740). The first twelve stars belong to the main sequence, and for these the computed radii come out much alike except for the first and the last. Thus it appears that the stars of the main sequence are of about the same size as the sun. The whiter ones are only a little larger, and owe their brightness more to their high temperature than to their sizes, while the faintness of the red dwarfs arises mainly from low temperature. The four giant stars have much larger diameters. The two red super-giants are enormous, — α Orionis larger than the orbit of the earth, and α Scorpii than that of Mars. The calculated diameters of the three white dwarfs, on the contrary, are all smaller than that of Uranus; indeed, the last one (6000 miles) is less than that of the earth.

The angular diameters are greatest for the bright red stars (as should obviously be true), but even these stars are too distant to show as perceptible disks in the greatest telescopes.

The masses of many of the stars listed in Table XXVIII are known, since they belong to binary systems, and those of the others may be estimated from the absolute magnitudes by means of the mass-luminosity curve (§ 772). The densities of the various stars may then be computed. The results are given in the last two columns of the table. Masses derived from the curve are placed in parentheses.

The densities computed for the stars of the main sequence from B to G are very similar to those already found for the eclipsing variables (§ 796), and those of the redder stars of the main sequence form a natural extension of the series. The low densities found for Capella, Arcturus, and even β Pegasi may be matched among eclipsing variables (RZ Ophiuchi, W Crucis).

But the gigantic diameter of a star like Antares, and its extraordinarily low mean density (only 1/3000 of that of air under standard conditions) are very remarkable; and confirmation of

TABLE XXVIII. CALCULATED STELLAR DIAMETERS

	m_v	sp	p	M_v	T	R	d	μ	ρ
Main sequence									
β Centauri . .	0.9	B1	0″.011	−3.8	21,000°	11	0″.001	(25)	0.018
ν Scorpii . . .	4.3	B3	.009	−0.8	17,000	3.2	.0003	(5.2)	0.16
β Aurigæ A . .	2.8	A0	.034	0.6	11,200	2.4	.0008	2.2	0.13
α Lyræ	0.1	A0	.124	0.6	11,200	2.4	.003	(3.0)	0.11
α Can. Maj. A .	−1.6	A0	.371	1.3	11,200	1.8	.006	2.4	0.42
α Aquilæ . . .	0.9	A5	.204	2.5	8,600	1.4	.003	(1.7)	0.6
α Can. Min. .	0.5	dF5	.312	3.0	6,500	1.9	.006	1.1	0.16
α Centauri A .	0.3	dG0	.758	4.7	6,000	1.0	.007	1.1	1.1
70 Ophiuchi A .	4.3	dK0	.192	5.7	5,100	1.0	.002	0.9	0.9
61 Cygni A . .	5.0	dK7	.300	8.4	3,800	0.7	.003	(0.45)	1.3
Krüger 60 A .	9.2	dM3	.257	11.2	3,300	0.34	.0008	0.26	9.
Barnard's star .	9.7	dM4	.538	13.4	3,100?	0.16	.0008	(0.18)	45:
Giants									
α Aurigæ A . .	0.9	gG0	.063	−0.1	5,500	12	.007	4.2	0.0024
α Boötis* . . .	0.2	gK0	.080	−0.3	4,100	30	.023	(8)	0.0003
α Tauri . . .	1.1	gK5	.057	−0.1	3,300	60	.034	(4)	2×10^{-5}
β Pegasi . . .	2.6	gM5	.016	−1.4	2,900	170	.025	(9)	2×10^{-6}
α Orionis . . .	0.9	cM0	.017	−2.9	3,100	290	.048	(15)	6×10^{-7}
α Scorpii A . .	1.2	cM0	.0095	−4.0	3,100	480	.042	(30)	3×10^{-7}
White dwarfs									
α Can. Maj. B .	8.4	F	.371	11.2	7,500	0.034	.00012	0.96	27,000
40 Eridani B .	9.7	A0	.203	11.2	11,000	0.019	.00004	0.44	64,000
van Maanen's star	12.6	F	.255	14.5	7,500	0.007	.00002	(0.14)	400,000

*See section 941.

NOTES. The letters A and B denote the brighter and fainter components, respectively, of binary stars.

Apparent (visual) magnitude is denoted by m_v, spectral class by *sp*, parallax in seconds of arc by p, absolute (visual) magnitude by M_v, absolute temperature (centigrade degrees) by T, radius in terms of the sun by R, apparent diameter in seconds of arc by d, mass in terms of the sun by μ, and density by ρ (in gm./cm.³).

these extraordinary values by other modes of observation is highly desirable. The white dwarfs are stranger still. There can be no doubt about the masses of the first two of these, but the computed densities are more than forty thousand times that of water and nearly two thousand times as great as that of any known form of matter. Some independent test of such an amazing conclusion is urgently required.

Fortunately, independent and conclusive tests are available in just the cases where they are wanted.

818. The Stellar Interferometer. The capacity of a telescope for revealing the true form of a small object — its "resolving power" (§ 53) — is limited by the size of the diffraction, or spurious,

disk produced by a point-source of light. The stellar interferometer is an instrument, invented by Michelson, which makes use of the very property of light which produces the diffraction disk to overcome the difficulty thus presented. It has been employed to measure the separation and position angle of the components of close double stars, but its primary use is to measure the angular diameters of isolated stars.

Essentially it consists of a telescope so arranged that light reaches the focus only through two small areas of the objective. These areas may be of any shape; they are on the same diameter, at opposite sides of the objective and equidistant from the center.

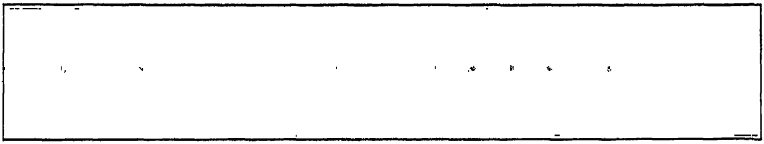


FIG. 252. Fringes as seen with the Interferometer

These photographs by Anderson are of images of an "artificial double star," taken with a small interferometer (the actual telescopic images are too small to photograph). The diffraction disks at the two ends show fringes of maximum visibility. For those in the middle the fringes of the two components are out of phase, and the visibility is nearly zero

Appearance of fringes. When this device is pointed at a star, the star-image is observed in the usual manner, through an eyepiece. If one of the open areas is temporarily screened, the image presents the normal appearance, just as though the whole objective were acting, except that with the diminished effective aperture the size of the diffraction disk is correspondingly increased (§ 53). When the screen is removed, however, the diffraction disk is seen, with a sufficiently powerful eyepiece, to be crossed by a number of straight equidistant, parallel dark "fringes" (Fig. 252) running at right angles to the direction of the line joining the two apertures. The light which normally would appear in the dark strips of the star-image is diverted into the intervening ones, so that there are alternate bright and dark fringes, with a bright one always crossing the exact center of the diffraction disk.

These are produced by the "interference" of light from the two unscreened areas of the objective. It is found that the distance x between adjacent bright and dark fringes is less

the greater the separation D of the centers of these apertures, according to the theoretical formula (§ 823)

$$\frac{x}{f}=\frac{1}{2}\frac{\lambda}{D}, \tag{16}$$

where f is the focal length of the objective and λ is the "effective" (or average) wave-length of the light.

Suitable apparatus is provided for varying the distance D, and thus for adjusting x to a desired value.

Fringes due to two stars. Now if two stars are separated in the sky by the small angle α (in radians), the distance y between the centers of their diffraction disks in the focal plane of the objective is given by elementary optical principles as

$$y/f=\alpha. \tag{17}$$

With the arrangement just described, the superposed images of the two stars are each crossed by a separate system of fringes. Since light from one source does not interfere with light from another source, the fringes in one star-image are formed independently of those in the other.

819. Method of Use on Double Stars. If the instrument is suitably adjusted, the bright fringes of one star may be made to fall on the dark fringes of the other (and of course the bright fringes of the second on the dark fringes of the first). When this condition is realized, the overlapping fringes disappear altogether if the stars are equally bright. The measurement with the interferometer of the angular separation of two close components of a double star involves the adjustment of the aperture-distance D until the fringes disappear owing to such coincidence (Fig. 252).

If the line joining the two unscreened areas is parallel to that joining the star-images, coincidence obviously will occur if the distance x between adjacent bright and dark fringes is made equal to the distance y between the central bright fringes of the two diffraction disks. Accordingly, if D is small at first and is gradually increased, the disappearance of the fringes is proof that x and y are equal, or, combining the two formulæ, that

$$\alpha=\frac{1}{2}\frac{\lambda}{D_0}, \tag{18}$$

where D_0 is the critical value of D (which is easily read from a suitable scale attached to the instrument).

As the value of λ can be determined from suitable observations of the starlight, that of α, the angular separation, is readily calculated.

Fig. 253 illustrates schematically the type of interferometer most suitable for the measurement of double stars. It was devised by J. A. Anderson and successfully employed by him in 1920 to measure Capella (separation of components about 0''.045), which previously had been observed only as a spectroscopic

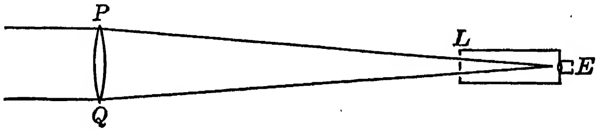


FIG. 253. Double-Star Interferometer

This consists of a plate placed in the converging beam of light from the objective and pierced with two parallel slits (at right angles to the plane of the figure). The distance of the slits can be varied, and the plate can be rotated about the axis of the telescope. When the slits are set in the position angle of the close pair, the fringes produced by the two stars are coincident. If the plate and the slits are rotated from this position in either direction, the fringes get out of step and neutralize one another when the plate has been turned through an angle θ such that $\alpha \sin \theta = \lambda / 2\, D_0$. The distance D_0, to be substituted in this formula, is not the actual comparatively small distance between the apertures but that between their projections on the objective; it is read from a convenient scale on the instrument

binary. Position angle (§ 764), as well as separation, may be conveniently determined with this device.

820. Results with Double Stars. The study of Capella was continued by Merrill, and the orbit was computed from the interferometric data. The agreement with the orbit spectroscopically observed was good; and a comparison of the two led to a determination of the parallax (0''.063), which agreed well with the mean trigonometric determination (0''.067) and with the spectroscopic parallax (0''.076). The two components of Capella are nearly equal in brightness — a favorable condition for measurement.

Merrill and Anderson (1921), while testing an interferometer, found that κ Ursæ Majoris was a binary; the brightness of the two components is the same within half a magnitude or so; and the separation at that time was measured as 0''.0836. Unknown to these observers, Aitken, in 1907, had found that this star was a

visual binary; the separation was then about twice as great. About forty other stars were examined by Merrill; only five or six of these gave evidence of binary character.

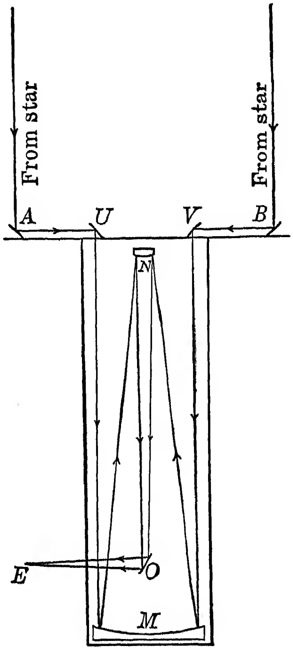


FIG. 254. Beam Interferometer

This diagram illustrates the optical arrangement used in the measurement of a star's diameter. Four plane mirrors A, B, U, V are mounted on a steel beam bolted across the tube of the 100-inch reflecting telescope. The light from the star follows the paths indicated and ultimately reaches the eye at E. The aperture-distance D is that between the outer mirrors A and B, which are movable along the beam

In 1925 Pease, with the 20-foot interferometer, resolved the interesting second-magnitude star ζ Ursæ Majoris.

This star, called Mizar by the medieval Arabian astronomers, possesses a fourth-magnitude companion at a few minutes' distance, called Alcor; the pair form a naked-eye double known from the earliest times. Mizar itself was the first star observed to be telescopically separable into a pair (§ 761). This telescopic pair was the first binary to be measured photographically, by Bond in 1857; and one of the components was the first star to be announced as a spectroscopic binary, in 1889, by E. C. Pickering. Thus Mizar itself is a triple star, — a visual double, with one component spectroscopically double.

Though the period of the close pair is only twenty days, it proved to be just within the limits of resolution of the beam interferometer. The observations showed a change of 45° in position angle in four days, with distances varying from 0″.013 to 0″.011. The agreement with the spectroscopic orbit, and with the parallax deduced from the group motion, was excellent.

821. The Measurement of Stellar Diameters. The application of the stellar interferometer to close double stars is merely secondary, since there are surprisingly few objects accessible to it and not to ordinary methods of measurement. The measurement of stellar diameters can be effected by no other instrument.

This problem is more difficult but not essentially different. The measurement is made in the same way, by increasing the distance between two apertures until the fringes disappear.

To understand how this happens, the disk of a single star may be thought of as two half-disks, and all the light in these thought of as concentrated at two points near the respective centers of area of the two half-disks. Thus the single star is, from this standpoint, roughly equivalent to an extremely close double star.

If the angular diameter is β, the angle between the effective centers of the two half-disks is $0.41\,\beta$. Writing this as the equivalent of α in the double-star formula (18),

$$\beta = 1.22\,\frac{\lambda}{D_0}, \qquad (19)$$

where D_0, as before, is the distance between the apertures when the fringes vanish.

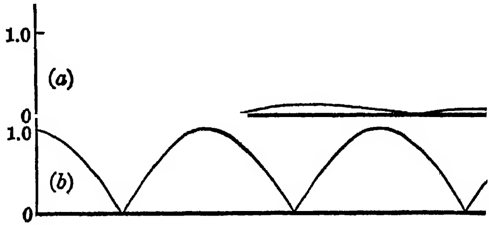


FIG. 255. Visibility of Interference Fringes for Different Separation of Apertures

The upper curve represents the "visibility" of the fringes, that is, the degree of contrast, for a uniformly luminous star-disk, as the separation of the interferometer apertures is increased. The lower curve shows the same relation for a double star of separation equal to the diameter of the same disk. In the latter case the visibility returns repeatedly to its initial value as the separation increases. In the former the fringes, after once disappearing, return with only feeble contrast

This formula applies to a star-disk of uniform brightness. If the disk is darkened at the limb, the effective centers of the half-disks are closer together. For darkening similar to the sun's the correct formula can be shown to be

$$\beta = 1.43\,\frac{\lambda}{D_0}. \qquad (20)$$

The beam interferometer. Almost all stellar disks are so small in apparent diameter that the condition of fringe disappearance requires a separation D_0 of the apertures considerably greater than the diameter of the largest existing objective,—the 100-inch at Mt. Wilson. Fortunately this separation is not restricted to the diameter of the objective but may be increased by mounting auxiliary mirrors on a long steel beam placed across the upper end of the telescope (Fig. 254). In this way, with an instrument in use at Mt. Wilson, D_0 can be increased 20 feet. (A second interferometer, much more powerful, with 18-inch mirrors and a 50-foot beam, is under construction.)

822. Visibility of Fringes. Even the beam interferometer permits observation of the actual disappearance of fringes for only a very few stars. As the distance between the apertures is increased, however, the fringes in the images of certain other stars become less and less distinct, indicating that with a longer beam their actual disappearances could be observed. Suitable photometric measurements, in such a case, yield a curve connecting the conspicuousness, or "visibility," of the fringes with the distance apart of the apertures; and a comparison of this observed curve with a similar theoretical curve (Fig. 255) permits a fair estimate of the setting at which the fringes would vanish, and so of the angular diameter.

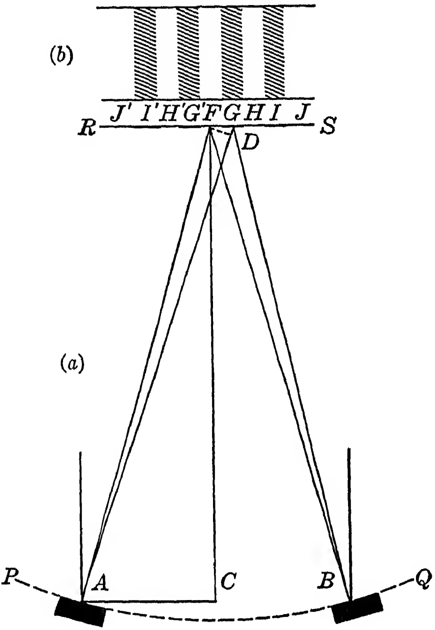


FIG. 256. Formation of Fringes in the Stellar Interferometer

RS in part (*a*) of the diagram represents the intersection of the focal plane with that of the paper. *G*, *G'*, *I*, *I'*, represent the intersections of dark fringes, and *F*, *H*, *H'*, *J*, *J'*, of bright fringes (due to a point-source lying at a great distance along *FC* prolonged). (*b*) represents these fringes as they lie in the focal plane, and as they are viewed in the eyepiece. In both figures, for convenience of drawing, the fringes are hugely magnified relatively to the objective. The limits of the diffraction disk are not indicated. (See section 823 for a fuller discussion)

823. Physical Theory. The measurement of an angle always depends upon the measurement of two distances, the ratio of which determines the value of the angle. If the angle is small, the distance which appears in the numerator of the ratio must be small. In the stellar interferometer this is made a wave-length of visible light, which is among the most minute of accurately known distances.

The formation of the fringes is illustrated in Fig. 256. The objective of a telescope is represented by *PCQ*; *C* is the center of the chord *AB*, *F* the principal focus, and *RS* the focal plane (perpendicular to the plane of the

paper). Suppose that the only light reaching the neighborhood of F comes from two small apertures, A and B, on the same diameter of the objective, at equal distances from C.

If a point-source of light lies at a great distance along FC prolonged, the light-waves reach A and B "in phase" (§ 550). That is, the light-vibrations of the two beams are synchronous at these points. Since the paths AF and BF are then equal, the waves reach the point F in phase and *reënforce* each other, forming a bright fringe. This will also happen whenever one of the wave-trains is a *whole number* of waves in advance of the other, so that the crests and troughs of one wave-train coincide, respectively, with the crests and troughs of the other, — that is, at any point F, H, J, . . . such that the path difference $AJ - BJ$, etc., is a whole number of wave-lengths.

If, however, this path difference is $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ . . . wave-lengths, the crests of one disturbance coincide with the troughs of the other; they completely neutralize each other, and a dark fringe is formed.

For the first dark fringe, at G,

$$AG - BG = \tfrac{1}{2}\lambda;$$

for the bright fringe at H,

$$AH - BH = \lambda;$$

etc.

It is easy to show that the successive distances FG, GH, HI, etc. are all equal, and that this distance x is given by the formula (16) stated in section 818. For when FD is drawn perpendicular to AG, the right triangles FDG and FCA are similar, so that

$$\frac{FG}{FA} = \frac{GD}{AC}.$$

But, with sufficient approximation,

$$AF = AD;$$

$$AG - AF = GD.$$

Similar reasoning shows, also, that

$$BF - BG = GD.$$

Hence

$$AG - BG = 2\,GD = \tfrac{1}{2}\lambda,$$

and

$$\frac{FG}{FA} = \frac{\frac{1}{4}\lambda}{AC},$$

or

$$\frac{x}{f} = \frac{1}{2}\frac{\lambda}{D},$$

writing $FG = x$, $FA = f$, $AC = \frac{1}{2}D$.

824. Advantages of the Stellar Interferometer. Although this instrument was invented as long ago as 1890 (and suggested even earlier by Fizeau), and successfully applied, about 1891, by Michelson (and also by Hamy) to the measurement of the angular diameter of Jupiter's satellites, astronomers failed to realize its

capabilities until, after long neglect, it was again brought to attention in 1920 by Michelson himself, aided by the Mt. Wilson observers.

In the measurement of small angular separations the interferometer possesses two great advantages over the ordinary method of telescope and micrometer. The resolving power for double stars is more than twice as great as that of a telescope with aperture equal to the separation D of the interferometer apertures. For star-disks the advantage, although more difficult to state numerically, is still considerable. With the beam interferometer, D can be made much larger than the aperture of any existing or projected telescope; and by using the visibility curve (Fig. 255), angular separations beyond the direct range of the beam can be determined.

Secondly, the fringes obtained with the interferometer are very much less disturbed by bad seeing than the direct image with the full aperture of the telescope. (The larger area collects more light, but also more trouble!) This difference is greater than anyone anticipated before 1920, and is not yet fully explained.

825. Results of Measurement of Diameters. The successful determination of the angular diameter of Betelgeuse, on December 13, 1920, was one of the modern triumphs of observational science. The angular diameters of seven stars (Table XXIX) have been well measured, — all by Pease, with the Michelson 20-foot beam interferometer, at Mt. Wilson. Of the stars listed in Table XXVIII (p. 740) the five red giants — α Boötis, α Tauri, β Pegasi, α Orionis, α Scorpii A — possess by far the largest calculated diameters. Naturally these were the stars first selected for measurement, and o Ceti was added because of its interest as a variable.

The agreement between the computed and observed angular diameters is very good (except in the case of Aldebaran), and accordingly the theory of sections 815 and 816 is well verified. The red giants, then, greatly exceed the sun in diameter; indeed, Antares and α Herculis are larger than the orbit of Mars. Their masses, however, according to the mass-luminosity curve, are not enormously greater than the sun's, so that the mean density of such a star is hundreds of times less than that of atmospheric air.

As indicated, the angular diameter of Betelgeuse has been found to vary from about 0″.047 to 0″.034. Further study will be required before this interesting phenomenon is well understood; the star may be pulsating slowly in the period of its light-variation (§ 843). (Betelgeuse is an irregular, slowly changing variable, of small range — showing indications of a period of about six years.)

826. Computation of Temperatures from Measured Angular Diameters. The procedure of section 816 may be reversed, and the color-index and temperature computed from the visual magnitude

TABLE XXIX. STAR DIAMETERS MEASURED WITH THE INTERFEROMETER

	Visual Magnitude	Sp.	Feet Separation	Measured Angular Diameter *	Parallax	R †	T (Computed by (21))	Computed Angular Diameter
α Boötis . . . (Arcturus)	0.2	K0	24	0″.020	0″.080 ± 0″.005	27	4300° K	0″.023
α Tauri . . . (Aldebaran)	1.1	K5	24	0 .020	0 .057 ± 0 .005	38	3800	0 .034
α Orionis . . . (Betelgeuse)	0.9	M0	10 14	0 .047 0 .034	0 .017 ± 0 .004	300 210	3200	0 .048
α Scorpii A . . (Antares)	1.2	M0	12	0 .040	0 .0095 ± 0 .002	450	3100	0 .042
β Pegasi . . . (Scheat)	2.6	M5	22 +	0 .021	0 .016 ± 0 .005	40	3100	0 .025
α Herculis . . (Ras Algethi)	3.5	M8	16 −	0 .030	0 .008:	400	2650	
o Ceti (max.) (Mira) (min.)	3.5 9.9	M7e	8.2	0 .056	0 .02:	300 (300)	2400 1700	

* The measured angular diameter was determined by Pease from the observed separation for fringe disappearance, using an equation of the type of (19) or (20), and employing appropriate values of the effective wave-length.

† The linear radii listed, in terms of the sun's as unity, are calculated from the measured angular diameters and adopted parallaxes by equation (13). For comparison: the radius of the sun is 6.95×10^5 km., the mean distance of Mercury from the sun is 83.2 solar radii, and of Mars 327.6 solar radii.

and angular diameter, assuming that the stars radiate like black bodies. Solving equation (14) for T,

$$T = \frac{5900}{3.05 + 0.20\, m_v + \log d''}. \tag{21}$$

The temperatures thus computed are those listed in Table XXIX. These independent determinations of the effective temperatures agree very closely with those previously found (§§ 810,

813, 814) for stars of the same spectral classes, except in the case of α Tauri, and the discordance here is not serious.

The same method may be applied to β Aurigæ, for which the spectrum is A0, the absolute magnitude of each component $+ 0.6$, and the radius 2.8 times the sun's (Tables XXIV and XXV). From equation (11) we then find $T = 10{,}200°$, in good agreement with the value previously found for Class A.

827. The White Dwarfs. We pass now to the other, still more remarkable, consequence of the hypothesis that the stars radiate like black bodies, — namely, the enormous computed densities of the white dwarfs. There is no hope of measuring the diameters of such tiny stars, but another line of attack is open. According to the general theory of relativity (§ 361), light-waves originating in a strong gravitational field should apparently be slowed, upon escaping into interstellar space, so that the lines in the spectrum of a massive star should be shifted slightly toward the red. The observed effect would be exactly similar to that produced by a radial velocity of recession, and, in the case of the sun, would correspond to a velocity of 0.6 km./sec. This displacement, as has already been said (§ 591), appears to be present in the solar spectrum, but is very hard to disentangle from other small shifts. For other stars the shift — being proportional to the gravitational potential at the surface — should vary as μ/R. Referring to Table XXVIII, we see that this quantity is nearly the same for all stars of the main sequence, and much smaller for the giants. It can reach large values only for the white dwarfs, where it may be twenty or thirty times as great as for the sun.

Such a displacement should easily be measurable, but can only be separated from the Doppler shift if the radial velocity is already known. The radial velocity of an isolated star cannot be independently determined, but nature has been generous in providing a case in point where this can be done.

828. The companion of Sirius satisfies all the requisite conditions. The theoretical shift is greater for it than for any other known star; and its true radial velocity at any time can be calculated, since that of Sirius has been accurately observed and the component of the relative orbital motion in the line of sight is known. The only difficulty, but a very serious one, is that of

photographing the spectrum of a star less than 12″ from another which is ten thousand times as bright; but Adams, with the great Mt. Wilson reflector, has secured several spectrograms of it.

Even at times of the best seeing, some scattered light from Sirius enters the slit, producing a wide, faint spectrum, on which the narrow one of the companion is superposed. The latter is of Class F or somewhat "earlier," with the hydrogen lines broad and the metallic lines unusually faint; but, with proper precautions, measurements of considerable accuracy can be made on it. The results show a definite displacement of all the lines toward the red. After allowance is made for the blending of the scattered light of Sirius with that of the companion (which diminishes the displacement for the lines at the violet end) and for the orbital motion of the companion, the average "Einstein shift" is found to correspond to a velocity of $+19$ km./sec. The predicted value, with the data of Table XXVIII, is $0.6 \times 0.96/0.034$, or 17 km./sec.

The observed displacement leads to a radius 0.030 times that of the sun, and (reckoning backward) to a surface temperature of 8000°, corresponding to a spectrum about A7; this agrees perfectly with Adams's statement that the observed spectrum is "earlier than F0."

The predictions of theory, therefore, have once more been fully confirmed by observation, and the small diameter and high density shown to be substantially correct. With $R = 0.030$ the mean density of the companion comes out 36,000 times that of the sun, or nearly 50,000 times that of water.

Until recently the very existence of so great a density would have appeared wholly incredible; but Eddington has shown that it is not merely possible but probable, in accordance with modern physical conceptions. As has already been remarked (§ 667), the atoms in the interior of the sun, or of any star, must be very greatly ionized, and have lost all but a few of their innermost electrons. The remaining residues of atoms are undoubtedly far smaller than the complete atoms with which we are ordinarily familiar, and a gas composed entirely of them, and of free electrons, which are still tinier, could be compressed to enormous densities and still leave the separate particles plenty of room to move about, so that the state of the matter would still be gaseous.

829. The Temperatures of the Stars. The determinations of stellar temperatures made in different ways are collected in Table XXX. The results agree in a really remarkable fashion, and furnish strong evidence that the stars actually radiate very much like black bodies. Even for the companion of Sirius the results are concordant. The only values that are not checked by two or more independent determinations are those for Class B, and in this case the theory of ionization (Chapter XXV) shows that the results here given are not far from the truth, and also that the effective temperature rises to 35,000° or even 40,000° in Class O. The difference in temperature between giants and dwarfs is also explained, and is confirmed quantitatively, by this theory.

The values adopted in the last column of Table XXX may therefore be regarded with considerable confidence. The absolute magnitudes of eclipsing variables for which the spectral types and true dimensions are known may be computed by equation (2), p. 732. In this way the following values are obtained for the stars of Table XXV, the brighter components alone being considered. The absolute magnitudes read from the mass-luminosity curve are given here for comparison.

	Computed	Curve	Difference		Computed	Curve	Difference
V Pup . .	− 3.2	− 3.5	+ 0.3	TV Cas	+ 0.4	+ 2.3	− 1.9
Y Cyg . .	− 1.9	− 3.2	+ 1.3	RS Vul .	− 1.0	− 0.4	− 0.6
u Her . .	− 1.7	− 1.6	− 0.1	β Aur . .	+ 0.4	+ 1.4	− 1.0
σ Aql . .	− 1.2	− 1.0	− 0.2	S Ant . .	+ 3.1	+ 5.7	− 2.6
Z Vul . .	− 1.5	− 0.7	− 0.8	Z Her . .	+ 3.2	+ 2.8	+ 0.4
U Oph . .	− 0.3	− 0.7	+ 0.4	W UMa	+ 5.5	+ 6.3	− 0.8
				TX Her	+ 1.9	+ 2.0	− 0.1

The average difference, regardless of sign, between the values of the absolute magnitude found in these two quite independent ways is $\pm 0^{m}.8$. The most discordant star is one for which the data are poor. Omitting it, the average difference is only $\pm 0^{m}.66$, corresponding to a difference of 30 per cent in the parallax. The general agreement for the B-stars indicates that the temperatures given in Table XXX, and the deduced surface brightness, are probably nearly correct even in this case where the calculated values are most liable to be in error.

The problem of determining the temperatures and diameters of the stars is therefore now in a very satisfactory state. Twenty years ago there was not a single star whose diameter was known with even approximate certainty; today the linear diameters of all stars of known parallax, and the angular diameters of all the stars in the sky, can be calculated more accurately than the distances of all but the nearest can be measured.

TABLE XXX. THE TEMPERATURES OF THE STARS*

SPECTRUM	COLOR-INDEX	HEAT-INDEX	ENERGY CURVES	DIAMETERS	ADOPTED TEMPERATURE
B0	23,000				23,000
B5	15,000				15,000
A0	11,200		12,500	10,000	11,000
A5	8,600			8,000 †	8,600
F0	7,400				7,400
F5	6,500		8,000		6,500
gG0	5,500	5,500	5,800		5,600
gG5	4,700				4,700
gK0	4,100	4,200		4,300	4,200
gK5	3,300		3,000	3,800	3,400
gM2	3,050	3,100	2,700	3,100	3,100
gM5		2,750	2,500	2,650	2,700
N	2,200	2,700			2,600
Me(max).‡		2,300		2,400	2,300
Me(min).‡		1,650		1,700	1,650
dG0	6,000				6,000
dG5	5,600				5,600
dK0	5,100				5,100
dK5	4,400				4,400
dM	3,400			3,650 §	3,400

* All these temperatures are centigrade absolute.
† The companion of Sirius. ‡ Long-period variables. § α Gem. C.

CHAPTER XXII

VARIABLE STARS

DISCOVERY AND OBSERVATION · NOMENCLATURE AND CLASSIFICATION · LIGHT CURVES, SPECTRA, MOTIONS, AND DISTRIBUTION · CEPHEID AND CLUSTER VARIABLES: CHARACTERISTICS, PERIOD-LUMINOSITY CURVE, PULSATION THEORY · LONG-PERIOD VARIABLES · IRREGULAR VARIABLES · NOVAE, SUGGESTED EXPLANATIONS

830. Discovery. Many stars change in brightness more or less and are known as *variable stars*. The change of brightness is sometimes very conspicuous, especially when a "new star," or *nova*, appears where nothing had previously been visible to the naked eye. The observation of such an object by Hipparchus, 134 B.C., led to the preparation of the first star-catalogue. The period of scientific observation of their changes may be dated from the observations of the Nova of 1572 by Tycho Brahe, and from the discovery of the periodic variation of Mira Ceti by Fabricius in 1596.

The older variables, for the most part, were discovered by accident in the course of other work, such as the search for asteroids. A single glance at the sky has often been sufficient for the discovery of a bright nova. The great superiority of photography in the *systematic* search for variables was first effectively realized by E. C. Pickering. The great store of plates at Harvard contains the history of light-variation of many stars over many years. The Harvard method of search consists in superposing a positive of a given star-field on a negative of the same field taken on some other date. Variation in any star is at once apparent from the difference in size of the two images. A number of variables are usually discovered with one pair of plates. Other plates of the same region add to the number of discoveries and tell the story of the type of variation. As a rule no one takes the trouble to identify the previously known variables, but they are simply rediscovered along with the rest. From the percentage of

known variables which, for one reason or another, are missed, it is possible to estimate the number of unknown variables in this field which have been passed over and still await discovery. Thus, in Harvard map 32 "there are probably 61 variables of which 32 have been found." Discoveries are easily made, also, by means of the "blink microscope" (§ 726).

Another fruitful method of discovery is by the inspection of spectrum photographs. Certain types of spectra with bright lines are practically certain to belong to variable stars (§ 846).

The photometric observation of spectroscopic binaries has netted the discovery of a number of eclipsing and Cepheid variables.

Finally, variables are only too often discovered in an unwelcome fashion, — when a comparison star, whose constancy has been relied upon as a standard of reference for a known variable, turns out to be variable itself. This happens often enough, particularly in very precise photoelectric observations, to indicate that probably at least 5 per cent of all the stars are slightly variable.

The number of stars which are actually known to be variable exceeds 5000, and discovery is proceeding rapidly.

831. Nomenclature, Catalogues, etc. The designation of variable stars is at present undertaken by the Astronomische Gesellschaft. A provisional number, indicating the order and the year of discovery, is assigned to each newly announced variable. Thus, 155.1904 was the one hundred and fifty-fifth variable discovered in 1904. After sufficient confirmation of its variability the star, if it has not already a letter or a number, is designated by one or two capital letters followed by the name of the constellation in which the star is located. The first variable discovered in a constellation receives the letter R, the next S, and so on to Z. After Z the letters are doubled, as, for example, RT Persei, SZ Herculis, etc. The combination ZZ has been reached in several constellations and is succeeded by AA. The number of variables which had been named on this system, in 1925, was 2671. Most of the others are faint objects in star-clusters or star-clouds which can only be identified on marked photographs. The novæ are not included in this scheme but are known by their constellation and year of appearance, — for example, Nova Aquilæ, 1918. Several other systems of nomenclature have been suggested, — for ex-

ample, the Harvard designation, which is a six-figure number giving the position of the star for 1900. The first two figures are the hours; the third and fourth, the minutes of right ascension; the fifth and sixth, the degrees of declination. Southern declinations are written in italics. RT Persei has the Harvard designation 031646, indicating that its position for 1900 is $\alpha = 3^h\ 16^m$; $\delta = +46°$.

The most comprehensive catalogue of variable stars is the *Geschichte und Literatur des Lichtwechsels der Sterne*, which summarizes all the information (to 1920) concerning those variables which have received names. A list of these stars, extended year by year, and giving the elements of the light-variation when known, is published by the Berlin-Babelsberg Observatory.

832. The Observation of Variable Stars. When, however, a variable star has been discovered, the observer's work has just begun. Its changes must be followed and their laws determined.

When the variation is strictly periodic and a mean light curve can be formed, the star may profitably be observed with the greatest care and by the most accurate photometric methods, — *visual*, *photo-electric*, and *photographic*. The observations consist in measuring the difference in brightness between the variable and one or more constant comparison stars, and they may be repeated until they define the entire course of the mean curve with high accuracy. Should one of the comparison stars prove to be a variable, its own type of variation must be found and allowed for, or the observations in which it is involved must be discarded. The varying atmospheric extinction introduces a troublesome error, and to secure good observations it is necessary to heed a variety of precautions, — for example, in visual work to keep the line of the eyes parallel to the line of the two stars, to avoid "position-angle error," and, perhaps most important of all, to avoid comparing stars of different color (§ 693).

When high precision is unnecessary, good results may be secured by the *Argelander method* of estimating the difference in magnitude of the variable and several comparison stars whose magnitudes are known. This may be carried out at the telescope or upon photographs. The observer usually estimates how many "steps" of just distinguishable difference of brightness would fill the intervals between the stars. His record may be a4v2b, mean-

ing that the variable v is 4 steps fainter than star a and 2 steps brighter than star b. If the magnitudes of the comparison stars are known from previous photometric observations, that of the variable star may be estimated directly.

The majority of variable stars are far from regular, either in period or in range, and the accuracy that is attainable by the Argelander method is amply sufficient for their observation. Observations by this method are also of great value in determining the character of variation (and the period in the case of regular variables), and so finding which will best repay precise photometric observation.

Experience has shown, however, that if, for example, the observer expects the star to be increasing in brightness, he is likely to record it as continuing to increase after its light has become stationary, the preconceived opinion having a subconscious influence upon his judgment (cf. § 400). Observers therefore take pains to observe a long list of stars, so that each particular estimate is forgotten before the same star is examined again. This method requires but very modest instrumental equipment, and valuable observations can be made at odd times by amateurs engaged in other occupations. The American Association of Variable Star Observers, and similar associations in other countries, have organized this amateur activity to a high degree of efficiency, and are publishing every year thousands of observations which, as they accumulate, become increasingly valuable for researches on the nature and cause of stellar variation (cf. § 845, Fig. 262).

833. Classification. There is no reliable evidence of steady secular change in the brightness of any star. The few serious differences between the estimates of Hipparchus and Ptolemy and the measures of the present day are no more than might be accounted for by errors of observation or copying.

The known changes appear to be of a fluctuating character, some periodic, others non-periodic. On this basis, variable stars may be divided into several rather definite groups:

I. Periodic stars
   1. Eclipsing variables, recognized by the form of the light curve
   2. Variables of short period, ranging from a few hours to two months
   3. Long-period variables, — from four or five months to two years

II. Non-periodic stars
 4. Irregular variables, a nondescript class containing several sub-groups
 5. Temporary stars, also called novæ

Variables of the first two classes are conspicuously periodic and regular in their changes; those of the third class are but roughly periodic and are more or less variable in range of light-variation.

834. General Properties. The one characteristic which is common to almost all variable stars is that they are of great luminosity. The eclipsing variables must for the most part be brighter than Sirius; the long-period and irregular variables at maximum are comparable with Arcturus or Capella in real brightness; the short-period variables are among the brightest stars known, ranging from 100 to 10,000 times the sun's luminosity; the relatively meager knowledge concerning novæ indicates that they rise to a brilliance even superior to this. Variable stars, therefore, at their best, are nearly all giants; only a few eclipsing variables and a very few of the irregular variables appear to be dwarfs.

All spectral types are represented among variable stars, but there are conspicuous differences among the different classes. Eclipsing variables are mostly of early type, B and A; short-period variables range through almost the entire sequence from B to M, with a strong preference for classes F and G; long-period variables are nearly all red, of classes M, N, R, and S; and the irregular variables belong mainly to these same classes, with a few scattering objects of earlier type. Finally, the novæ have peculiar spectra which undergo characteristic changes as the light fades. Almost all variables exhibit changes in color-index and in heat-index and in spectrum along with the changes in brightness, and also concomitant variations in radial velocity.

The variations are so different in different stars that it is evident that they cannot have any one common cause. A fully satisfactory theory permitting the exact calculation of the manner in which the brightness varies exists only for the *eclipsing variables*, and in this case it is based on the fundamental assumption that the two components are not variable at all. These stars should therefore be classified and discussed among binary stars, as has been done in Chapter XX.

For the remaining classes it is almost certain that the brightness of the stars is really variable, and the changes in brightness, color, and spectra are so related as to make it very probable that the surface temperature changes.

PERIODIC VARIABLES

835. If the eclipsing variables are eliminated, there remain about 2400 stars, listed in the *Vierteljahrsschrift* for 1925, which are presumably inherently variable. Of these about 1000 are

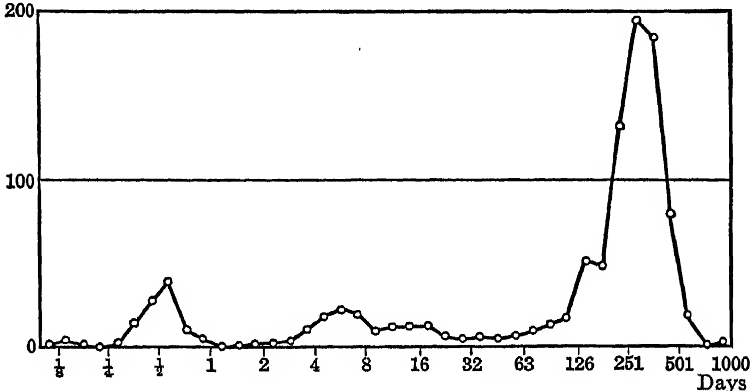


FIG. 257. Distribution of Periods of Variable Stars

The plotted points give the number of stars for which the logarithm of the period lies between limits differing by 0.1. This method of graphical representation is very convenient when it is desired to represent large and small numbers upon the same diagram

recognized as definitely periodic, while about 150 are known to vary irregularly. Most of the remainder have been insufficiently observed.

If the numbers of variables which have periods lying between specified limits are plotted, the curve shown in Fig. 257 is obtained. The variables fall into three obvious groups, with periods clustered about values near 12 hours, a week, and 300 days, and containing, respectively, about 100, 150, and 750 stars.

The stars of the first two groups have so many characteristics in common that they are usually classified together as short-period variables. Those of the third group have very different properties.

836. The Cepheid Variables. Stars with periods less than a day are often called *cluster variables,* since they are found in great numbers in the globular star-clusters. Those of periods near a week are named *Cepheids,* after the typical star δ Cephei (Fig. 258). The name "Cepheid," however, is very often used to include the short-period variables of both types. We shall follow this usage, referring, when necessary, to the group of longer period as "typical Cepheids."

The Cepheid variables have been more carefully studied and more is known about them than about any other stars that are intrinsically variable, but much still remains to be learned.

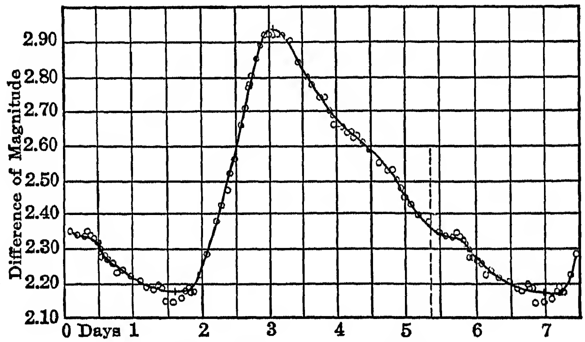


FIG. 258. Light Curve of δ Cephei
By Joel Stebbins

837. The groups of longer and shorter period have numerous characteristics in common, of which the most conspicuous are:

(1) The variation is very regular, and the period and form of the light curve change very little even in the few cases where minor fluctuations have been observed.

(2) The variation is continuous and smooth, the light curve resembling the velocity curve of a spectroscopic binary. The rise to maximum is usually more rapid than the fall, and the maximum usually more sharply marked than the minimum. In a few cases (ζ Geminorum, etc.) the rise and fall of brightness are about equally rapid, but a slow rise and rapid fall are unknown. Minor waves are authenticated in the light curves of some. For many cluster variables the rise is extremely rapid, the light sometimes doubling in less than an hour (Fig. 259).

(3) The visual range of variation very rarely exceeds 1.5 magnitudes and is usually less than a magnitude.

(4) The photographic curve is similar to the visual, but shows, on the average, a 50 per cent greater range in brightness. These stars are therefore much redder at minimum than at maximum.

(5) The spectra vary periodically along with the light. The hydrogen lines and the enhanced metallic lines are much stronger at maximum light than at minimum. The arc lines show much less change.

(6) The spectra are correlated with the periods. Stars with periods of $\frac{1}{2}$ day are, on the average, of Class A; those with

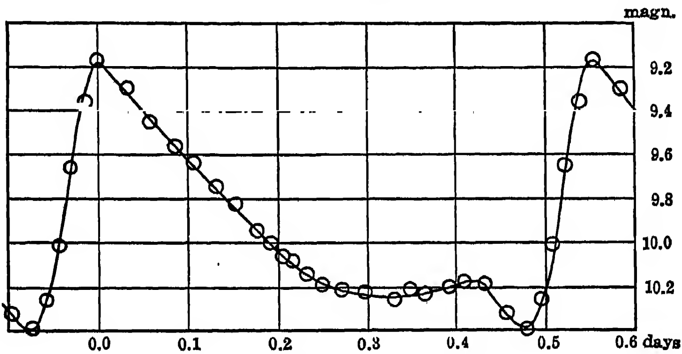


FIG. 259. Light Curve of RR Ceti

By C. L. Stearns, Yale University Observatory

periods of about 4 days, of Class F5; of 8 days, of Class G0; and those with periods of over 20 days cluster about Class G5.

(7) All these stars show variations in radial velocity with the period of the light-variation. The velocity range is small and nearly proportional to the range of variation in magnitude. The maximum velocity of approach invariably occurs at or near the time of maximum brightness, and the greatest velocity of recession near the time of minimum light.

When orbits are computed from the velocity curves, the value of $a \sin i$ is in all cases extremely small, rarely exceeding 2,000,000 kilometers, and the computed mass functions are very small, usually less than 0.001. In several cases spectroscopic binaries which show the above characteristics have been found, on photo-

metric study, to be Cepheids of small range. The most notable case is the polestar, which has a period of 3.968 days and a range in visual magnitude of only 0.08. The photographic range is twice as great.

838. In certain characteristics, however, the typical Cepheids differ from the variables of the cluster type. The former include a number of fairly bright stars. δ Cephei, η Aquilæ, and ζ Geminorum are, on the average, of about the fourth magnitude, and eight or ten others are visible to the naked eye. They show a very strong galactic concentration and have small proper motions, which nevertheless show a definite parallactic drift due to the solar motion. The mean parallax found from this is very small (only $0''.004$ for stars averaging about the fifth magnitude), which gives an absolute magnitude of -2 and shows that these variables are of very great luminosity. This conclusion is fully confirmed by the fact that their spectra show the "c" characteristics in a very marked degree, such as is met with only among stars of the very brightest absolute magnitudes. Their peculiar velocities are small (about 12 km./sec. according to Strömberg), and the τ components of proper motion confirm this conclusion.

Those *cluster variables* which do not occur in clusters are apparently much fainter, the brightest two (R Muscæ and RR Lyræ) being of the seventh magnitude. They show very little galactic concentration, being distributed almost uniformly over the sky. Proper motions are known for only a few, but these are much larger in proportion to the apparent brightness than for the typical Cepheids. Radial velocities also are remarkably great, the peculiar velocity in this direction averaging about 70 km./sec. As might be expected for stars of such high velocity, the group as a whole is moving relatively to the sun with a velocity of fully 100 km./sec. (cf. § 759). The mean parallax can be only very roughly determined, but the data indicate a value of about $0''.001$ for stars of the tenth magnitude, which makes the absolute magnitude about 0, not far from the average for stars of Class A.

839. Cepheids in Clusters. The discovery by Bailey, in 1895, of numerous variable stars in the globular star-clusters (§ 859) has led to consequences of great interest and importance. Variables have now been discovered in nearly thirty globular clusters

(Fig. 260). The great majority of them have periods not far from half a day, and in each cluster the median magnitude (that is, the average of the maximum and minimum values) is almost exactly the same for all the variables of this type. There can be no doubt that these stars are actually members of the cluster and substantially at the same distance from us. It may be concluded that the cluster variables in any given cluster are all of the same absolute magnitude, and it is very probable that the variables in different clusters have really the same luminosity and owe their differences in apparent brightness to differences in distance. A few typical

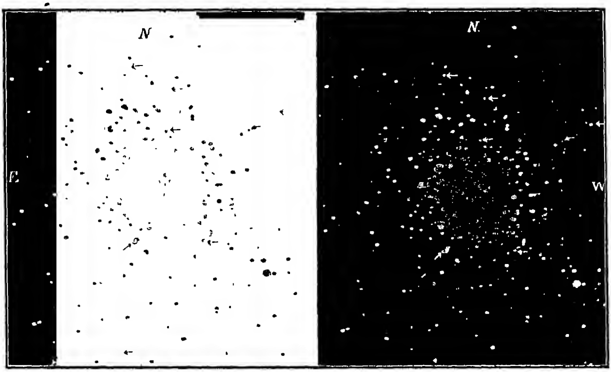


FIG. 260. Variable Stars in Messier 5

These two photographs were taken at Arequipa two hours apart. The little arrows point out some of the stars which have changed in brightness in that short time

Cepheids are found in globular clusters, and these are all considerably brighter than the cluster variables in the same cluster.

840. The Period-Luminosity Curve. The Magellanic Clouds, which are isolated star-clouds in the southern heavens, resembling those of the Milky Way in general characteristics, contain great numbers of variable stars. Miss Leavitt, at Harvard, has discovered nearly 1800. There are few variables in the neighboring sky, so that almost all those in the Clouds must belong to them. Periods and light curves have been determined for more than a hundred stars in the smaller Cloud. The periods range from fifteen hours [1] to more than a hundred days, and the light curves

[1] Variables of still shorter period are known to exist, but their magnitudes have not yet been accurately measured.

are definitely of the Cepheid type. The median photographic magnitudes of these stars are very closely correlated with their periods, as is shown in Fig. 261. The stars of any given period are substantially of the same apparent magnitude and must also be of the same absolute magnitude. By combining these results with those obtained from the globular clusters a "period-luminosity

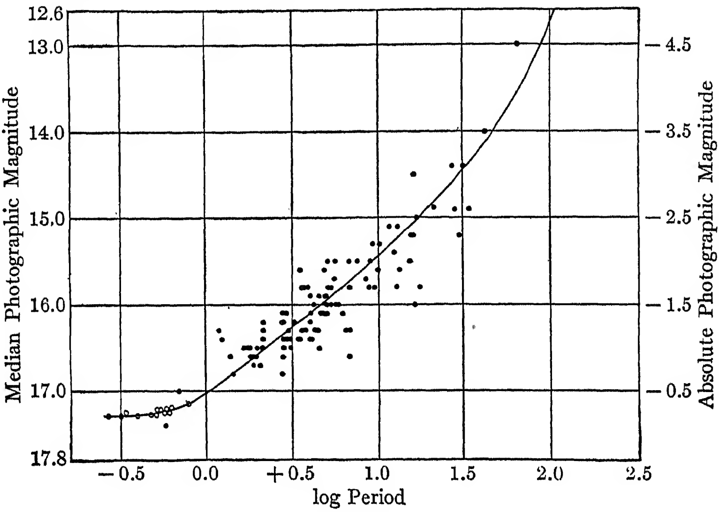


FIG. 261. Period-Luminosity Curve for Cepheid Variables

The black dots represent variable stars in the small Magellanic Cloud. The horizontal coordinate is the logarithm (base 10) of the period in days; the vertical, the observed median photographic magnitude. The open circles represent variables of shorter period in various globular clusters — reduced to the same scale on the assumption that the absolute magnitudes of Cepheids of the same period are the same. The vertical scale is a large one, and the accordance of the individual observations with the curve is therefore much better than appears at first glance. (This curve is copied from one by Shapley.) The visual magnitudes are nearly equal to the photographic for the stars of shortest period, and about one magnitude brighter for those of longest period. The absolute magnitudes have been added on the right

curve" can be obtained which covers the whole known range of period, and from which the *relative luminosity* of Cepheids of different periods may be determined with great precision.

Why this remarkable relation should exist is not yet explained, but the evidence that it actually does exist is very strong. If, then, the absolute magnitude of any one Cepheid or of a small group can be determined, the "zero point" of the period-luminosity curve can be fixed, and it will then be possible to read from the

curve the absolute magnitude of any Cepheid whose period is known. This calibration can be made with the aid of the brighter Cepheids which are found in the sky at large, using the absolute magnitudes derived from the proper motions. In this way Shapley has derived the scale given in Fig. 261. As the number of proper-motion stars available for the determination of mean parallax and absolute magnitude is rather small for the application of the statistical method, the values of the absolute magnitude there indicated may be somewhat in error, possibly by as much as half a magnitude.

The typical Cepheids of long period are among the brightest known stars; one of period 100 days would have an absolute visual magnitude of about -6 and be 20,000 times as bright as the sun.

841. Application of the Period-Luminosity Curve. Although this relation between period and absolute magnitude still remains purely empirical, it has had applications of very great importance. For example, it permits an immediate calculation of the distances and positions in space of all known Cepheid and cluster variables. In this way Shapley has found that the typical Cepheids lie at distances ranging from 60 to nearly 6000 parsecs. They extend far in all directions along the galactic plane, but lie relatively close to it, their average distance from it being only 150 parsecs and hardly any being farther from it than 500 parsecs. The isolated cluster variables extend to equally great distances, but, unlike the others, are scattered widely on each side of the galactic plane, one third of them being more than 1000 parsecs from it.

Of still greater interest is the possibility of determining the distance of any star-cluster or star-cloud, however remote, in which Cepheid variables can be recognized. The results of this, by far the most powerful known method of sounding the depths of space, will be discussed in Chapter XXIII.

842. The cause of Cepheid variation has been much discussed, and a fairly satisfactory theory is available. The concomitant changes in brightness, color, and spectrum indicate clearly that the immediate cause of all these is a variation in the surface temperature of the star, or at least in the portions visible to us at different times.

The form of the light curve shows that the variations cannot be due to eclipses, and it may be proved also that, since the rise is much more rapid than the decline, it could not be produced by the rotation of any convex body, brighter on one side than on the other.

The magnitude and rapidity of the change (a cluster variable may increase its brightness by fifty times the sun's whole light in two hours, and lose all this again in six hours) indicate a rapid periodic transformation of heat energy to some other form of energy and back again, the loss by radiation being small in so short an interval. The regularity of the oscillation makes it probable that the other form of energy is gravitational energy.

These facts and inferences, and also the observed changes in radial velocity, are consistently explained by the *pulsation theory*, first proposed by Shapley and developed mathematically by Eddington. This theory attributes all the changes to a *periodic expansion and contraction of the star* under the combined influence of gravitation and the elasticity of the gases of which it is composed.

843. Pulsations of a Gaseous Star. A normal star, like the sun, is doubtless in internal equilibrium, the pressure at any point due to the weight of the overlying layers being balanced by that of the incandescent gas (aided by radiation pressure). Suppose that, by some external constraint, such a star should be compressed till its radius became, for example, 10 per cent smaller. The gravitational forces and the resulting pressure at any point would now be greater, but the compression would heat the gases and increase the gas pressure. Calculation shows that the increase in gas pressure would be the greater of the two, so that the star, in its altered state, would tend to expand and return to its original size. When it reached it, however, it would still be expanding rapidly, and the inertia of the moving matter would carry it beyond the position of equilibrium into a state in which the internal gas pressure would be too low and the star would tend to contract. A disturbance of the initial equilibrium would therefore set up a series of regular oscillations about the original configuration. The period of this "pulsation" should be inversely proportional to the square root of the density, and should also

depend upon the law according to which the pressure of the gas increases when it is diminished in volume.

On this theory the observed changes in radial velocity are interpreted as due to the actual motion of the surface toward or from the earth as the star expands and contracts. The temperature in the interior of the star should be highest when the diameter is smallest; but since the surface layers do not share much in the compression, the heat will take some time to work out to the surface. The greatest surface temperature and the maximum light will therefore come some time later, when the star is expanding rapidly, as is observed to be true. Similarly, the minimum temperature will come while the star is contracting. The greater the range of the pulsation, the greater should be the changes both in light and in velocity; this again agrees with observation.

Eddington has shown that when the amplitude is considerable, the interval from the time of most rapid contraction to that of most rapid expansion (that is, from minimum to maximum light) should be shorter than that from maximum to the following minimum; but the mathematical discussion here becomes so complicated that it is not practicable to follow it far enough to compute the actual forms of the velocity and light curves.

Qualitatively, therefore, the pulsation theory accounts satisfactorily for the phenomena. An important quantitative test remains: the distance through which the surface moves on each side of its mean position, as calculated from the radial velocity, should be only a moderate fraction of the radius of the star. The former can be found from the values of $a \sin i$, derived by treating the radial velocity as due to orbital motion. If all the light which we receive came from the point of the surface which is moving directly toward us, these two quantities would be equal; allowance for the fact that parts of the visible surface are moving obliquely can be made by increasing the calculated values of $a \sin i$ by about 40 per cent. The radius can be calculated from the absolute magnitude and spectral type.

The results for a number of Cepheids of different periods are given in Table XXXI. The assigned spectral classes are average values during the variation; the absolute magnitudes represent the median visual brightness and are derived from Shapley's

period-luminosity curve. The mean radii are computed from these and the spectra with the formula of section 816, but are here given in millions of kilometers, as are the values of $a \sin i$. The maximum value of $a \sin i/R$ is only 0.13, and the mean for the nine stars is 0.05, or 0.06 excluding the exceptional case of Polaris. The spectroscopic observations thus indicate a change in radius averaging about 8 per cent on each side of the mean, and this is not exorbitant. The larger values of this quantity go with the larger range of variation.

If the stars were of fixed radius, the changes in visual magnitude (averaging about $0^m.4$ on each side of the median) could be brought about by changing a median temperature of 6000° by some 500° each way, or by 8 per cent. The actual changes of surface temperature have probably about the same range.

Finally, if the masses of the stars are taken from the mass-luminosity curve, the densities given in the last column but one are obtained. They are low, but no lower than for many other giant stars. The product $P\sqrt{\rho}$ is nearly constant, as it ought to be theoretically; indeed, the gradual increase for the stars of longer period was predicted theoretically by Eddington.

TABLE XXXI. CEPHEID VARIABLES

STAR	P	MAX.	MIN.	SP.	ABS. MAG. M	$a \sin i$ MILLIONS OF KM.	RAD. R MILLIONS OF KM.	$a \sin i / R$	MASS $\odot = 1$	DENS. ρ $\odot = 1$	$P\sqrt{\rho}$
RR Lyr	$0^d.57$	$7^m.1$	$7^m.8$	A6	−0.4	0.17	4	0.04	4.6	0.022	0.09
SV Cas .	1 .95	5 .9	6 .3	F5	−1.2	0.30	9	0.03	6.3	0.003	0.10
α UMi .	3 .97	2 .1	2 .2	F8	−1.8	0.15	15	0.01	8.5	0.0008	0.11
δ Cep . .	5 .37	3 .7	4 .6	F8	−2.2	1.27	18	0.07	10.5	0.0006	0.13
η Aql . .	7 .18	3 .7	4 .6	F9	−2.6	1.77	24	0.07	13	0.0003	0.13
ζ Gem .	10 .15	3 .7	4 .3	F9	−3.2	1.80	30	0.06	18	0.0002	0.15
X Cyg .	16 .38	6 .0	7 .0	G0	−3.9	6.12	48	0.13	26	0.00008	0.15
Y Oph .	17 .12	6 .1	6 .5	G0	−4.0	1.79	50	0.04	28	0.00008	0.15
l Car . .	35 .52	3 .6	5 .0	G0	−5.1	8.59	80	0.10	50	0.00003	0.19

The pulsation theory gives so good an account of almost all the characteristics of the Cepheid variables that it is at present widely accepted. It has not yet been proved (or disproved) that

pulsations of this sort can account for the very rapid rise in brightness which is observed in some of the cluster variables, nor has a satisfactory explanation of the remarkable relation between period and luminosity yet been reached. Moreover, observations at the University of Michigan show different velocity curves for different spectral lines, as if different layers in the star's atmosphere underwent oscillations of different amplitude and phase. These matters must be left to the future.

844. Jeans's Theory of Cepheids. An alternative theory, recently suggested by Jeans, regards the Cepheids as rotating stars in which the centrifugal force is so great, compared with gravity, that they are on the point of dividing into two parts, or actually in process of doing so. Such a mass would resemble an elongated ellipsoid rotating about its shortest axis, and with one equatorial axis more than twice as long as the other. It would be subject to oscillations, making it alternately more and less elongated, and very slowly increasing till it breaks in two. Both the oscillation and the rotation of the elongated mass would cause variations in light, thus accounting for complex light curves like that of RV Tauri (§ 849) as well as those of long-period variables and Cepheids, while the final state, after fission into parts, might give rise to an eclipsing variable. Jeans shows that this theory will account for the approximate constancy of $P\sqrt{\rho}$; but it has not yet explained the peculiarities of the velocity curves, and may still be regarded as tentative, though promising.

An older theory may be briefly mentioned, which attributes the variation to the orbital motion of the visible component through some sort of "resisting medium," which, by its friction, heats up the preceding face of the star. It is now clear that the stars themselves must be much larger than the orbits in which, on this theory, their centers of gravity must move, — so large, indeed, that the hypothetical companions would collide with them at periastron, so that the "orbital" theory now appears to be untenable.

845. The Long-Period Variables. These stars form a numerous class with periods ranging from 50 or 60 days to more than 2 years; but very few periods are less than 100 or more than 600

days, and more than half of them lie between 250 and 400 days. The range of variation is usually much larger than for the periodic variables of other classes, often exceeding four magnitudes and sometimes reaching eight or nine. The light-variation is only roughly periodic; the times of maxima are only approximately predictable, and the form of the light curve differs very considerably at different times.

Fig. 262, which is plotted from the observations of the American and French variable-star associations, represents the variation of χ Cygni from 1920 through 1925. This star is so far north of the ecliptic that it can be observed in the United States

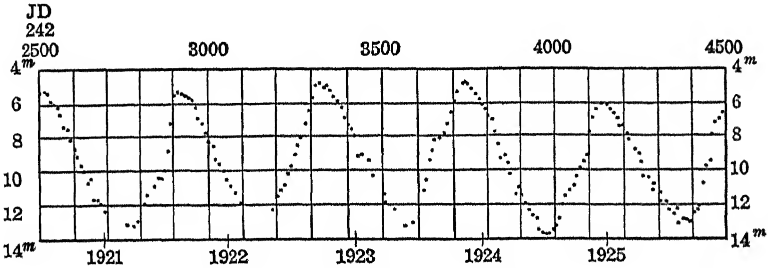


FIG. 262. The Light Curve of χ Cygni

The points are means of observations over ten-day intervals. The Julian date is given above, the calendar date below. (American Association of Variable-Star Observers)

almost continuously throughout the year, and the record is practically continuous. Successive maxima or minima may differ in brightness by a magnitude or more, and no two curves have exactly the same shape. The intervals differ a little in this case, but may do so by a month or more in the case of other stars. During the steady drop from magnitude 4.8, on November 6, 1923, to 13.7, on July 13, 1924, the star diminished to 1/35,000 of its brightness on the earlier date.

The mean light curve is usually smooth, and secondary maxima, or "still-stands," during the rise or fall are rare. The increase of brightness is usually more rapid than the decrease, but very rarely more than twice as rapid, as often happens among the Cepheids.

The best-known long-period variable is *o* Ceti, or *Mira* ("the Wonderful"), the first star to be recognized as a variable (Fabri-

cius, 1596). The variations are very conspicuous. At maximum it is usually of magnitude 3 or 4, but it occasionally reaches the second, and is the brightest star in that part of the sky. At minimum it falls to the ninth magnitude. The period averages 330 days, but the interval between successive maxima varies between 320 and 370 days. The predicted maximum for 1926 is on November 7, — in a convenient month for observation.

It has recently been discovered that Mira is a double star, the companion being of the tenth magnitude and $0''.9$ distant, and sharing the proper motion of the variable ($0''.23$). The pair is unquestionably physical. The companion is a remarkable white dwarf with a spectrum of Class B8 with bright lines of hydrogen and helium, enhanced iron, etc. Its existence was first suspected from the fact that these lines at the time of minimum were shifted laterally with respect to those of the M-type spectrum.

846. Spectra of Long-Period Variables. The long-period variables are all red stars and nearly all of them show banded spectra. A large majority are of Class M, but classes N, R, and S are also represented.

In typical stars, such as *o* Ceti, the bands of titanium oixde are very strong, as are also the low-temperature arc lines of the metals (Fig. 263). Both of these become still stronger as the light fades. These characteristics indicate an unusually low temperature. This is fully confirmed by the radiometric measures which have been discussed in Chapter XXI. Most long-period variables, when near maximum, show bright hydrogen lines, which are the most noteworthy feature of their spectra. Stars with banded spectra and bright hydrogen lines are practically sure to be variable, and great numbers of variables have been discovered at Harvard in this way.

In spectra of Class Me, that is, of Class M with emission lines, the violet hydrogen lines $H\gamma$ and $H\delta$ are brighter than the blue line $H\beta$. The next line of the series, $H\epsilon$, is absent. It falls in the great dark (H) line of $Ca+$, and it would seem as if the calcium vapor rose higher in the star's atmosphere than the luminous hydrogen, and absorbed its light. In spectra of Class Se, $H\beta$ is stronger than the other hydrogen lines. The bright hydrogen lines are usually sharp and somewhat displaced toward the violet,

the displacement being greatest for the stars of longest period. As the star's light diminishes, the bright hydrogen lines gradually fade. Other bright lines appear in the spectra, especially when the light is declining toward minimum. These are all low-temperature lines of elements of difficult ionization,—

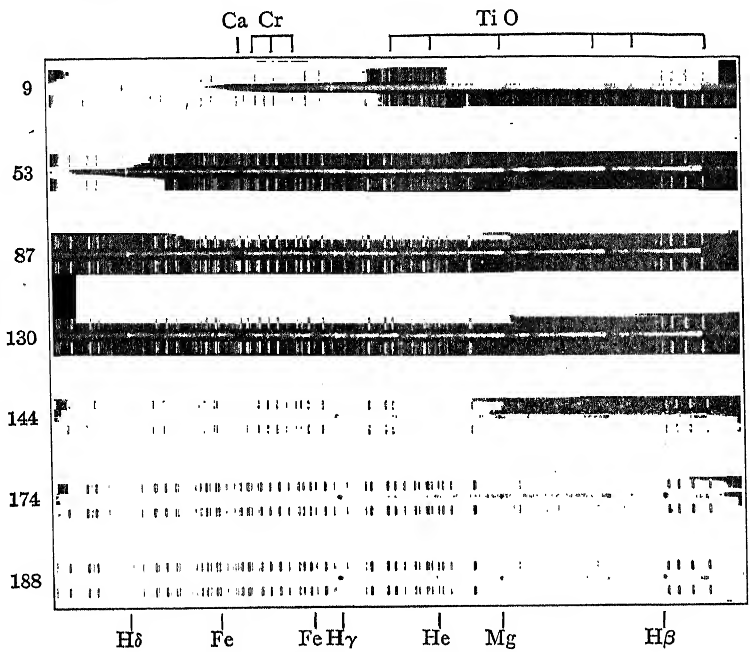


FIG. 263. Changes in the Spectrum of *o* Ceti

Slit-spectrograms, with iron-arc comparison, taken at intervals of from 9 to 188 days after maximum (as indicated on the margin). The bright hydrogen lines Hγ and Hδ are stronger than Hβ. As the light diminishes, the low-temperature lines of magnesium and iron (marked at the bottom) become bright, while those of calcium and chromium, and the bands of titanium oxide, remain dark but grow stronger. The wide fuzzy lines of hydrogen and helium at minimum come from the spectrum of the companion. The most important dark lines (Ca, λ 4227; Cr, 4254, 4274, 4289; and the bands of titanium oxide) are indicated at the top. The more prominent bright lines are shown at the bottom (Hβ, Hγ, Hδ; He, 4471; Mg, 4571; Fe, 4202 and 4308). (A. H. Joy, Mt. Wilson Observatory)

magnesium, silicon, and iron. The similar lines of metals of easy ionization, such as calcium and chromium, are dark.

Among the long-period variables of Class Me, which are the most numerous group, the average range of variation increases from about 3.5 magnitudes, for a period of 150 days, to 7 magni-

tudes, for a period of 500 days. The general average is 4.9. Ludendorff, who discovered this fact, has also pointed out that those variables which have no bright lines in their spectra have, on the average, a range about 2 magnitudes less. The mean period increases with increasing redness, being 183 days for stars of spectra from K to M5, inclusive, 271 for M8, and 387 for N. For the stars with emission lines the mean periods are 259, 297, and 361 days when the underlying spectra are M0, M5, and M8. The rise to maximum is most rapid in the Me stars of long period (over 410 days). These stars show a strong concentration toward the Milky Way, while those of shorter period do not.

847. The proper motions of long-period variables are small, averaging about 0″.03 for stars of magnitude 7.5 at maximum, while their radial velocities are large, the mean peculiar radial velocity being 35 km./sec. The velocity of S Libræ is + 385 km./sec. The solar motion with respect to these stars also comes out exceptionally great (65 km./sec.). From these data and other evidence Merrill and Strömberg conclude that the absolute magnitude of long-period variables of Class M, at maximum, is close to 0. The stars of Class S appear to be of about the same brightness. Long-period variables may therefore be regarded as giant stars which periodically grow faint. If the radiometric magnitudes were to be considered, they would be classed not merely as giant stars but as very bright giants at all times, for their heat radiation is always large in proportion to their light, and diminishes by only 1 or 2 magnitudes when the light falls by 5 magnitudes or more.

These measures show, as has been explained in Chapter XXI, that the superficial temperatures of the long-period variables are much lower than those of any other stars, and this is confirmed in the case of *o* Ceti by the direct measures of its diameter (§ 825). The radial velocity of *o* Ceti, as derived from the absorption lines, shows a definite variation in the period of the light-variation, which, when interpreted as orbital motion, gives $a \sin i = 28{,}000{,}000$ kilometers. The form of the velocity curve resembles that of a Cepheid, but the maximum velocity of *recession* comes close to the time of maximum light. Several other variables show evidence of similar change in velocity.

848. The cause of long-period variation is still obscure. It is now clear that these stars are giants of very large diameter and exceptionally low surface temperature. Thus a moderate change of temperature (perhaps 20 per cent on each side of a mean of 2000°) is sufficient to account for the great changes in visual magnitude and heat-index, accompanying, as these do, relatively small ones in the total radiation, on account of the rapid change in luminous efficiency at these low temperatures.

The great increase in intensity of the titanium oxide bands at minimum is also important. Joy, from direct measures, finds that these bands diminish the visual brightness of *o* Ceti by about two magnitudes even at maximum, and estimates that this effect is twice as great at minimum. The violet region is little influenced by the bands. This explains why the color-index is much less than for most stars of low temperature, and does not increase at minimum. Joy estimates that the actual range of temperature is from 2300° to 1800° K, which should change the radiometric brightness of a black body by $1^m.1$ and the visual brightness by $3^m.3$. The rest of the observed visual range (which averages $5^m.5$) may be attributed to the increased band absorption.

The cause of these temperature changes is uncertain. The velocity variations suggest a *pulsation*. Assuming that *o* Ceti, at an average maximum, has the absolute magnitude 0, its parallax comes out $0''.02$, and its radius, 210×10^6 km. The observed $a \sin i$ is 13 per cent of this, suggesting a change in size of about 18 per cent each way, which harmonizes with the 20 per cent range in temperature. The mass may be roughly estimated as 5 or 10 times the sun's, which would give a density of about 3×10^{-7} and make $P\sqrt{\rho}$ about 0.2, in line with the values found for the Cepheids. But why such a pulsation, if present, should make the star bright in the stage when the Cepheids are faint is unexplained, and the irregularities in the period are hard to account for on this hypothesis.

The "*veil*" *theory* assumes that the outer regions of the star become periodically clouded, owing to an accumulation of opaque material of some sort, and that heat accumulates below this "veil," until the obstructing haze is dissipated, and then streams

out rapidly. This accounts admirably for the irregularities of periodicity, but it seems very improbable that an atmosphere of such excessively low density could become opaque in this fashion. Neither theory can account, at present, for the bright spectral lines and their remarkable changes, and it may be that the true interpretation is quite different from either.

IRREGULAR VARIABLES

849. Many stars vary in brightness in a manner which shows so little systematic character and is so unpredictable that they are necessarily classed as irregular variables. They are almost all red stars (classes M and N), and very few of them vary more than two magnitudes.

In spectral peculiarities and in semblance of periodicity some of them resemble the long-period variables. In fact, the boundary between these two classes is very ill-defined, some authorities assigning a star to the one class, some to the other. Some of the brightest stars, such as α Orionis and α Herculis, are irregular variables. They are stars of great size and low density. The cause of their variation is unknown. The interferometer measures of α Orionis indicate changes of diameter from year to year which may indicate some kind of pulsation.

A few sub-groups of stars with definite characteristics of variation may be recognized. They are customarily designated by the stars which are typical of the groups.

(1) RV Tauri. The variation of these stars resembles, to some extent, but in an irregular manner, that of the Cepheids. A shallow minimum often occurs between two deep ones. The average period of RV Tauri is 78 days. Their spectra, when known, are of types G and K. The range in brightness rarely exceeds two magnitudes.

(2) R Coronæ Borealis. This star remains, often for years, constantly of the sixth magnitude, then rapidly drops several magnitudes. After an interval which may be short or may be several years, and during which there take place minor oscillations, the star brightens again to its normal brightness. Two or three other very similar variables are known.

The spectrum at the constant phase resembles that of the brightest Cepheids, and has been classified by Adams and Joy as cG0. Bright enhanced metallic lines appear at minimum.

This change eliminates the otherwise attractive explanation that obscuring clouds (§ 893) of opaque matter sometimes drift between us and the star. This explanation, however, is apparently permissible in the case of numerous faint, irregular variable stars which occur in and near the great nebula in Orion. These stars are dwarfs in absolute magnitude, — the only dwarf variables known, except a few eclipsing binaries like W Ursæ Majoris.

(3) U Geminorum. Stars of this type are normally faint, but brighten up abruptly at irregular intervals of a few months, to drop back again after a few days. The most notable of them are SS Cygni, 12^m brightening to $8^m.5$; U Geminorum, $13^m.8$ to $8^m.8$, and SS Aurigæ, $14^m.7$ to $10^m.5$. The spectrum of SS Cygni at maximum is characterized by wide, shallow, dark bands of hydrogen etc.; at minimum it is of the Wolf-Rayet type (§ 686). Both the light-variations and the spectroscopic peculiarities show some analogy to novæ.

NOVÆ

850. Temporary stars occasionally blaze up with startling rapidity and gradually fade out again. The photographic records show that they are not new creations, but that most of them were previously visible as very faint stars. Consequently the designation "temporary star" is more accurate than "new star."

The brighter ones have usually been discovered with the naked eye by some amateur observer familiar with the constellations, a glance at the sky revealing the intruder. Many fainter ones have been found, long after their appearance, by comparison of photographs or by their spectra. Nova Geminorum 1903 was discovered as a result of being used as a guide star for one of the Oxford astrographic plates. Investigation as to why the plate was wrongly centered led to the recognition of a nova.

The number of these objects must be very great, since in the first quarter of the twentieth century five have been conspicuous to the naked eye: Nova Persei 1901 rose from $13^m.5$ to $0^m.1$; Nova Geminorum 1912, from 15^m to $3^m.7$; Nova Aquilæ 1918,

from 11^m to $-1^m.1$; Nova Cygni 1920, from below 15^m to $1^m.8$; and Nova Pictoris 1925, from $12^m.7$ to $1^m.2$. Nova Lacertæ 1910 rose from $13^m.5$ to $5^m.0$, but was not discovered till it had dropped to the seventh magnitude. This number may be exceptional, as no bright nova was recorded in the first half of the nineteenth century, and only three in the second half. The fainter ones are much more numerous. Bailey, from a study of the Harvard plates, concludes that at least ten, and probably twenty, brighter than the ninth magnitude, appear every year. Very few of these happen to be detected, and the total number of novæ which are so far definitely authenticated is about fifty.

851. Distribution and Brightness. The novæ show a very strong galactic concentration, a large majority being within 10° of the galactic equator, and very few more than 20° from it. The fainter ones are much more numerous in the neighborhood of Sagittarius than in the opposite part of the Milky Way.

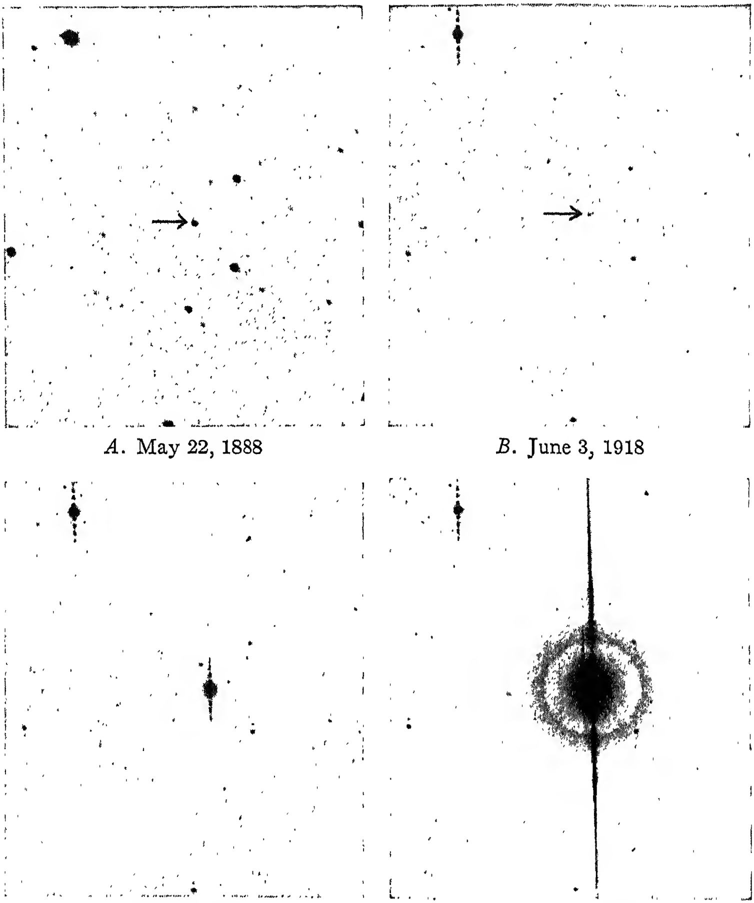
In addition to these galactic novæ, numerous very faint novæ and two brighter ones have been observed in certain spiral nebulæ. The brightest stars ever recorded have been novæ. Nova B Cassiopeiæ, known as "Tycho's star," which appeared in November, 1572, was for some days as bright as Venus at her best (visible in the daytime), and then gradually waned until, at the end of sixteen months, it became invisible, for there were no telescopes then. It is not certain whether it still exists as a telescopic star; it may be any one of several which are near the place determined by Tycho.

Another temporary star was observed by Kepler in 1604, which for some weeks was as bright as the planet Jupiter and remained visible for nearly two years. The brightest since then was Nova Aquilæ 1918, which exceeded all the stars but Sirius and perhaps Canopus.

852. Light Curve. Thanks to the photographic records, it is possible to follow the history of the novæ for many years preceding their discovery as well as subsequently. Many of them were visible before the outburst as very faint stars, which showed only small fluctuations in brightness.

The rise to maximum is exceedingly rapid, usually taking place in two or three days or even less. Nova Aquilæ, on June 5, 1918,

was of the eleventh magnitude, as it had been for thirty years previously. On June 7 it was of the sixth magnitude and increased



A. May 22, 1888 *B*. June 3, 1918

C. June 7, 1918 *D*. June 8, 1918

FIG. 264. Nova Aquilæ, No. 3

The images above and below each bright star are formed by a grating placed in front of the objective, for the purpose of determining photographic magnitudes (§ 695). The halation ring about the bright image of the nova on June 8 was produced by reflection from the back of the plate. (Harvard Circular 210)

half a magnitude in half an hour (Fig. 264). The next night it was of the first magnitude and was *discovered* with the naked eye by dozens of people. On June 9 its magnitude was – 0.5.

Nova Persei 1901 rose from below the eleventh magnitude to above the third in twenty-eight hours.

After attaining its maximum brightness the typical nova begins almost immediately to fade. The fall is at first rapid; Nova Aquilæ had declined to the third magnitude eighteen days after its discovery (Fig. 265). The decrease in brightness gradually slackens, and fluctuations, amounting sometimes to nearly a magnitude, take place, which are roughly periodic and slowly decrease in amplitude.

A year after the outburst the brightness is from a hundredth to a thousandth of that at maximum, and the star returns to

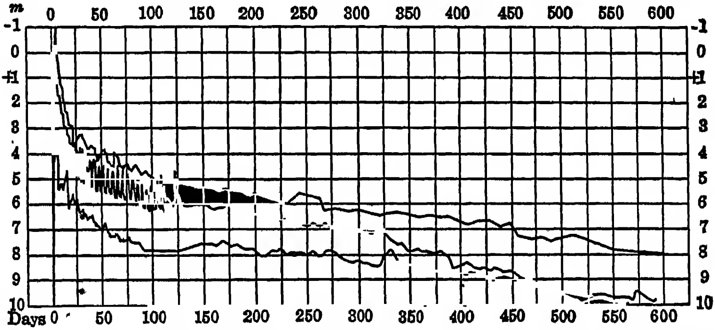


FIG. 265. Light Curves of Nova Aquilæ, 1918; Nova Persei, 1901; and Nova Geminorum, 1912

Courtesy of Harvard College Observatory

normal brightness in from ten to twenty years. Sometimes, however, the changes are slower, as in Nova Aurigæ 1892, which remained bright for about four months, and in Nova Pictoris 1925, which remained brighter than the fourth magnitude for nearly six months.

A still more remarkable case is that of T Pyxidis, which is ordinarily of the fourteenth magnitude, but rose abruptly above the eighth in 1890, 1902, and 1920. The light curve on all three occasions, and the spectrum on the last, resemble that of the novæ so strongly as to leave little doubt of the similarity of the changes.

Still more exceptional is η Carinæ. It is situated in a remarkable nebulous region (Fig. 217) and has a very peculiar spectrum, full of sharp bright lines, many of them of unidentified origin.

P Cygni, which appeared in 1600 as a star of the third magnitude, has been, for many years past, constant at $4^m.5$. Its spectrum shows remarkable bright lines accompanied by dark components on the violet side, but does not appear to change. A few other stars with similar spectra are known.

Whether these last two stars should be counted as novæ or as irregular variables of a peculiar sort is not certain.

853. Parallax and Absolute Magnitude. The parallaxes of several novæ have been carefully observed and found to be very small. They are as given in Table XXXII.

TABLE XXXII. PARALLAXES OF NOVAE

	Date	Parallaxes	Number of Observers
Nova Persei	1901	$0''.011 \pm 0''.003$	6
Nova Lacertæ	1910	$0\ .012 \pm 0\ .014$	1
Nova Geminorum	1912	$0\ .002 \pm 0\ .008$	2
Nova Aquilæ	1918	$-0\ .003 \pm 0\ .004$	5
Nova Cygni	1920	$0\ .026 \pm 0\ .008$	1

The mean for all five is $0''.010$, while the mean of the visual magnitudes at maximum is 1.9, and before the outburst not brighter than $13^m.5$. The corresponding absolute magnitudes are -3.1 and $+8.5$, which indicates, in a general way, that novæ, before the outburst, are faint stars, while at maximum they are comparable with the very brightest. Further evidence of the latter fact is given below. (But see page 789.)

854. Spectrum. The novæ follow, with their variation in brightness, a sequence of spectral characteristics which, in the main, are common to all. The account which follows is taken mainly from Wright, 1924.

During the interval of a new star's rise to maximum luminosity the spectrum is continuous, with a few faint and diffuse emission and absorption bands. In some cases, as in Nova Aquilæ and Nova Pictoris, the details at this time were more pronounced. In the first case the spectrum strongly resembled that of α Cygni; in the second, that of Canopus or ι Scorpii, — all c-stars of very great luminosity.

Immediately after maximum the spectrum changes abruptly to one containing broad bright "bands," not like those which appear in band spectra. These bands appear to be ordinary emission lines enormously widened, bordered on the violet side by absorption lines which are sometimes double. Many isolated absorption lines also appear. The absorption lines are identifiable with familiar stellar lines shifted toward the violet by amounts corresponding to an enormous radial velocity of approach, which for the two sets of lines in Nova Aquilæ (Fig. 266) amounted to

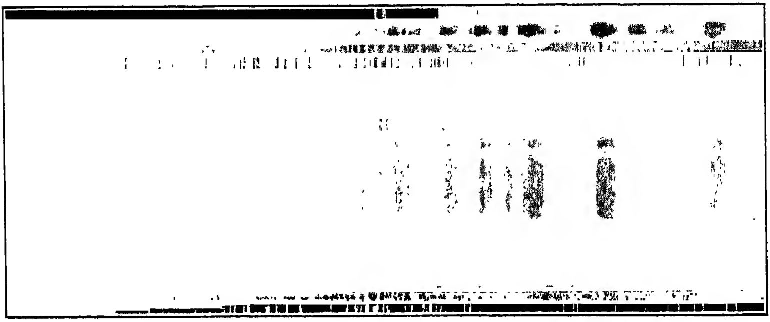


FIG. 266. Spectrum of Nova Aquilæ, June 12, 1918

These photographs were made three days after the nova had reached its maximum. The comparison spectrum is that of α Cygni. In the upper section the narrow spectra are shown as photographed. To each of the narrow dark hydrogen lines in α Cygni (below) corresponds a wide bright band in the nova, with two dark lines on the violet side of it. In the lower section the spectra are artificially widened, and that of α Cygni is deliberately displaced toward the violet. Almost every line in it is now seen to coincide with one in the spectrum of the nova. The numerous lines in the ultra-violet are due to ionized metals: chromium, vanadium, etc. (From photographs by W. H. Wright, Lick Observatory)

1700 km./sec. and 2300 km./sec. The displacements increased for the first two days and then remained nearly constant. The hydrogen lines are always present. The others are sometimes the enhanced lines of the metals, such as appear in α Cygni (cA2), and at other times, and usually later, lines of helium, oxygen, and nitrogen, such as are found in spectra of Class B. The bright bands represent some of the same lines, especially those of hydrogen, widened symmetrically on both sides of their normal position. These wide bands usually show considerable structure, containing numerous subordinate maxima, which are similar for different bands and retain their places, with little alteration, for months.

The (H) and (K) lines of calcium, and the (D) lines of sodium, are usually sharp and narrow, and show small radial velocities resembling the "stationary" lines found in spectroscopic binaries. At certain times the band of N + at λ 4640 becomes extremely wide, while the enhanced lines of helium disappear.

Later, when the light has faded considerably, the bright lines which are characteristic of the gaseous nebulæ appear, widened like the others into bands. After a few years these fade out and leave a spectrum of the Wolf-Rayet type (Class O). It is probable that this also changes in the course of time, for T Coronæ

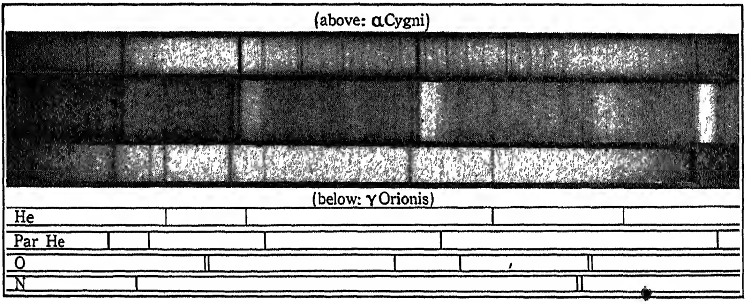


FIG. 267. Spectrum of Nova Geminorum

Photographed March 30, 1912, in comparison with α Cygni (Class A2) and γ Orionis (B2). The less-displaced hydrogen lines of the central spectrum (of the nova) belong to a spectrum similar to the first of these; the more displaced, to one similar to the second. Many fainter lines correspond to lines in one or the other. The positions of lines of helium, "parhelium," and of ionized oxygen and nitrogen are marked below ("parhelium" is a term now almost obsolete, which was once applied to the lines of the singlet series of helium. (From photograph by W. H. Wright, Lick Observatory)

Borealis, which was of the tenth magnitude before its brief outburst in 1866 and soon returned to it, now shows a spectrum of Class gM, except that the bright line of He + at λ 4686, which is characteristic of the O-stars, is present. The present absolute magnitude of this star, as determined from its spectrum, is not far from 0, whence it follows that, at its brief maximum in 1866, it was of absolute magnitude − 7.

Nova Aquilæ 1918 is the only one whose spectrum has been caught before the outbreak. It is shown on objective prism plates taken some years before the rise, and, though very faint on the plates, appears to have been of Class A.

855. Origin of the Bands. The Expanding Shell of Gas. The observation of Nova Aquilæ in 1918–1919 showed clearly that the radiations producing the wide bright bands originated, not in the star itself, but in a shell of gas surrounding it and moving outward in all directions with great velocity. This shell became visible in the telescope, within six months of the outburst, as a faint, greenish, nebulous envelope surrounding the star, which evidently started from it at the time of the disturbance and increased steadily in diameter at an almost uniform rate of 2″ per year. As this disk expanded and its image covered a larger part of the slit of the spectrograph the bright bands in the spectrum widened on each side of the narrow linear spectrum of the star, showing

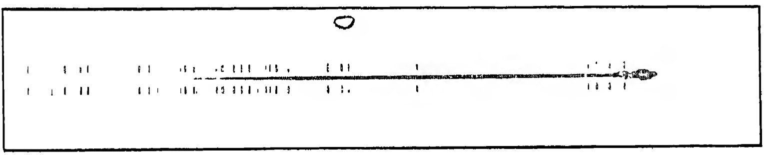


Fig. 268. Spectrum of Nova Aquilæ and its Nebulous Envelope

The elliptical images of the nebular lines (§ 905) are conspicuous at the right. (From photograph, Mt. Wilson Observatory)

that their light came from the expanding nebula. Some other bright lines were confined to the central star. After about four years the wide bright bands in the spectrum (Fig. 266) faded out; but the gaseous envelope was still there in 1926, and had reached a diameter of 16″. On the later spectrograms the bright bands have a conspicuously elliptical shape, as shown in Fig. 268. This is just what might be expected, as appears from Fig. 269, which represents a cross-section of the expanding shell of gas by a plane passing through the earth and the star. At the points A_1 and A_2, where the shell appears farthest from the star, the outward motion is at right angles to the line of sight, and the lines are not displaced by Doppler effect.

For a part of the shell apparently nearer the star, as indicated by the dotted line, the moving gases are either at B_1 or C_1, and the bright lines will be strongly displaced to the violet and the red. Finally, close to the star, near D and E, the displacements will be at maximum, and equal in both directions. This explanation (due

to Adams) accounts for the phenomena very well. For example, the bright details in the bands (visible in Fig. 268) indicate that the gases in some particular portions of the shell (for example, at B_1) shine more brightly than elsewhere. As the shell expands, this portion will move with constant velocity. The region in the band with displacement corresponding to the radial velocity of B will be, and remain, unusually bright.

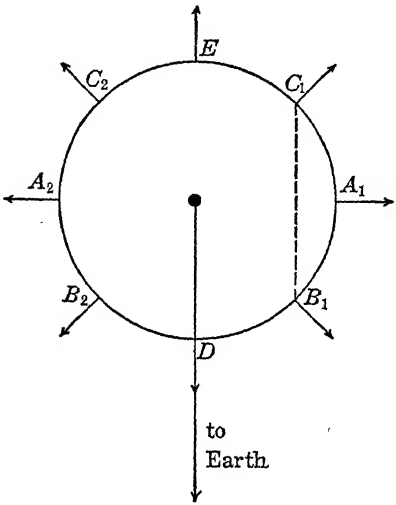


FIG. 269. The Expanding Shell of Gas around a Nova

The Doppler shifts in the spectrum of a nova are thus explained (§ 855)

Absorption lines can be produced by the shell only at D, where it lies in front of the star, which alone gives a continuous spectrum. These lines will be shifted toward the violet by an amount corresponding to the full velocity of expansion; that is, they must lie on the short-wave edge of the bright bands, as is observed to be the case. Many lines appear to be absorbed by the shell but not emitted with any strength, — notably the enhanced metallic lines. The two sets of displaced hydrogen lines of Nova Aquilæ can be explained as due to absorption in two shells, one expanding at 1700 km./sec., another at 2300 km./sec. In Nova Cygni 1920 there were for a time five sets of displaced hydrogen lines, indicating velocities from 800 to 1800 km./sec. The metallic lines shared the smallest displacement. The more displaced lines usually fade out sooner than the others.

Though the expanding disk has not actually been seen, except in Nova Aquilæ, the spectroscopic phenomena in other novæ are so similar as to make it very probable that they have a similar origin.

The principal shell around Nova Aquilæ showed a radial velocity of 1700 km./sec., or about 360 astronomical units per

year. Comparing this with the observed angular rate of increase of the radius, $1''.0$ per annum, the parallax comes out $0''.0028$, the distance 1200 light-years, and the absolute magnitude at maximum about -8.8, corresponding to almost 300,000 times the sun's luminosity. The absolute magnitude before the catastrophe comes out $+3$, — nearly normal for a "late" A-star.

One serious difficulty, however, attaches to this explanation. The images produced by the nebular lines and by the hydrogen lines show the same radial velocities, but the lateral extension (that is, the size of the disk) is much larger for the first than for the second. Until this matter is cleared up the estimate just given for the distance of the nova must be received with caution.

856. The Nebula near Nova Persei. A nebula of a quite different sort was observed near Nova Persei in 1901, — a diffuse, extended cloud of faint light (Fig. 270). On September 20, seven months after the nova appeared, the most conspicuous features of the nebula were from $6'$ to $7'$ from the nova. Six weeks later they had all moved outward by distances ranging from $35''$ to $65''$, and laterally as well by considerable amounts. Such rapid motions in objects at stellar distances were, and still remain, quite unprecedented. Further observations showed that the nebulosity was moving outward in all directions from the star, at a rate of about ten minutes of arc per year. Measures of parallax soon showed that the star was remote, and that the motions must be extraordinarily rapid; but even before this happened, Kapteyn suggested the explanation which has been generally accepted. At the time of maximum the star was intensely luminous for a few days; then it faded rapidly. The outgoing light from it must therefore have formed, six months later, a hollow sphere one light-year in diameter and only a few hundredths of a light-year thick; outside was darkness, for the light of the outburst had not arrived, and inside there was only the fainter light of the fading star. Suppose now that, near the nova, there was a sheet of scattered clouds of diffuse matter in space — dark nebulæ — invisible unless lighted up from the outside. As the expanding spherical shell of light passed over these, one after another would be lighted up for a few days, and an observer at a great distance would see an expanding ring of light, — irregular, since the clouds

and wisps of reflecting matter were not uniformly distributed, and fading out as the light grew fainter and passed beyond the clouded region. The spectrum of this nebula (photographed by Perrine) was quite unlike that of any other, and was consistent with what might have been expected with reflected light from the nova.

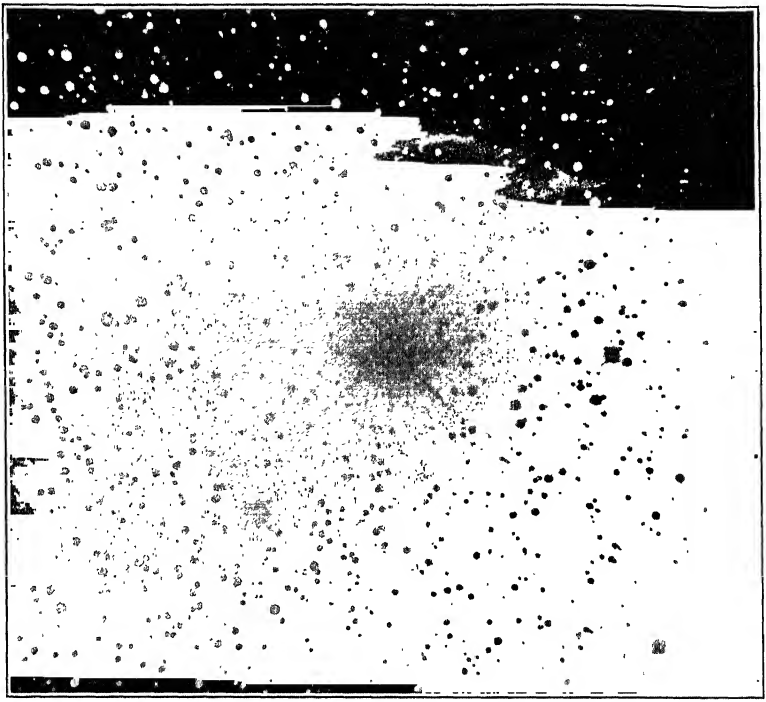


FIG. 270 *A*. The Moving Nebula near Nova Persei, September 20, 1901
Photograph by G. W. Ritchey, Yerkes Observatory

It might seem at first that this should give a very accurate determination of the distance of the nebula, and hence of the nova, but things are not quite so simple. At any given date, say one year after the star flared up, reflected light would reach us from any points, *B*, *C*, or *D* (Fig. 271), such that the sum of the distances *NC* and *CE* was greater by one light-year than the distance *NE* from the nova to the earth. These points lie (in the plane of the figure) on an ellipse with foci at *N* and *E*; in space they are on the ellipsoid produced by rotating this ellipse

about *NE*. With any assumed distance for the nova this ellipsoid can be located in space, and points on it can be found, such that dust-clouds, situated there, would have reflected light to us at the observed time. If, however, the distance is assumed too great or too small, these hypothetical clouds lie behind the nova, as at *B*,

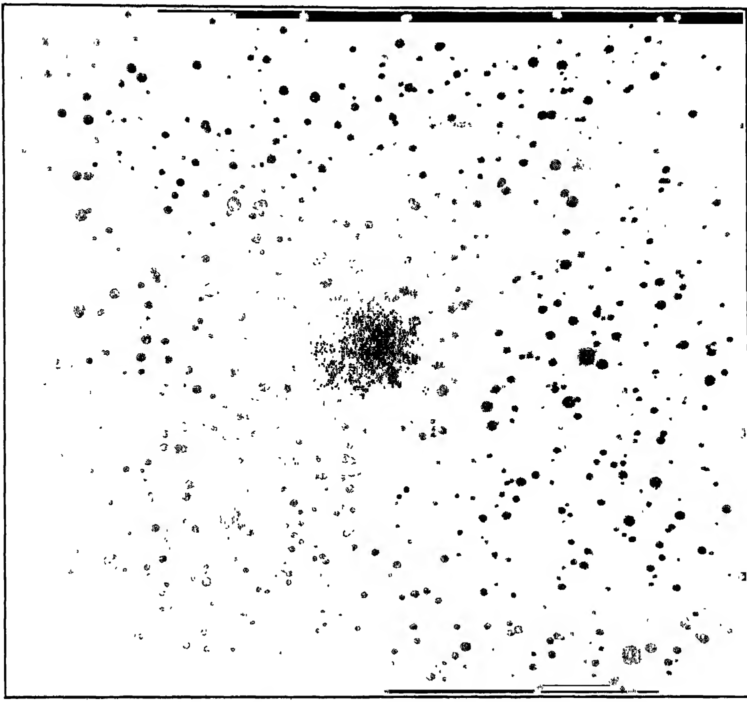


FIG. 270*B*. The Moving Nebula near Nova Persei, November 13, 1901
Photograph by G. W. Ritchey, Yerkes Observatory

or in front of it, at *D*. On the plausible but not demonstrable assumption that they actually lie at the same distance as the nova, like *C*, its distance comes out about 350 light-years, the parallax $0''.009$, and the absolute magnitude at maximum -5, about 10,000 times as bright as the sun. A much smaller, but more permanent nebulous envelope, about $30''$ in diameter in 1926, and slowly expanding, may be analogous to the gaseous shell observed around Nova Aquilæ, but its motion is slower than would be inferred from the parallax and radial velocity.

857. Cause of the Phenomena. It is evident that the outburst of a nova transcends in magnitude all other known physical catastrophes. The fact that the outbreak is short-lived, and that the star is left much as before after it is over, shows, however, that the disturbance cannot be very deep-seated.

These outbursts, moreover, are by no means of rare occurrence. Allowing ten galactic novæ per year, and assuming that the number of stars which are capable of becoming novæ is ten billion (ten times the total number observable with the greatest telescope), the catastrophe should happen to a given star, on the average, once in a billion years, and the stars are probably much older than this. Many, perhaps most, of the stars may therefore have been novæ at some time during their careers.

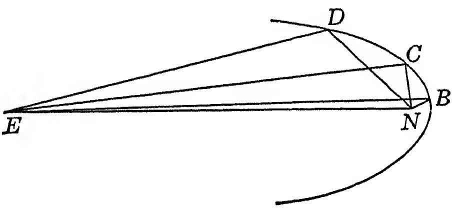


FIG. 271. The Illumination of a Dark Nebula by Nova Persei (see section 856)

The immediate cause of the cataclysm appears to be a tremendous development of energy, somewhere beneath the star's surface, which heats the stellar gases to such a degree as to cause them to expand violently into space in all directions at speeds far too great to be held back by the star's gravitation.

For the first few days the expanding mass of gas is dense enough to give a continuous spectrum, and the light increases enormously. The expansion in Nova Aquilæ was at the rate of about one astronomical unit per day, so that in two days it would have been almost as large as Antares, and, being much hotter, may well have been as bright as estimated above. As the expanding mass becomes more rarefied its spectrum becomes discontinuous, and the gaseous shell described in section 855 is formed. Meanwhile the main mass, highly heated by the outbreak, settles back, at first tumultuously, and then gradually, into a hot star (Wolf-Rayet type). The disappearance of this spectrum after a few decades indicates that the heating was only skin deep.

Only two known sources of energy appear to be competent to produce such tremendous consequences. The first is a collision

between two stars; but an easy calculation shows that, with stars strewn in space as thinly as they are, such collisions should be exceedingly rare, — millions of times less frequent than the appearance of even bright novæ. The second is the release of the energy which is now believed to be stored within the atoms. This release might be precipitated either by some internal condition of the star or (more probably) by the collision of the star with a wandering body of small dimensions, — perhaps no larger than an asteroid, as W. H. Pickering has suggested. This hypothesis appears to be capable of accounting for the main facts, and will be discussed further in Chapter XXVI. It must remain, however, a speculative one for the present.

EXERCISES

1. Convert the magnitudes of χ Cygni (Fig. 262) into light-intensities and draw the light curve. Notice how this differs from the curve in magnitudes. Do the same for RT Persei (Fig. 246).

2. Why are estimates by the Argelander method less suited to the careful study of eclipsing variables?

3. What evidence of variation in the period and range of χ Cygni is to be found in Fig. 262?

4. What is the magnitude of a variable star from the observations $a4v1c$ and $b1v4d$ if the magnitudes of the comparison stars are $a = 9.2$, $b = 9.4$, $c = 9.8$, $d = 9.9$? *Ans.* 9.6.

5. Why cannot the variation of a Cepheid variable be interpreted as due to eclipse?

6. What is the period of RR Ceti (Fig. 259)? What is the range in magnitude and in light-intensity?

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G. Müller and E. Hartwig, *Geschichte und Literatur des Lichtwechsels der Sterne.* Poesche and Trepte, Leipzig.

C. E. Furness, *An Introduction to the Study of Variable Stars.* Houghton Mifflin Company, Boston and New York.

Karl Schiller, *Einführung in das Studium der veränderlichen Sterne.* J. A. Barth, Leipzig.

Note. For Nova Sagittæ (1913), which was $7^m.2$ at maximum, van Maanen finds $\mu = 0''.080$, $p = 0''.012 \pm 0.005$, giving $M = +2.6$ at maximum. Unless this star has a very large tangential velocity, it must have been much fainter than the other novæ.

CHAPTER XXIII

STAR-CLUSTERS AND THE MILKY WAY

OPEN CLUSTERS · THEIR DISTANCES AND DIMENSIONS · GLOBULAR CLUSTERS · THEIR DISTANCES, DIMENSIONS, BRIGHTNESS, MOTIONS, AND DISTRIBUTION · THE MAGELLANIC CLOUDS · THE GALAXY, OR MILKY WAY · GALACTIC COÖRDINATES · SHAPE AND DIMENSIONS OF THE GALAXY · THE "LOCAL SYSTEM" · STATISTICAL STUDIES OF THE GALAXY · DENSITY, LUMINOSITY, AND VELOCITY FUNCTIONS

858. Star-Clusters, Star-Clouds, and Nebulæ. The stars are by no means scattered at random in the sky, but show a decidedly gregarious tendency. Here and there obvious *star-clusters* appear, some of which are conspicuous to the naked eye and were recognized and named in ancient times, as the Pleiades and the Hyades (both in Taurus), the group which forms the constellation of Coma Berenices, and the smaller one in Cancer known as Præsepe. The first three of these are "resolvable" by the naked eye (that is, the separate stars which compose them can be seen); the last, without optical aid, looks merely like a hazy spot of light, though the individual stars show clearly with a field-glass. Hundreds of fainter clusters are revealed by the telescope, most of which can be seen with small instruments, although larger ones are often required in order to resolve them into stars.

Besides these there are many luminous patches in the heavens, sometimes several degrees in extent, which, with a powerful telescope (or, still better, on a photograph of long exposure) are resolved into multitudes of faint stars. Most of these *star-clouds* are found in the Milky Way. The finest, which lie in the region of Sagittarius, in the southern summer sky, are familiar to everyone. Two isolated patches, of similar nature, which lie about 45° from the Milky Way and 20° from the south celestial pole, are known as the *Magellanic Clouds* (after the great navigator, who was thus honored after his death). They are conspicuous to the naked eye in southern latitudes.

Telescopic observations also reveal great numbers of small patches of light not resolvable into stars, which are called *nebulæ*, and still larger numbers of these can be detected by photography.

All these objects are permanent features of the heavens, and evidently are situated at distances which must be comparable with those of the individual stars, and may be greater. Relatively little was known about them even as late as 1910; but progress has since been remarkably rapid and has led to an extraordinary expansion of our ideas of the extent of the material universe.

859. Types of Star-Clusters. Clusters fall naturally into two classes: *open clusters* and *globular clusters.* The former contain usually a few hundred, or at most a few thousand, stars far enough apart to be easily separated telescopically; the brighter ones are all visible in small telescopes, and some of them to the naked eye. Globular clusters are composed of many thousands of very faint stars, densely packed into a close, globular swarm, though with no sharp boundary. Figs. 272 and 273 give an excellent idea of the appearance of the two types.

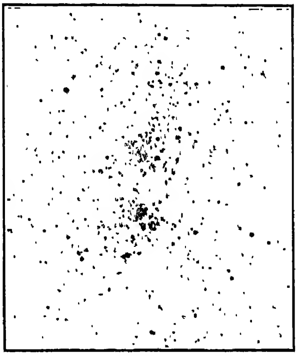

FIG. 272. The Clusters *h* and χ Persei

Rich open clusters, in a Milky Way region full of stars. (From photograph, Yerkes Observatory)

Clusters are usually known by their number in some catalogue, — either Messier's catalogue (M) of 103 clusters and nebulæ (1784), which contains the most conspicuous objects, or Dreyer's *New General Catalogue* (N.G.C.) (1887), which contains 7840 objects (mostly nebulæ), extended by the *Index Catalogue* (I.C.), in two parts (1894 and 1908), containing 5386 more.

860. Open Clusters. The number of open clusters which have so far been catalogued is about 200. All the clusters which are conspicuous to the naked eye belong to this type. Next to the four already mentioned, the most conspicuous are the great double cluster in Perseus (known as *h* and χ Persei) (Fig. 272) and Messier 7, which lies between Sagittarius and Scorpius. Both are easily visible to the naked eye and resolvable with a good field-glass.

Open clusters are distinctly *galactic* objects, the great majority of them lying within the Milky Way. Coma Berenices is an exception, close to the north galactic pole. Nearly all of them contain stars distributed over a wide range of magnitude. They vary greatly as regards the number of members, from groups containing thousands, like those in Perseus, to those so scanty that they can hardly be distinguished from chance agglomerations of unrelated

FIG. 273. The Globular Cluster Messier 13 in Hercules

Photographed by Ritchey with the 60-inch reflector, 11 hours' exposure. This picture corresponds to a much higher magnifying power than Fig. 272. (Mt. Wilson Observatory)

stars. They also differ widely in the degree of concentration toward the center, from compact groups like the Pleiades to very loose ones like Coma Berenices, and to the groups in Ursa Major and Scorpius (recognized by their proper motions), which are so scattered that they would not otherwise be called clusters at all. Some recent work of Shapley's indicates that the stars down to the twelfth magnitude show, in general, a much more patchy distribution than would be anticipated on the basis of mere chance, and it may be that extremely loose clusters are very numerous.

861. Non-Cluster Stars. In the region of the sky occupied by a cluster there are sure to be other stars which lie in the line of sight but in front of the cluster or behind it, and which have in reality nothing to do with it. These "non-cluster" stars should be excluded from any study of the cluster, but it is not easy to find which they are.

When the proper motions are known, there is no great difficulty, for the true cluster stars are all moving together (§ 737). Interlopers, in general, have different proper motions and can be picked out. For example, the bright star Aldebaran, though apparently almost in the middle of the Hyades, has a quite different proper motion and does not really belong to the group, being, in fact, much nearer.

Measures of radial velocity, when available, serve a similar purpose.

For faint stars these data are usually not available, and recourse must be had to statistical methods. The number of non-cluster stars should be approximately the same as in other neighboring regions of the sky of the same apparent area, and only the excess of stars above this number can be assigned to the cluster. Though this method does not tell us which stars belong to the cluster, it often gives very valuable information about its extent. Thus, in the case of the Pleiades, Trumpler finds, by the B.D. catalogue, that the number of stars brighter than $9^m.0$ in successive circular zones around the cluster is as follows:

Distance from Center	Number	Number per Square Degree	Probable Number	
			Non-Cluster Stars	Cluster Stars
0 –1°	63	20.0	12	51
1°–2°	56	5.9	36	20
2°–3°	79	5.0	60	19
3°–4°	87	4.0	84	3
4°–5°	104	3.7	108	(– 4)

Dividing the number of stars in each zone by its area in square degrees, the apparent "star density" is found. In the outer zones the nearly uniform density indicates what may be expected for the non-cluster stars. Assuming an average of 3.8 stars per square degree, the numbers in the fourth column are calculated. The excess of the numbers actually observed indicates the number of cluster stars, as given in the fifth column. It appears that the cluster is really about 3° in radius, much larger than it appears to the eye, but much condensed toward the center. With the aid of photographically

determined proper motions Trumpler has succeeded in picking out the cluster stars with considerable certainty, and has identified almost 250 (ranging from the third to the fourteenth magnitude) as members of the group.

862. The spectra of stars in open clusters belong to the usual classes and show a very marked correlation with the apparent magnitudes, which, as they are all about equally distant, differ from the absolute magnitudes by a fixed amount.

The fainter cluster stars of any given magnitude are all of about the same spectral class, growing steadily "later" for the fainter magnitudes, at the rate which is characteristic of stars belonging to the main sequence (§ 799). Among the brighter members, stars of early spectral class, belonging to the upper part of the main sequence, are always to be found, and in about half the clusters bright yellow and red stars (evidently giants) also occur. When these are present the brightest stars of the main sequence are of Class A (Hyades, Præsepe, etc.). In other cases (Pleiades, Perseus cluster, etc.) the brightest stars are of Class B, and there are no red giants. The color-indices confirm these conclusions. Non-cluster stars can often be identified by their failure to conform to these regularities.

In the Perseus cluster the brightest members are c-stars and must be very luminous.

863. Distances and Dimensions of Open Clusters. The distance of the Hyades has been accurately measured by the moving-cluster method. Spectroscopic parallaxes are available for a few other clusters. A more powerful method consists in assuming that those stars in the cluster which are of a given spectral class have the same mean absolute magnitude as other known stars of the same spectral class, belonging to the same sequence. When the main sequence can be clearly defined, and the giants, if any, separated, this method gives good values whenever it can be checked, and appears reliable. The colors of the stars can be used instead of their spectra, with some loss of accuracy.

In this way the distances of more than fifty open clusters have been estimated. The Hyades, at 40 parsecs, are much the nearest. Then follow the Pleiades and Coma Berenices, at about 100 parsecs, Præsepe at about 180, and so on. Most of them, according to Trumpler, lie between distances of 500 and 2000

parsecs, but the faintest and remotest are as much as 3000 parsecs, or 10,000 light-years, away. The diameters of the clusters can of course be only very roughly stated, as their outer boundaries are far from definite. The diameter of the Hyades is about 10 parsecs; that of the Pleiades, about the same.

The density with which the stars are scattered in space is very different for different clusters. In the faint galactic cluster Messier 11, Trumpler finds, at the center, 80 stars per cubic parsec, which is more than a thousand times greater than the star density in the vicinity of the sun (§ 722). In the Hyades, however, the hundred or so of stars which are known to belong to the cluster are spread through a region so large that it would probably contain more than thirty stars on the basis of random distribution. Thus there are probably many interlopers not merely in line between us and the cluster but actually within its limits — though not permanently, on account of their different motions. Most of these stars, however, must be fainter than the cluster stars.

864. Globular Clusters. About seventy of these remarkable objects (Fig. 273) are known, and it is probable that the list is substantially complete, except perhaps for very faint clusters. They may be recognized at once by their globular form, great central condensation, and the absence of any but very faint stars within them. The brightest of these clusters appears to the naked eye as a hazy star of about the fourth magnitude, and is known as ω Centauri. It is in declination $-47°$, too far south to be seen to advantage from stations north of the tropics. The next in order of brightness, 47 Tucanæ (Fig. 275), is in declination $-73°$. Three or four others are just visible to the naked eye. Among these is Messier 13 in Hercules (Fig. 273), — the finest globular cluster observable in northern latitudes.

The *number of stars* in the brighter clusters of this type must be at least fifty thousand, though they cannot all be counted, since on long-exposure photographs the star-images at the center run together. There is good reason to believe that even on these photographs only the brighter stars of the cluster are recorded, and that the total number of stars is much greater. Their *distribution* in the heavens is very peculiar, for they are almost con-

fined to one half of the celestial sphere. Only 4 of the 86 listed by Shapley are more than 90° distant from a point in the constellation Sagittarius in R. A. $17^h\ 30^m$, Decl. − 31° (galactic longitude 325° and latitude − 2°), and 44 of them are within 30° of this point.

865. Magnitudes, Colors, and Spectra of the Component Stars, — Variable Stars. The brightest stars in ω Centauri are of the photographic magnitude 12.3, those in M13 are about 13.5,

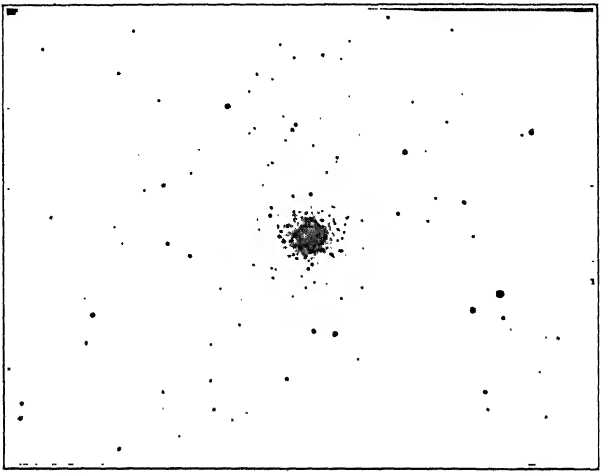

FIG. 274. The Globular Cluster N.G.C. 7006

From photograph by Mt. Wilson Observatory

while in the faintest clusters, such as N.G.C. 7006 (Fig. 274) there are no stars brighter than the seventeenth magnitude. The brighter clusters have been very carefully studied at Mt. Wilson by Shapley, who has determined both photographic and photovisual magnitudes, and finds that the stars exhibit a wide range of color-index, from about $-0^m.3$ to $+1^m.8$,— that is, from the color of stars of Class B to that of Class M. The brightest stars are all red, with average color-index about $+1^m.3$. The stars two or three magnitudes fainter are mostly white. Confirmation of these color differences is found in spectrograms made with a "focal-plane" slitless instrument, which show spectra of all classes from A to K. The integrated spectra of the clusters, produced by the

combined light of many stars, resemble Class F or Class G, as might be expected from a composite of all the types photographed with low dispersion.

The discovery of variable stars in the globular clusters has already been described (§ 839). They are all of the Cepheid, or cluster, type (except a few in 47 Tucanæ, which are apparently long-period variables) and show light curves, color-indices, variations in color, etc. of exactly the same sort as the galactic variables of the same type. The part which they have played in the determination of the period-luminosity curve has also been described.

866. Determination of Distance. When a cluster contains Cepheid variables, its distance may be found as soon as their periods and their apparent magnitudes on the photometric scale have been determined. The absolute magnitude may be read from the period-luminosity curve. The difference $m - M$ between the apparent and absolute magnitudes (which is often called the *modulus*) then gives the parallax by the equation

$$\log p = -1 - \frac{1}{5}(m - M).$$

For example, the median apparent photographic magnitude of the short-period variables in Messier 3 is 15.50. The median absolute magnitude, according to Shapley's curve, is -0.23. Hence the modulus is $+15^{m}.73$, and

$$\log p = -1 - 3.14 = 5.86 - 10;$$

whence $p = 0''.000072$, giving a distance of 14,000 parsecs, or 45,000 light-years.

Only a minority of the globular clusters contain known variables, but Shapley has extended his determinations of distance to the rest in a very ingenious fashion. In all the clusters which contain variables the brightest stars of all are found, on the average, to be $1^{m}.3$ brighter (photographically) than the cluster variables, with extraordinary uniformity from cluster to cluster. Their absolute photographic magnitude is therefore -1.5. (Visually it would be about -2.8, as these stars are all red.) This makes it possible to extend the photometric determination of distance to all globular clusters in which the magnitudes of the

brighter stars have been determined. In practice the five brightest stars of all in a given cluster are rejected (as some of them may be "foreground" stars accidentally projected on the cluster), and the mean is taken for the next twenty.

For the remaining clusters Shapley took advantage of the fact that if the apparent diameters of the clusters of known distance were plotted against the parallaxes, a smooth curve was obtained. From this curve the parallax of any other cluster could be read off, given its diameter as measured on a uniform series of plates like the Franklin-Adams charts.

867. Dimensions of Globular Clusters. The computed distances are vast beyond all precedent, and have led to an entirely new conception of the extent of the sidereal universe. For ω Centauri, the nearest and brightest of the clusters, the resulting parallax is $0''.00015$, and the distance 6500 parsecs, or 21,000 light-years.

This is the nearest of the clusters; Messier 13, the brightest in the northern hemisphere, is at a distance of 11,000 parsecs. Most of them are two or three times farther off, and the remotest so far known (N.G.C. 7006) (Fig. 274) has the enormous distance of 70,000 parsecs, or 230,000 light-years.[1]

At 186,000 miles per second, the light by which we see the nearest of these clusters has been traveling for a time far greater than the duration of human civilization; the light from the remotest ones has been traveling perhaps longer than the human species has existed.

The dimensions of these clusters are correspondingly great. The denser central portion of such a cluster is some 5 parsecs in diameter, while the thinner outlying regions extend to a diameter of fully 100 light-years, or perhaps farther, as is shown by the occurrence of occasional cluster variables beyond these limits.

The different clusters are extraordinarily alike, not merely in size but in all other characteristics. Photographs of them, taken on the same linear scale, and with exposures bringing out stars of

[1] These distances, and all quantities depending on them, are based on Shapley's period-luminosity curve. Some other investigators believe that these distances should be reduced by about one third. The matter is not yet decisively settled.

the same real brightness, would be distinguishable from one another only by the minor details of arrangement. The reason for this remarkable similarity is still unknown.

The law of density of distribution of the stars is apparently the same in all, the number of stars per unit volume varying nearly as $(r^2 + a^2)^{-\frac{5}{2}}$ when r is the distance from the center and a a constant. The fainter stars extend farther from the center than the brighter ones. This density law probably depends upon some sort of statistical equilibrium, according to which the numbers of stars that move from one region to another, under the attraction of the whole mass, are so adjusted as to preserve the general form, size, and structure.

The actual star density is very high. Within ten parsecs of the center of Messier 3 there are at least 15,000 stars brighter than the twentieth magnitude, and thus more than four times as bright (photographically) as the sun, while within a similar distance from the sun there are probably only about ten stars of equal brightness. Even so, the average distance of a bright star from its nearest neighbors in the cluster would be some 50,000 astronomical units, and, with a liberal allowance for fainter stars, there is plenty of room for them to move freely without collisions.

868. Brightness and Motions. The brightest stars are all red super-giants, comparable with Betelgeuse or Antares in brightness and color, and doubtless also in size. Among those which are two or three magnitudes fainter, white stars are found in abundance. Whether the still fainter ones grow yellower is not known, for stars as faint as the sun have not yet been photographed in any globular cluster, though they might be reached in ω Centauri with long exposures.

The total light of a globular cluster corresponds approximately to the absolute visual magnitude -9, that is, to about 300,000 times the sun's light. The total mass can only be roughly guessed at, owing to our ignorance of the fainter stars, but is probably several hundred thousand times that of the sun.

The stars of the cluster are doubtless moving in orbits under the attraction of the whole mass. A simple calculation shows that their speeds should be comparable with that of the earth in its orbit, which, even in ω Centauri, should displace a star by only $0''.1$ per century. It is not surprising, therefore, that no perceptible internal motions have yet been observed.

That the clusters as a whole are moving rapidly in space is shown by the radial velocities, which Slipher and others have

measured for 18 of them. These run up to 350 km./sec. and show a solar motion, relative to the system of clusters, of about 300 km./sec. toward $20^h\ 24^m$, $+62°$. When allowance is made for this, the average peculiar radial velocity of a cluster is found to be 117 km./sec. The corresponding proper motions, which, with Shapley's distances, should average less than $0''.5$ per century, have not yet been detected.

869. Distribution in Space. The distribution of these globular clusters in space is very remarkable. As Shapley has shown, they form a vast group, somewhat ellipsoidal in shape, and fully 75,000 parsecs in maximum diameter. The equatorial plane of this group practically coincides with that of the Milky Way, but its center is nowhere near the sun, being some 20,000 parsecs distant in galactic longitude 325° (in the direction of Sagittarius). The sun is indeed almost at the edge of the aggregation — which explains why the globular clusters appear to be practically confined to one half of the celestial sphere.

Though the clusters show, in their arrangement, a definite relation to the galactic plane, they are not concentrated close to it; indeed, there is a conspicuous absence of globular clusters near the central line of the Milky Way in the sky, and within about 2000 parsecs from the galactic plane in space. There is no known reason why globular clusters should not exist near this plane, and it is probable that those which are there are hidden from us (cf. § 894).

870. The Magellanic Clouds are among the most interesting objects in the heavens. The Large Cloud is in the constellation Dorado, with its center in R.A. $5^h 26^m$, Decl. $-69°$, or 33° from the galactic equator, and consists of a dense portion about $3°.6 \times 1°.2$, with extensions which increase its apparent diameter to $7°.2$. The Small Cloud (Fig. 275) is in Tucana, in R.A. $0^h\ 56^m$, Decl. $-73°$, and galactic latitude $-44°$. Its inner portion is $1°.8 \times 0°.9$ and its outer diameter about $3°.6$. Its integrated light is about equal to that of a star of the second magnitude. The main mass of the Clouds is composed of a multitude of faint stars, from the eleventh magnitude downward. Shapley estimates there are 500,000 stars brighter than the eighteenth magnitude in the Small Cloud. Innumerable fainter stars doubtless escape observation.

Many objects, of kinds found elsewhere only in the Milky Way, appear in the Clouds, — for example, stars of Class O, stars of the P Cygni type (§ 852), and gaseous nebulæ. Thus the Clouds

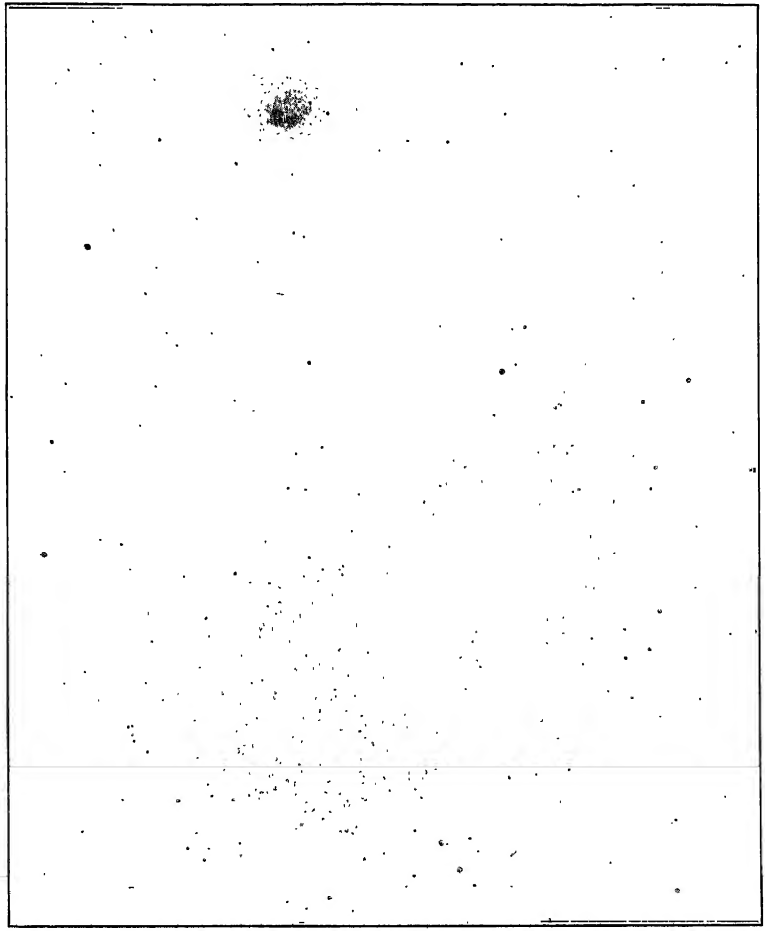

FIG. 275. The Small Magellanic Cloud, and the Globular Cluster 47 Tucanæ

Photographed with the 24-inch Bruce doublet at the Arequipa station of the Harvard College Observatory

appear in all respects like detached portions of the Galaxy. There are also five globular clusters apparently connected with the Large Cloud, and scores of open clusters in it.

Both Clouds are very rich in variable stars. These variables are all faint, from the eleventh magnitude down, and are confined to the Clouds, almost none of the same sort being found in the sky outside, so that there can be no doubt that they are really members of the Clouds and not projected upon them. Most of them are of the Cepheid type, though some may be of long period or irregular. Those for which periods have been determined conform very closely to the period-luminosity relation, the average deviation being only $0^m.23$. More than 1500 of these variables remain to be studied.

871. Distances, Dimensions, and Motions of the Clouds. The Cepheid variables permit an excellent determination of the distances of the Clouds. Using the latest determination of the absolute magnitudes, Shapley obtains the modulus $17^m.70$ for the Large Cloud, and the distance 34,500 parsecs, or 112,000 light-years. The distance determined from the globular clusters in the Large Cloud agrees exactly with this. The Small Cloud is somewhat nearer, with a modulus of $17^m.55$ and a distance of 32,000 parsecs. The distance between the centers of the Clouds is 12,000 parsecs.

The outer diameter of the Large Cloud comes out 4300 parsecs, and that of the smaller, 2000. The denser central portions are some 2100 by 700 parsecs in the first case, and 1100 by 500 in the second. These dimensions are greater than those which were commonly assigned to the whole sidereal universe at the end of the last century.

The radial velocities of both Clouds have been found, with the aid of the gaseous nebulæ within them. These show high speeds of recession, averaging + 276 km./sec., from 17 nebulæ in the Large Cloud, while 1 nebula in the Small Cloud gives + 170 km./sec. The individual values in the first case differ from the mean by 17 km./sec. (without regard to sign), indicating internal motions of the same order as for galactic stars.

Radial velocities of individual stars in the Clouds have not yet been measured, but would be within the reach of instruments like the greatest northern telescopes (to which the Clouds are unfortunately inaccessible), and nothing is yet known about the proper motions.

872. Bright Objects in the Clouds. The Magellanic Clouds present a unique opportunity for the study of celestial objects of great luminosity. Such objects occur nearer the sun, but even the nearest, like Canopus and α Cygni, are usually too far off for reliable determinations of parallax and absolute magnitude. The fortunate concentration of the more distant ones in isolated clouds makes the determination possible. In this way it has been found that the stars of Class O in the Large Cloud range between the absolute magnitudes -8 and -3, with a mean of $-5^m.1$ and an average deviation from this mean of $\pm 0^m.9$. These figures refer to the stars in which the bright bands in the spectrum are conspicuous, those which are distinguished merely by the presence of dark lines of He + being unrecognizable with the low dispersion which has so far been necessarily employed. The stars of the P Cygni type are even brighter, with average $M = -6$. The brightest of them, S Doradus, is an irregular variable ranging from $8^m.2$ to $9^m.4$ (photographic), and has the extraordinary mean absolute magnitude of -8.9. At maximum it gives off *500,000 times as much light as the sun.*

Other stars in the Cloud may be even brighter, but it is hard to separate them from the "foreground" stars, which are much nearer. Measures of radial velocity, when practicable, will settle the question. All told, there appear to be about 10,000 stars in the Small Cloud which are more than a thousand times as bright (photographically) as the sun, and some 400 which exceed the sun ten thousandfold. The brightest stars in the Small Cloud appear to be about 200,000 times as bright as the sun, and the brightest stars in the Large Cloud still brighter.

Shapley, to whom these figures are due, finds also that some of the gaseous nebulæ in the clouds have absolute magnitudes (photographic, and including the light of the stars contained in them) as bright as -10. The largest of these (the "looped nebula," 30 Doradus) is 40 parsecs in diameter. If it could be brought as near to us as the Orion nebula, it would extend over most of the constellation.

873. Other Star-Clouds. There are star-clouds even more distant. A faint, hazy object, discovered by Barnard and known as N.G.C. 6822, is resolved, on photographs with the most powerful

instruments, into a cloud of stars of the nineteenth magnitude and fainter (Fig. 276). Hubble has found Cepheids in this cloud which exhibit the period-luminosity relation, with a modulus of $21^m.65$, corresponding to a distance of 214,000 parsecs, or 700,000 light-years (Fig. 277). The central part of the cloud is $8' \times 3'$, and its outer portions $20' \times 10'$, corresponding to a real size of 1250×625 parsecs. The brightest stars in the cloud are of absolute magnitude -6, and it contains several gaseous nebulæ, — the largest being 80 parsecs in diameter. The radial velocity, from observations of one nebula, is only $+25$ km./sec.

FIG. 276. The Remote Star-Cloud N.G.C. 6822

Gaseous nebulosity is shown around the stars in the lower left-hand corner. (From photograph by E. P. Hubble, Mt. Wilson Observatory)

874. The Galaxy, or Milky Way. This is a luminous belt, of irregular width and outline, which surrounds the heavens nearly in a great circle, passing through the constellations Cassiopeia, Perseus, Auriga, Monoceros, Argo, Crux, Centaurus, Scorpius, Sagittarius, Aquila, and Cygnus (and many others). It is very different in brightness in different parts, and in many places is marked by dark patches (Fig. 217). For about a third of its length, from Cygnus to Centaurus, it is divided into two nearly parallel streams by an irregular dark central band. In spite of this it is a good deal brighter in this region than in the opposite part of the sky, in Auriga and Monoceros.

The Milky Way is composed almost entirely of faint stars, invisible to the naked eye but contributing, by their combined light,

to the brightness of the apparently continuous background. In the richer regions the stars are very densely crowded.

According to van Rhijn's measures, the average apparent brightness of the Milky Way is less than twice that of the sky remote from it. (It appears greater because the eye gets used to regarding the less luminous parts of the sky as "dark.") If the sky illumination due to the zodiacal light and to permanent aurora could be removed, this ratio would rise to about eight to one, and the Galaxy would be far more conspicuous. The brightest patches are much more luminous.

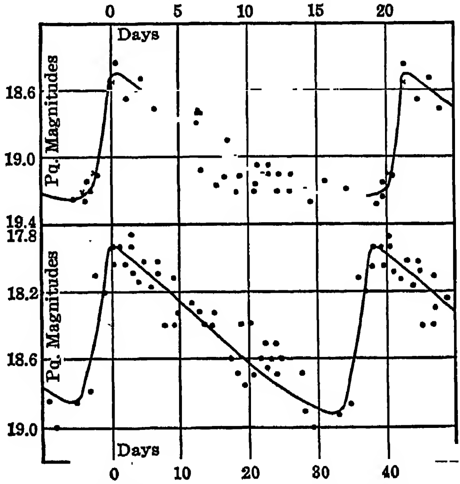

FIG. 277. Light Curves for Two Cepheids in N.G.C. 6822

Upper curve, variable No. 6. Period, 21.06 days; range, 18.5–19.25. Lower curve, variable No. 2. Period, 37.45 days; range, 17.9–18.9. The three crosses on the rising slope of the upper curve represent observations on successive days, and illustrate the rapid brightening of the variables. (By E. P. Hubble)

875. The Galactic Circle and Galactic Plane. The central line of the Milky Way cannot, of course, be defined with precision, but by plotting the estimated positions of the centers of numerous prominent features a surprisingly good determination can be made. It is thus found that this line is nearly a great circle, inclined about 63° to the equator. The poles of this circle—the "galactic poles"—are in R.A. $12^h 40^m$, Decl. $+28°$, in Coma Berenices, and in $0^h 40^m$, $-28°$, in Sculptor. The values here given are those adopted at Harvard from statistical studies. Hertzsprung, in 1912, put the north galactic pole in $12^h 43^m$, $+27°.2$; Graff (1920), in $12^h 49^m$, $+26°.8$.

The *galactic plane*, which intersects the celestial sphere midway between these poles, is by far the most fundamental plane of reference known at present. As Herschel remarks, "the galactic plane is to the sidereal universe much what the plane of the ecliptic is to the solar system," and all studies of

the distribution of the stars must be based upon it. For statistical purposes, therefore, a system of *galactic longitudes and latitudes* is adopted, the latter being measured from the galactic equator, and the former along it, starting from its ascending node on the celestial equator (more precisely, on the equator of 1900, to avoid complications due to precession), which is in $18^h\ 40^m$ R.A. in the constellation Aquila. For obvious reasons there is no advantage in getting these coördinates more closely than to the nearest tenth of a degree (or, usually, to the nearest degree), and the transformation from R.A. and declination into them can be made accurately enough with the aid of a diagram.

The observed central line of the Milky Way does not lie exactly along the galactic equator, but, on the average, about 1° south of it, which evidently means that the sun is a little to the north of the plane in which the centers of the actual star groups lie.

876. Galactic Concentration of Objects of Various Types. As has already been said (§§ 701–706), the stars are concentrated toward the Milky Way to an extent which varies greatly for those of different sorts. The concentration increases for the fainter stars — all sorts taken together — and is very great for bodies of certain kinds, such as the stars of Class O, the fainter stars of Class B, the c-stars, the typical Cepheid variables, the novæ, the open clusters, and the gaseous nebulæ. Objects of most of these sorts are also found in the Magellanic Clouds. On the other hand, certain objects show very little concentration, such as the naked-eye stars of class M (all giants), the stars of considerable proper motion, the long-period variables, and the short-period Cepheids, while the globular clusters and the spiral nebulæ are more numerous away from the galactic equator than near it.

877. Its Significance. From the days of the elder Herschel it has been recognized that the concentration of stars toward the Galaxy in the heavens means that they are actually grouped in space in a region having roughly the shape of a flattened disk, somewhat like a thin watch, extending much farther in the direction of the galactic plane than at right angles to it, as is illustrated in cross-section in Fig. 278. Outside this region the stars are more thinly scattered, though not absent. It is clear from the figure that if we are dealing with a class of stars which lie relatively near us, — within the sphere *a*, for example, — we shall find almost equal numbers in all directions. The stars of large

proper motion, almost all of which lie within 100 parsecs of the sun, form a good example. If, on the contrary, we take a class of stars most of which lie at a considerable distance, between the spheres *b* and *c*, — for instance, eighth-magnitude stars of Class B, — they will show a strong concentration toward the galactic equator.

In most cases the range of distance within which the stars lie is greater than is illustrated in the figure, and the results are not quite so clear-cut; but it is easy to see that in general the galactic concentration will increase with the average distance of the class

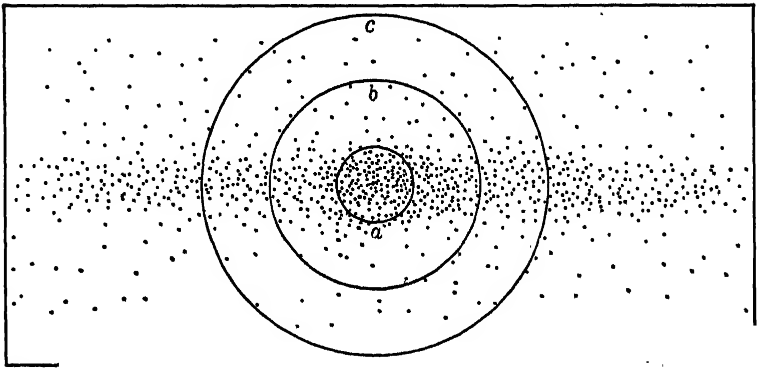

FIG. 278. Galactic Concentration

of stars concerned. Stars of the fifteenth magnitude, which, on the average, are more distant than those of the tenth, should show a greater concentration; and the concentration is great for the O-stars, c-stars, and Cepheids, which, being of great absolute brightness, are visible at great distances.

Certain other stars which show very little galactic concentration — notably the long-period variables and the Cepheids with periods less than a day — are of fairly high luminosity and can be seen a long way off. In these cases it is practically certain that these stars are widely scattered on either side of the galactic plane, while others, such as the Cepheids of longer period and the O-stars, lie relatively close to it.

878. Dimensions of the Galaxy. Within another decade or so it is probable that the extent of the Galaxy will have been fairly

well mapped out by the powerful photometric methods which have been so successful with the globular clusters and the Magellanic Clouds. The known Cepheids of longer period, according to Shapley, are to be found in the galactic plane, in all directions, up to a distance of more than 5000 parsecs, while their average distance from this plane is only 150 parsecs. The Cepheids of short period, on the contrary, have an average distance of 960 parsecs from this plane and are found up to distances of nearly 6000 parsecs. It is probable that the limits of the system have not yet been reached in either case. Searches for fainter variable stars, which are already in progress, indicate that the star-clouds in Sagittarius extend to a distance of at least 10,000 parsecs. When these studies are completed, they may tell us how far away the actual limits are, and whether they are more remote in some directions than in others. This is probably the case; for among the novæ, which are undoubtedly of very high luminosity, those which are apparently fainter, and probably, on the average, more distant, show a strong preference for the half of the Milky Way centering in Sagittarius, in the same region which is favored by the globular clusters. The stars brighter than the tenth magnitude are about equally numerous in all galactic longitudes, but this may mean (and probably does mean) that the great majority of them are not far enough away to reach the limit in any direction. Our soundings all appear the same because they all give "no bottom."

It is evident that there are many star-clouds within the Galaxy; some, as in Cygnus, Scutum, and Sagittarius, are conspicuous. How large a proportion of all the galactic stars are gathered into such clouds, and how many remain to form the general "field" of more thinly and uniformly scattered stars, we do not yet know.

879. The "Local System." One of these star-clouds appears to be in our immediate neighborhood; indeed, the sun is almost in the middle of it. Many years ago Gould noticed that a large number of bright stars lay in a fairly narrow belt, inclined somewhat to the Milky Way and passing through the constellations of Orion, Canis Major, and Scorpius. Shapley has recently pointed out that the stars of Class B brighter than the sixth magnitude are

concentrated, not toward the galactic equator, but toward a great circle, inclined about 12° to it and intersecting it near galactic longitudes 70° and 250°, which passes nearly along "Gould's belt"; the B-stars fainter than $7^m.5$ follow the galactic equator; those of intermediate magnitude, a line between the two. This indicates that the brighter B-stars (that is, the nearer ones) belong mainly to a localized star-cloud, or cluster, of flattened shape, like a thin cake of soap, but with its equatorial plane inclined to that of the Galaxy. A considerable fraction of the A-stars also belong to this cluster, and doubtless many of later classes, but the latter are hard to pick out from the majority of "field-stars." A considerable number, though a minority, of the bright B-stars may also belong to the "field."

The fact that the B-stars visible to the naked eye show a mean motion in space which falls outside of Strömberg's general relation (§ 759) is strong evidence of the reality of this cluster.

Charlier, who has also investigated the distribution of the brighter B-stars, finds that the cluster is of a flattened spheroidal form. The denser portion has an equatorial diameter of six or seven hundred parsecs, and a thickness about one third as great. The center is some ninety parsecs from the sun toward $7^h 40^m$, $-55°$, in the constellation Carina. Shapley calls this the "local system," — which shows how astronomical ideas of nearness and remoteness have altered during the last quarter of a century.

880. Statistical Methods. The form and size of the Galaxy can also be investigated in quite a different way, namely, by statistical studies based on such data as the numbers of stars between different limits of magnitude, the mean parallaxes of stars of different magnitudes, the number of stars of a given magnitude which have proper motions between certain limits, etc. Until the development of the new photometric methods these statistical investigations afforded the only hope of solving the problem, and they are still of great importance.

The problem of stellar statistics is to deduce from the apparent distribution of the stars in the heavens with respect to magnitude, proper motion, radial velocity, parallax, galactic concentration, etc. (all of which are observable or easily derived from observation) what is the true distribution of the stars in space, and the distribution of their real luminosities and velocities.

These desired data may be expressed in terms of three statistical functions: the *density function*, which gives the total number of stars per unit volume (for example, per million cubic parsecs); the *luminosity function*, which shows what proportions of these stars have absolute magnitudes lying in successive equal intervals; and the *velocity function*, which defines the similar distribution of their velocities in space.

An exact specification of these functions would be extremely complicated. The density function varies with the distance from the sun (our arbitrary starting-point), with the galactic latitude, and in all probability with the galactic longitude as well; the luminosity function may be different in different regions, and probably is different near the galactic plane and far from it; the velocity function is different for stars of different absolute magnitude and also for stars moving in different directions, on account of star streaming, and is further complicated by the solar motion. Most important of all, all these functions are undoubtedly very different for the stars of different spectral classes, and even for the giants and dwarfs of the same class.

A complete solution of this intricate problem is at present out of the question. The best that can be done is to seek a partial solution, in which only the most important features are considered and the others are neglected. Thus, for example, all spectral classes are lumped together, variations with galactic longitude are neglected (which amounts to taking average values around each circle of galactic latitude), star streaming is ignored, the luminosity function is assumed to be the same everywhere, and for this and the other functions such mathematical forms are adopted that the complicated equations connecting them with the observed distributions are soluble.

Even when thus simplified, the problem is too difficult to be discussed in detail here; but the general principles involved are not hard to understand.

881. Evidence that the density is not uniform has already been presented, in section 702. It follows from the argument there given that if the density were the same at all distances, and the luminosity function unaltered, the stars brighter than magnitude $m + 1$ would be 3.98 times more numerous than those brighter than m, and also that the mean distance of stars of the former magnitude would be 1.59 times as great, and the mean parallax 0.63 times as great, as for those a magnitude brighter. The actual ratio of increase in number is 2.8 for the naked-eye stars, and drops gradually to 1.8 for those of the twentieth magnitude, while the mean parallaxes derived from observation (§ 749) decrease by the factor 0.68 from one magnitude to the next.

On the assumption of uniform density, moreover, the stars between the m^{th} and $m + 1^{\text{th}}$ magnitudes should send us 1.59 times as much light as the stars a magnitude brighter; and the total light of all the stars down to an indefinitely faint magnitude, should be indefinitely great — an absurd conclusion.

882. Limitation of the Visible Universe. All these contradictions disppear if it is assumed that the star density diminishes with increasing istance. Though the volume of space within which the stars of absolute nagnitude *M* must lie, if they are to appear to us as between the apparent nagnitudes *m* and *m* + 1, is the same as before, the number of stars in this pace will be much smaller. The brighter the absolute magnitude, or the ainter the apparent magnitude, the greater will be this falling off in number. The ratio of increase of numbers will now steadily diminish for the fainter tars. The proportion of stars of great absolute brightness among those of the ainter apparent magnitudes will also diminish, and in consequence the mean arallaxes will decrease more slowly than in the case of uniform density. These phenomena will be more conspicuous in high galactic latitudes than n low (as is illustrated in principle in Fig. 278, and in fact in Table XI), and ikewise more conspicuous for stars of great luminosity (such as those of Class B) than for those of smaller luminosity (Class F) (cf. Table XII).

Among the naked-eye stars the giants far outnumber the dwarfs, since we an see them at much greater distances and are hunting for them in a much reater volume of space (§ 802). For the fainter magnitudes the case is lifferent. The distance at which a giant star must be if it is to appear, for xample, to be between the sixteenth and seventeenth magnitudes is so great hat the star density is exceedingly low and very few giant stars are included. For the dwarfs, which may be ten or even fifty times nearer, the diminution f density is less drastic. The percentage of dwarf stars therefore steadily ncreases as the apparent magnitude grows fainter. As a natural consequence the faint stars average redder than the brighter ones; indeed, among hose of the seventeenth magnitude hardly any have color-indices less than).7 (except in the Milky Way or in other star-clouds).

883. Evidence against Absorption of Light in Space. This diminution f the density of star distribution at great distances refers, of course, to the *isible* stars. It might arise either from an actual sparseness of stars in these egions or from a slight absorption of light in interstellar space, which made he distant stars look fainter and concealed all but the brighter ones. Mere tatistical studies cannot distinguish between the two. Even if we knew the arallax of every star in the heavens, we could not tell, without using some hysical method to find their real luminosity (such as is available for the Cepheid variables), whether the distant ones looked faint because they were aint or because their light was partly lost on the way to us.

The best actual evidence is derived from Shapley's study of the globular lusters, in which the distances are derived by photometric means. If light s lost on the way, the distant clusters should suffer most, and they should ppear fainter in proportion to their diameters than the nearer ones, or arger in proportion to the brightness of the individual stars. This is not the ase, and it is, therefore, improbable that there is any sensible loss of light n transit, even over 100,000 light-years. We shall see later, however, that here are local regions in which strong obscuration of light occurs.

884. The Gravitational Argument. The conclusion that the density with which the stars are scattered in space actually falls off at great distances finds strong support in an argument of quite a different type, first made by Lord Kelvin.

Suppose (to illustrate by a simple case) that there was a vast sphere, in which stars were uniformly scattered so that there was on the average a mass equal to that of the sun in a volume equal to that of a sphere of 1 parsec radius.

A very simple calculation[1] shows that the period of a star revolving just outside this sphere under the attraction of the whole mass would be 93,000,000 years, whatever the radius of the sphere. It can also be shown that the velocity of escape for a star at the center of the sphere would be $\sqrt{3}$ times that of one moving in this circular orbit. Now, if the sphere were 100 parsecs in radius, the orbital velocity would be 6.5 km./sec., and the escape velocity 11.2 km. For a sphere 1000 parsecs in radius the escape velocity would be 112 km.; for one of radius 10,000 parsecs, 1120 km.

Comparing these figures with the actual velocities of the stars, it appears that if the "universe" of stars has an effective gravitational influence (measured by the potential at its center) comparable with that of a sphere of the assumed density, and is less than 1000 parsecs in radius, the faster-moving stars must be escaping from it. If this influence were comparable with that of a sphere 10,000 parsecs or more in radius, the fastest-moving stars would be turned back by its attraction long before they reached its outer limit, and it would be necessary to assume that the stars of the outer regions were moving in such a way that none of their paths would come anywhere near our part of the universe. Neither of these deductions appears to be at all probable, and it may be concluded, therefore, that the gravitational potential in the vicinity of the sun due to the whole universe of stars is comparable to that within a sphere of the assumed density and a few thousand parsecs in radius.

This argument does not demand that the stellar universe be of spherical form (which was merely assumed for simplicity in calculation) or that it have any sharp boundary. Stars may

[1] Let the radius of the sphere be R parsecs. The total mass inside it is R^3 times that of the sun. A star moving in a circular orbit just outside it would have a mean distance of $2.06 \times 10^5 R$ astronomical units. Hence, since $m = a^3/P^2$, or $P^2 = a^3/m$, we find $P^2 = 8.75 \times 10^{15}$ (R^3 canceling out).

continue to be strewn, with increasing sparseness, to an indefinite distance. But it does show that the dimensions of the region within which they are thickly strewn must unquestionably be finite.

Most important of all, it takes into account the attraction of all matter, whether luminous or dark, and hence we may regard it as the most cogent argument in favor of the limitation of the "universe," using this term in the technical sense which has become current in astronomy, that is, to denote the aggregation of stars and other matter which constitutes the Milky Way.

885. The "Kapteyn Universe." The best determination which has yet been made of the size and form of this limited universe of stars is that by Kapteyn (1922), — the last achievement of a lifetime of masterly statistical investigation. According to Kapteyn the Galaxy is a vast cluster of stars, in the form of a flattened ellipsoid of revolution, of diameter about five times its thickness. The stars are closest together at the center, and thin out gradually toward the periphery, with no definite boundary. The density diminishes to 1/10 of the central value at a distance of about 550 parsecs along the axis, and, in the equatorial plane, at 2800 parsecs from the center. For 1/100 of the central density these values are respectively 1700 and 8500 parsecs. The number of stars per thousand cubic parsecs, at the center, is given by Kapteyn as 45, and the whole number of stars belonging to the cluster, including those thinly scattered at greater distances, is estimated as 47,000,000,000. This distribution of stars (together with the luminosity and velocity functions discussed below) represents the observed number of stars of different magnitudes, the galactic concentration and its variation with magnitude, and the mean parallaxes and proper motions, in a satisfactory manner, and presumably gives a good picture of the nature of the Milky Way.

It is, however, only a generalized picture, giving the main outlines but not the details. For example, no account is taken of local clusters or clouds of stars, or of differences of extent of the Galaxy in different directions in its own plane. Consideration of the latter important question was deferred to a second approximation, which the distinguished author did not live to make.

886. The luminosity function, which defines the relative number of stars of different absolute magnitudes in *a given volume of space*, was determined by Kapteyn in the course of his work. His conclusions, including modifications by Seares in the case of the fainter stars, are summarized below.

THE LUMINOSITY FUNCTION

ABSOLUTE MAGNITUDE	RELATIVE NUMBER	ABSOLUTE MAGNITUDE	RELATIVE NUMBER
-5	1	$+5$	200,000
-2.5	90	$+7.5$	350,000
0	3,300	$+10$	500,000
$+2.5$	42,000	$+12.5$	600,000

That is, for every star of absolute magnitude -5 there should be 90 of $M = -2.5$, 3300 of $M = 0$, and so on. The stars of small luminosity are enormously more numerous than the brighter ones.

Kapteyn's original formula indicated that the number of stars per unit volume decreased for absolute magnitudes fainter than 8. Later work by Seares shows that this is not the case. These faint dwarfs can be seen at such small distances that they contribute only a negligible fraction of the total number of stars, even of the fainter apparent magnitudes, so that the calculated numbers of stars of given magnitude, etc., are very little altered by the change. The number of stars in a given volume of space, however, is probably at least two or three times as much as Kapteyn estimated, and the whole number of stars in the universe may be correspondingly greater. The number of stars of the fainter absolute magnitudes cannot go on increasing indefinitely, for, if it did, space would be packed full of stars; but at what point it begins to fall off is not yet known.

For the stars which appear brighter than a given magnitude, things are very different. If the limit is set at the visual magnitude 1.5, for example, stars of absolute magnitude 10 will be included only if they are nearer the sun than 0.2 parsec, so that none are found. For absolute magnitude 5 the limiting distance is 2 parsecs; for $M = 0$, 20 parsecs; for $M = -5$, 200 parsecs. The volume of the limiting sphere increases a thousandfold from each step to the next. Hence if the star density were uniform, there should, when selected in this way, be sixteen stars of absolute magnitude 0, and five of absolute magnitude -5, to every one of absolute magnitude 5 (though in any given region of space the disparity is 60 to 1 the other way in the first case, and 200,000 to 1 in the second). This represents roughly what is actually found among the first-magnitude stars. Most of the stars visible to the naked eye are therefore giants, though most of those in any given region are dwarfs.

887. The velocity function can be determined, almost directly, from measures of radial velocity. To pass from these measures of the component of velocity in one direction to the velocities in space is not difficult. Since,

however, the stars which have been observed for radial velocity are of the brighter apparent magnitudes, or else have been selected for some such characteristic as proper motion, considerable care is required to get rid of the effects of selection and obtain values representative of the stars in a given region of space.

The Mt. Wilson investigators find that the distribution of the *logarithms* of the space velocities is well represented by the "law of errors," — half of them deviating from the mean by less than 0.18, — which means that half the stars have velocities lying between 0.55 and 1.25 times the arithmetical mean of all (which in this case is 1.21 times the geometrical mean). This assumption gives too few very small velocities, but this is unimportant except in dealing with stars of very small proper motion, and can be allowed for there. The mean velocity increases steadily for the fainter absolute magnitudes, being about twice as great for $M = 10$ as for $M = 0$.

The stars of high velocity — more than 100 km./sec. — are not taken into account by this formula and evidently belong to a special group of some sort, for which there is plenty of other evidence (§ 759). When the density, luminosity, and velocity functions are known, the formula for the mean parallax of stars of given magnitude and proper motion can be calculated theoretically. The results agree very closely with that derived empirically from observation (§ 752).

888. Star Streaming in the Kapteyn Universe. Kapteyn's model of the universe is of the type in which Eddington has shown that star streaming may exist, the preferential direction being at right angles to the radius. Applying his equations, Kapteyn has shown that the average space velocity of the stars and the average amount of streaming may be accounted for quantitatively, on a dynamical basis, provided that the average density of gravitating matter in the central region is 0.1 times the sun's mass per cubic parsec, and also provided that the sun is about 650 parsecs from the center (which he finds to be in galactic longitude 257°, in the constellation Vela).

The average mass per star throughout the universe is given by Kapteyn as 1.6 times the sun's mass, — a very important conclusion, since it indicates that the total mass of dark matter cannot be much greater than that of the visible stars. Changes in the calculated dimensions and density of the system would alter these numbers but do not seem likely to upset this conclusion. The velocity of escape from the center of the Galaxy is given by Kapteyn as 100 km./sec. The great bulk of the stars have smaller velocities than this and probably belong to the "Kapteyn

universe," but the fast-moving stars, which show the asymmetrical distribution of velocities described in section 757, must be visitors from outside.

It is probable that the stars of high velocity are members of some great system, of much lower density and very much greater size than Kapteyn's, within which the latter may be only a "local cluster"; but further work is necessary before this can be established.

The mapping of the outer parts of the visible universe, with the aid of the faint Cepheid variables, which may be hoped for in a few years, should settle many of these points, and may considerably modify some of the statistical conclusions stated above.

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CHAPTER XXIV

THE NEBULÆ

GENERAL CLASSIFICATION · GALACTIC NEBULAE: DARK, DIFFUSE, AND PLANETARY · EXTRA-GALACTIC NEBULAE: SPIRALS, ELONGATED, GLOBULAR, AND IRREGULAR · APPEARANCE, DISTANCES, DIMENSIONS, MOTIONS, BRIGHTNESS, MASSES, AND NATURE

889. The nebulæ, which were mentioned incidentally in the previous chapter, deserve one to themselves. They appear as faint, hazy clouds of light, sometimes of great extent, sometimes so small that it is not easy to distinguish them from stars. A very few of the brightest are visible to the naked eye. Many more can be seen with small telescopes, but the majority are so faint that they can be studied only by photography.

Their number is very great. All but a few hundred of the thirteen thousand objects contained in Dreyer's catalogues (§ 859) are nebulæ, and the number revealed by photography with long exposures is enormous. Seares (1925) estimates that about 300,000 could be reached with the 60-inch reflector and an hour's exposure.

The brighter nebulæ are designated by their numbers in some catalogue; the fainter, by their positions on photographs.

890. Observation of Nebulæ; Advantages of Photography. The general outlines of the brighter nebulæ can be fairly well recorded by visual observation, but there is no other field in which photography has such great advantages over the eye.

The principal reason for this is that the nebulæ appear as luminous *surfaces*; and no optical device whatever can make an extended surface appear brighter than it does to the unaided eye, or even quite as bright. A suitable magnifying power makes it look larger, and this helps greatly in seeing small objects; but a surface which is large enough to be visible, although too faint, cannot be brought to sight by any optical aid. Photographic action, however, is cumulative, and, with long exposures, clearly

reveals details which, from their faintness, are inherently invisible to the eye. This advantage is intensified by the fact that many nebulæ emit much violet and ultra-violet light, which is feebly, if at all, visible, but has a strong photographic action.

A hardly less important advantage of photography is that it gives, in a single exposure, a record, correct as regards both position and brightness, of a multitude of details, usually far too complex to be drawn with accuracy even by the most skillful draftsman. Nearly all observation of nebulæ is therefore done photographically.

It is important for such work that the aperture should be large compared with the focal length, for under these circumstances more light is concentrated upon each square millimeter of the plate. Reflectors and doublets are therefore usually employed.

891. General Classification of Nebulæ. The best scheme of classification of nebulæ appears to be that which has recently been suggested by Hubble. According to this they are divided into two great groups, the *galactic* and *extra-galactic* nebulæ. The former are strongly concentrated toward the Milky Way, and are shown by a variety of evidence to be really members of the galactic system, at distances comparable with those of the stars, and in many cases related to them. The latter avoid the Milky Way more completely than any other class of celestial objects, and there is now evidence that some of them lie at enormous distances and actually far outside the galactic system. Various differences in the appearance and other characteristics of nebulæ of the two classes make it usually easy to tell to which group a given object belongs. The extra-galactic nebulæ are much the more numerous, only a few hundred out of the many thousands known being of the galactic type.

The nebulæ also fall sharply into two groups with regard to their spectra, as was found by Huggins in 1867, those of one sort giving a nearly continuous spectrum (usually found, when well observed, to contain dark lines), and the others a spectrum of isolated, sharp bright lines, such as are produced by a rarefied gas. The latter are usually called *gaseous nebulæ*. They are practically all of the galactic type. Some galactic nebulæ, however, and practically all the extra-galactic nebulæ which have been inves-

tigated, give continuous spectra. Patches of gaseous nebulosity occasionally occur in extra-galactic nebulæ.

The gaseous nebulæ are sometimes called *green* nebulæ, and those with continuous spectra *white* nebulæ, since, when bright enough, they show these colors to the visual observer.

GALACTIC NEBULÆ

892. Types of Galactic Nebulæ. The galactic nebulæ may be subdivided into three main groups, according to their appearance:

Planetary nebulæ, which are roundish and sharply defined at the edge, and in a small telescope often resemble a faint planetary disk (whence their name).

Diffuse nebulæ, which are of irregular outline and often very irregular in shape.

Dark nebulæ, which do not shine, and are detected only because they obscure the stars which lie behind them.

All the planetary nebulæ and some of the diffuse nebulæ have gaseous spectra; the remainder of the diffuse nebulæ have continuous spectra. In discussing these objects it is convenient to reverse the above order and to begin with the most recently discovered type.

893. Dark Nebulæ. In many parts of the Milky Way there are dark patches which look very much like small clouds, but which are really permanent features of the heavens. A conspicuous example is the dark marking in the Southern Cross called by mariners the "coalsack" (Fig. 217).

It had often been suspected that these were regions of obscuration in which the more distant stars were hidden by some opaque obstacle, but the credit of establishing this beyond reasonable doubt belongs to Barnard, who studied them long and carefully, and finally published a catalogue of one hundred and eighty-two of them. A large and nearly round dark spot like the "coalsack" might possibly be an opening in a relatively thin layer of stars, through which the dark background beyond is seen; but many of the dark markings discovered by Barnard are so small, so sharply defined, and of such remarkable shape that this explanation becomes absurd, and it becomes clear that between us and

the stars of the Milky Way there actually exist opaque clouds, sometimes small and sharply defined, and again of enormous extent, like the dark "lanes" in Ophiuchus (Fig. 279).

In most cases these obscuring clouds seem to be quite opaque, the few stars which appear in the obscured region being those which lie in front of the cloud; but sometimes it looks as if the stars behind were merely dimmed and not obliterated. Great

FIG. 279. Vacant Lanes running East from ρ Ophiuchi

$\alpha = 16^h\ 50^m$; $\delta = -25°$. Exposure, $3^h\ 40^m$, June 3, 1915, with the 10-inch Bruce doublet of the Yerkes Observatory. (From photograph by E. E. Barnard, Yerkes Observatory)

regions of obscuration are found in Taurus, Orion, Ophiuchus, Scorpius, and Crux, and smaller ones all along the Galaxy. Outside the Milky Way, where there is hardly any background of distant stars against which such clouds might be seen, few have been detected.

894. Their Distances and Dimensions. Some of the greatest of these dark regions are connected with bright stars in such a way as to make it certain that the obscuring clouds must really lie near these stars in space. In the neighborhood of some of these stars, such as ρ Ophiuchi and ν Scorpii, the obscuration brightens

up into visible (or, more accurately, photographable) nebulosity, centered round the star in such a way as to suggest at once that the dark clouds are lighted up by the star's light (Fig. 279).

In this way it is found that the great obscuring clouds in the region of Scorpius and Ophiuchus are at the same distance as the stars of the Scorpius cluster; some of those in Taurus are associated with the Pleiades, and those in Orion with the bright stars of that constellation. The distance is from 100 to 150 parsecs in the first case, about 100 in the second, and from 150 to 200 in the third.

These are among the largest of the obscured regions, and are probably the nearest dark nebulæ. Many of the smaller ones may be much farther off, but it is clear that they are not only within the galactic system but within the nearer parts of it. Any remoter cloud, in fact, would have so many stars in front of it that it would not be conspicuous.

The actual dimensions of these clouds must be exceedingly great. The dark "lane" in Ophiuchus, for example, is some seven degrees long by half a degree wide. Kapteyn finds for the distance of ρ Ophiuchi (which is evidently connected with the nebula) the value 170 parsecs. This makes the obscuring mass 20 parsecs long by about a parsec and a half wide.

Still greater clouds may exist. The Milky Way is divided for nearly 120° of longitude, from Cygnus to Centaurus, into two "streams" by a central band of very irregular width, which is almost as dark as the non-galactic sky. This band stretches right across the center of the region of the heavens in which the globular clusters are most numerous; yet, as Charlier has pointed out, not a single one of these clusters appears within it, though they are found right up to its edges. If the dark division were a clear place between two belts of stars, through which we saw into the depths beyond, this could be explained only by assuming a very singular distribution of the globular clusters in space. It appears much more probable that the whole dark band represents an enormous obscuring cloud, or collection of clouds, which conceals what may well be the brightest part of the whole Milky Way and lets us see only its edges. This vast cloud must be many hundreds of parsecs long.

895. Nature of Dark Nebulæ. There appears to be little doubt that the main obscuring agent in these clouds is *fine dust*. This does not mean that the clouds are composed entirely or mainly of such dust. They may contain particles of all other sizes, from separate molecules to masses as large as planets or even stars; but it is easy to show that, ton for ton, the fine dust has far the greater light-stopping power.

If a cubic inch of opaque material is divided into cubes 1/100 of an inch on a side, there will be 1,000,000 of them, and their combined cross-section will be 100 square inches, so that they will obscure 100 times as much light as the original cube. This increase of obscuring power with subdivision goes on until the particles become so small that their circumference is equal to a wave-length of light. Beyond this point the light-waves go by them without much disturbance, and the obscuring power falls very rapidly.

Dust of the critical size (a few millionths of an inch in diameter) has extraordinary obscuring power. A layer containing about one tenth of a milligram of such dust per square centimeter of superficial area would be entirely opaque, whatever its thickness. On this basis the whole huge cloud in Ophiuchus could be produced with a quantity of dust equal to about a dozen times the sun's mass. This extraordinary opacity of a cloud of fine particles is familiarly illustrated by smoke or condensed steam, which is nearly opaque in a layer a few inches thick, although the suspended particles form but a small portion of the mass of the layer of air in which they lie.

The sharp outlines of many of the dark nebulæ suggest that they are held together by some force (presumably by their own gravitation) while the particles move fast enough to prevent them from falling together at the center. In such a cloud, collisions must be frequent, and if no fine dust were originally present it would probably soon be produced by the disruption of the colliding particles.

896. General and Selective Absorption. The nature of the obscuration differs according as the particles are larger or smaller than the critical size which gives the greatest effect. When they are larger, all wave-lengths are scattered equally, and the transmitted light is weakened but is not changed

in color. When they are smaller, the shorter wave-lengths are most scattered, according to Rayleigh's law (§ 602), and the transmitted light is reddened. This happens in a gas, but dust a little smaller than the critical size may be as much as a hundred million times more effective (pound for pound) than gas molecules in producing this selective scattering.

In general there is little evidence that stars seen through obscuring clouds are changed in color. Seares and Hubble, however, have found that stars which are immersed in nebulosity are almost always redder than the average for their spectral class, the excess in color-index being sometimes as great as $0^m.5$. In such a case, some at least of the obscuring matter must be extremely fine dust or gas.

In the case of a completely opaque cloud the test cannot be made, and it would be premature to conclude that most dust particles in the dark nebulæ are larger than the critical size. Various attempts have been made to detect a possible selective absorption of light in interstellar space at large. Shapley's work on the globular clusters, which shows that the range of color-indices within them is substantially the same as among the neighboring stars, is very strong evidence that absorption of this sort is negligible even at a distance of 100,000 light-years. It also indicates that the general absorption must likewise be small. One globular cluster (N.G.C. 4372) appears too faint for its diameter, but this is on the border of an obscured region. In general, interstellar space, outside the recognizable obscuring clouds, appears to be remarkably transparent.

897. The diffuse galactic nebulæ are usually very irregular in form and outline. They vary in apparent size and brightness from tiny wisps, which can just be photographed with long exposures, to the great nebula in Orion, which covers an area larger than the full moon and is visible to the unaided eye. They are definitely galactic, though the larger ones tend to follow the plane of the "local system" and a number are found in the Magellanic Clouds, which in this respect, as in others, appear to resemble detached portions of the Galaxy.

In many cases — as in Orion, the Pleiades, the region of Ophiuchus and Scorpius, etc. — diffuse nebulæ are associated with stars in such a manner as to make it probable that they are really related to them, and this suspicion has been turned into practical certainty by recent researches.

898. Relation between Stars and these Nebulæ. This was first proved in the case of the nebulosities in the Pleiades (Fig. 280*A*). Only the brightest portions of these are visible to the eye; but long exposures bring out fairly strong streaky nebulosity all

through the cluster, and fainter extensions to a distance of several degrees. The very appearance of these nebulæ suggests that they are clouds of some sort, lighted up by the light of the stars; and this was settled in 1912, when Slipher photographed the spectrum of the brighter regions and found it to be continuous, crossed by fairly strong dark lines of hydrogen and faint ones of helium, — in other words, of a late B type, just like that of the bright stars of the cluster and quite unlike that of any other nebula then known. Hertzsprung, in 1913, showed, by photometric measures, that the nebulosity was not more than 5 per cent as bright as a white surface, set up near the stars and reflecting their light, would appear to be. It is evident, therefore, that the Pleiades cluster is accompanied by such translucent clouds, probably of dust, which reflect the light of the stars and so become visible. Further confirmation is found in a marked diminution of the number of faint stars, of the thirteenth magnitude and beyond, within the Pleiades region, — which indicates a considerable, though not complete, obscuration of the stars which lie behind the cluster.

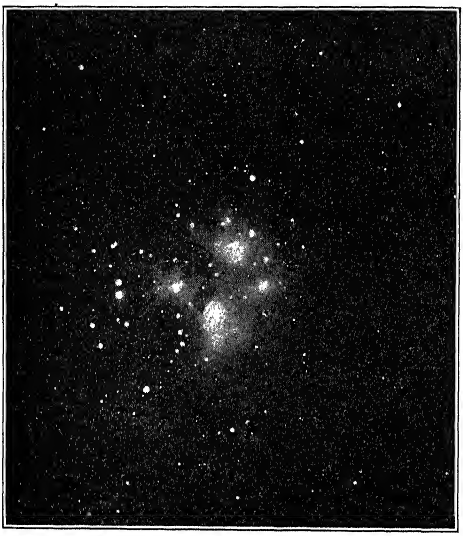


FIG. 280 *A*. The Pleiades

September 7, 1905. Exposure, $3^h\ 48^m$, with the 10-inch Bruce doublet of the Yerkes Observatory. (From photograph by E. E. Barnard, Yerkes Observatory)

In several other cases dark nebulæ brighten up around certain stars in such a way as to make it almost certain that they shine by reflected light. For three of these Slipher has found that the spectrum of the nebula is again like that of the star.

899. Recent Investigations of Diffuse Nebulæ. Still more recently Hubble has shown, after a thorough survey of the galactic

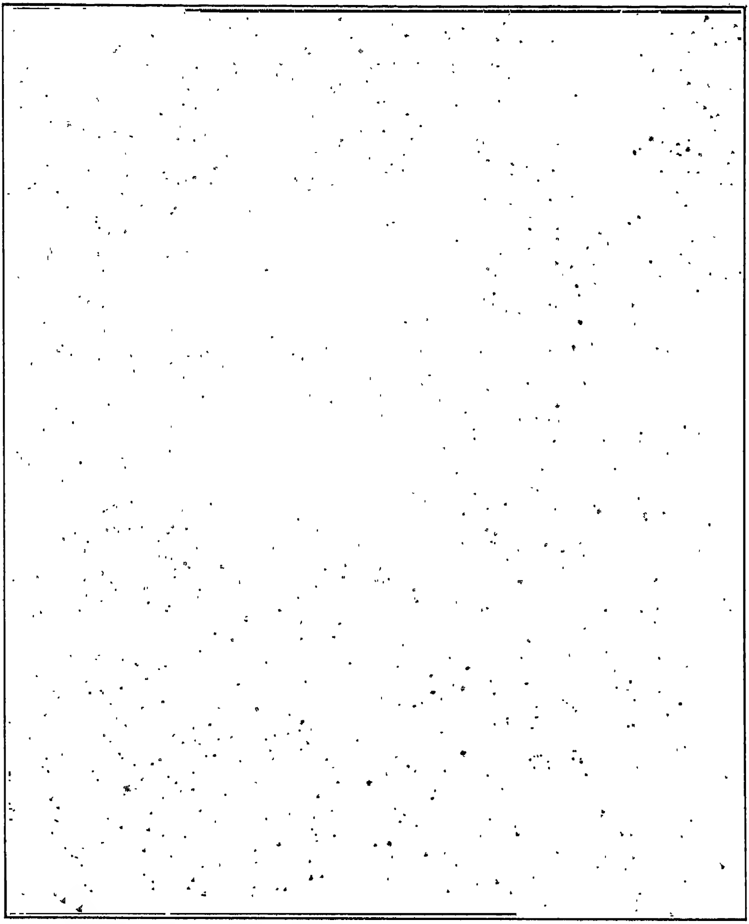


FIG. 280 *B*. The Constellation Orion embedded in Nebulosity

A photograph extending from declination + 23° to − 25°, and from right ascension $4^h\ 12^m$ to $6^h\ 44^m$ (nearly 1/20 of the whole sky), taken with a lens a little over 1 inch in diameter and an exposure of 10 hours, which shows, at the center, stars to the seventeenth magnitude. The identification of the brighter stars by means of a star map and a comparison with the appearance of the sky to the eye will be found a useful exercise. The large color-indices of Betelgeuse and of Aldebaran are especially impressive. (From photograph by E. P. Hubble, Mt. Wilson Observatory)

nebulæ, that every diffuse nebula, with one exception, is associated with one or more stars which appear to be really connected with it, and in some way responsible for its luminosity.

The stars which thus "excite" the nebulæ to shine are of all magnitudes from the first to the thirteenth, and of all spectral classes from O to K8, including at least one dwarf. Stars of the first magnitude are found to set nebulæ shining at a distance of several degrees; those of the thirteenth, at less than half a

FIG. 281. Nebula in Sagittarius (N.G.C. 6523; M. 8)

Irregular nebula. Exposure, three hours, June 27, 1919, with 60-inch reflector. (From photograph by J. C. Duncan at Mt. Wilson Observatory)

minute of arc. The maximum distance at which excitation is observed very rarely exceeds that at which a cloud, reflecting all of the incident light and illuminated by the star, would be just bright enough to be photographed with the exposure concerned, but often approaches this limit. This relation still holds true even if the nebula shows a gaseous spectrum, — a very remarkable fact.

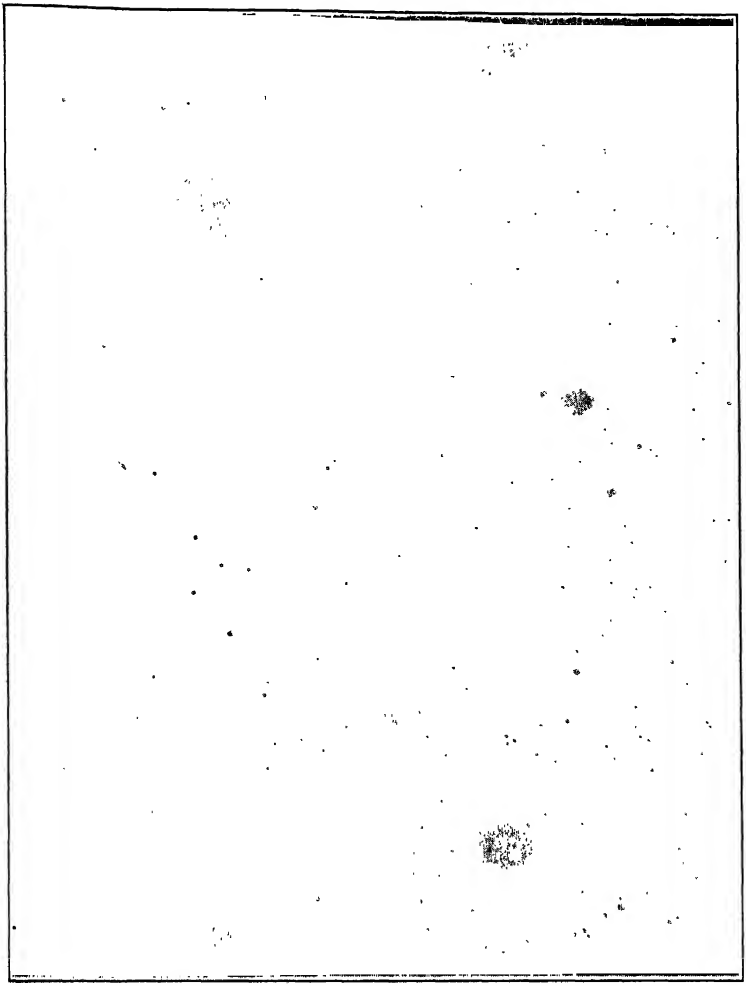


FIG. 282. Nebula South of ζ Orionis, containing Dark Bay (I.C. 434)

Exposure, three hours, November 13, 1920, with 100-inch reflector. (From photograph by J. C. Duncan, Mt. Wilson Observatory)

The few instances in which nebulosity is visible at a greater distance can be explained by a moderate obscuration of the principal star. In a single instance (a great oval ring of gaseous nebulosity in Cygnus, about $2\frac{1}{2}$ degrees in diameter, of which N.G.C. 6960 and 6992 form the brightest parts) no bright star

seems to be associated with the nebula; but there may be one hidden by a patch of dense obscuration between us and it. That this should happen once, in the eighty cases which have been studied, is not unreasonable.

The data are so complete and consistent as to put it beyond question that the diffuse nebulæ are not self-luminous and do not supply from within themselves the energy by which they shine, but get it from neighboring stars. If these stars were not near them, they would be dark and could be detected only if they obscured stars behind them.

Obscuration of this sort is very common in the vicinity of diffuse nebulæ,— not only the distant stars but parts of the nebula itself being often hidden, as in the case of N.G.C. 6523 (Fig. 281) and, still more strikingly, near ζ Orionis (Fig. 282), where the well-known "dark bay" is evidently a projecting mass of opaque cloud and the sky on one side of the nebula is full of faint stars which are all utterly obscured on the other, leaving only the relatively bright stars which lie in front of it. It is, indeed, practically certain that by far the greater part of the galactic nebulosity is dark and that the luminous galactic nebulæ represent the scattered portions of this dark matter which are set shining by the influence of neighboring stars.

900. Distances, Dimensions, and Motions. The distances of the diffuse nebulæ are substantially equal to those of the stars which illuminate them. In several cases the latter are known, ranging from 100 parsecs for the Pleiades and about 180 for the Orion nebula to more than 200,000 parsecs in N.G.C. 6822, in which the stars responsible for the light of the included nebulæ can still be found. The dimensions of these nebulæ are exceedingly large. The Orion nebula, which is about a degree across, must have a real extent of at least 3 parsecs, or 10 light-years. The nebulosity of the Pleiades, whose outer parts extend to 5° from the cluster, must be some 17 parsecs in diameter. The greatest known nebula is the "Looped Nebula" 30 Doradus, in the Large Magellanic Cloud, for which Shapley finds a diameter of 40 parsecs (about as large as the whole constellation of Orion).

Proper motion in a diffuse nebula has so far been detected in very few cases,— which is not surprising, as the details of the

nebulosity are rarely sharp enough to be measured with accuracy. In the "Crab Nebula" in Taurus (Fig. 283) the nebulous wisps are moving outward in all directions from the center at an average rate of 0″.13 per year. The "Network Nebula" in Cygnus (Fig. 284) shows a motion at right angles to its own length, — toward the bright star which is apparently, though probably not really, near it. The motion is most rapid at the narrow end, where it amounts to 0″.05 per year. In this case too the motion is away from the center of the great "loop" of which this nebula forms a part. The opposite side of this loop (N.G.C. 6992) is moving in the opposite direction, and it is probable that the whole huge oval ring is expanding in all directions. At the observed rate of motion it would take more than 100,000 years for the nebula to reach its present size, and it is tempting to speculate that it may have been expelled from some great nova which blazed up at that remote epoch.

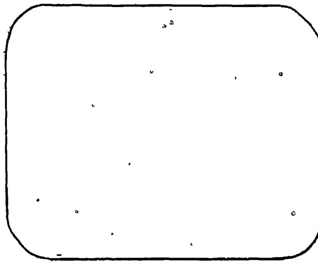


FIG. 283. The "Crab Nebula" in Taurus

N.G.C. 1952 (Messier 1). (From photograph by Lowell Observatory)

The radial velocities of gaseous nebulæ — even faint ones — can be accurately measured, since the light is concentrated in a few bright lines. After allowance is made for the sun's motion, the residual radial velocities of the diffuse nebulæ are small; Campbell and Moore find 11 km./sec. (for only five nebulæ, however). The diffuse nebulæ in the Magellanic Clouds give very high velocities, — doubtless those of the Clouds as a whole (§ 871).

The brightest parts of the great oval loop in Cygnus, N.G.C. 6960 and 6992, have radial velocities of −18 and +100 km./sec.

In the great nebula in Orion (Fig. 285) the observed radial velocity varies from point to point in a very complicated fashion. On the average the nebula is receding at the rate of 17.5 km./sec., but some portions (the long strip in the upper right-hand part of the figure and the isolated bright patch in the lower right-hand part) are receding, relatively to the mean, at the rate of from 4 to 7 km./sec., while velocities of approach preponderate

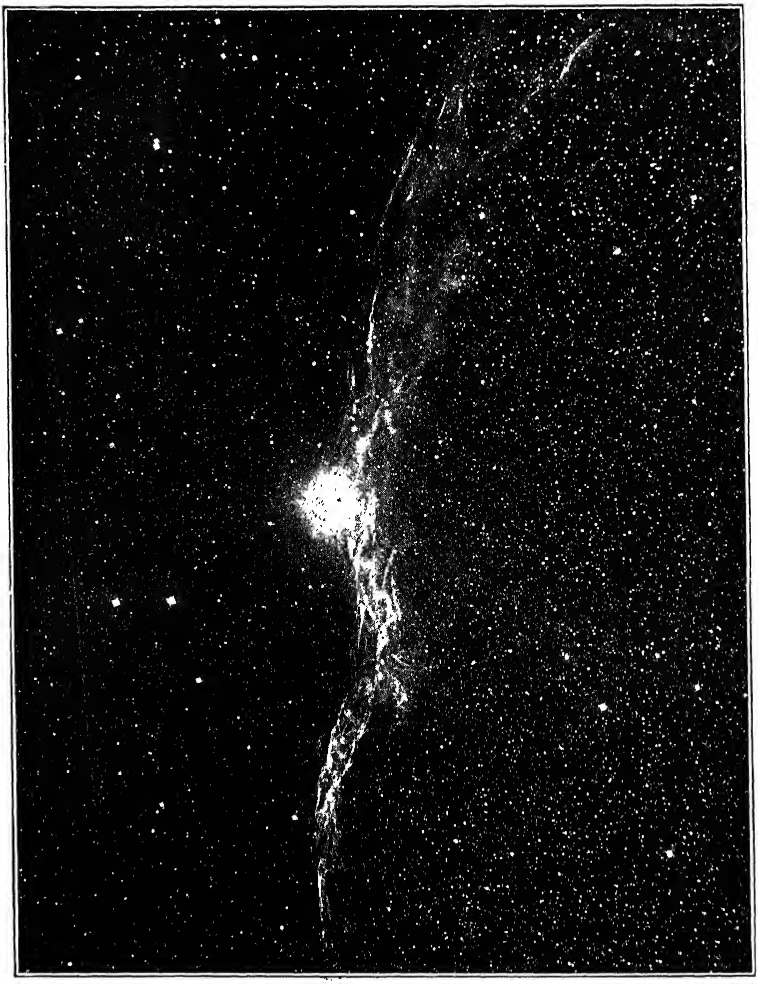


FIG. 284. The Filamentary Nebula in Cygnus, N.G.C. 6960

Photographed with the 100-inch reflector and an exposure of seven hours. The conspicuous difference in the number of faint stars on opposite sides of the nebula suggests partial obscuration in the region on the left. The trail of a meteor appears at the extreme left. (From photograph by J. C. Duncan at Mt. Wilson Observatory)

on the opposite side of the nebula. Even in the central region, however, differences of velocity of 10 km./sec. are found. The proper motions corresponding to similar velocities would be too small to be detected.

The lines of the nebular spectrum are sharp, but measurements with an interferometer of suitable type show that they possess a perceptible width. In this way Fabry and his colleagues have found that the width of the hydrogen line Hγ is as great as it would be if the emitting atoms had an average speed of 19 km./sec.; as they would from the thermal motion in a gas at 15,000° K. It would be wrong to conclude, however, that this is the temperature of the nebula, for a variety of other causes might produce widening of the lines.

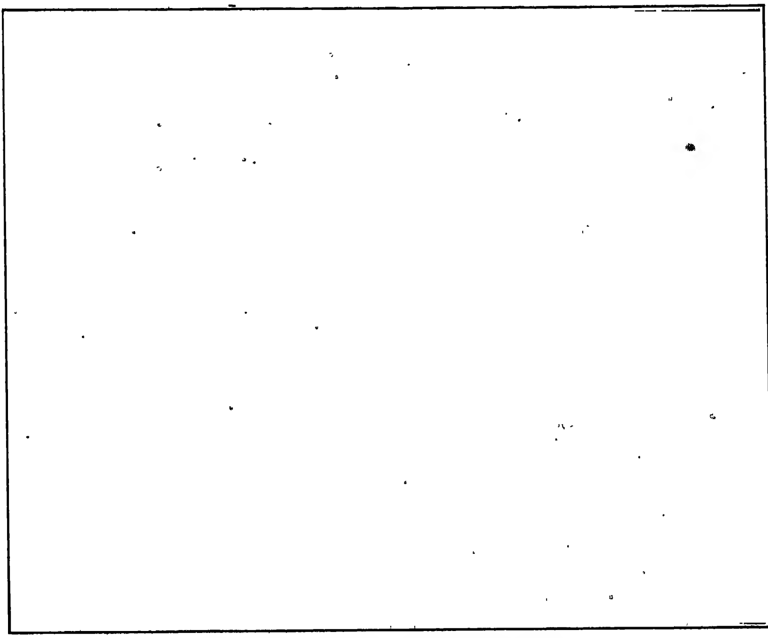


FIG. 285. The Great Nebula in Orion

(M. 42; N.G.C. 1976.) Exposure, three hours, November 19, 1920. The central portion has been reduced in intensity in order to bring out the detail of the bright portions. (Photographed with the 100-inch reflector at Mt. Wilson Observatory)

901. Probable Densities and Masses. Huge as these bodies are, it is certain that their densities must be exceedingly low. The parallax of the Orion nebula, according to Kapteyn, is 0″.0055. The bright part of the nebula is about one parsec in diameter, and its outer portions are something like 100,000 astronomical units from the center. If the observed radial velocities correspond to orbital motion, the period of revolution would be of the order of 300,000 years; and the whole mass of the nebula (applying

Kepler's third law) ten thousand times that of the sun. Even with this great mass the mean density comes out 10^{-18} times that of the sun, or one millionth of a billionth of that of air under ordinary conditions. At this rate a sphere as large as the orbit of Neptune would contain less than one tenth of the earth's mass. It is by no means certain, and perhaps not probable, that the attraction of the nebula is enough to retain the outer portions in orbital motion around it, so that the actual mass and density may be much less.

Though the mean density of the nebula is vastly less than that of the most perfect artificial vacuum, it may contain great numbers of solid particles, perhaps of large size, if only they are far enough apart. Its total mass must be great, for a circle 1 parsec in diameter has an area of about 7×10^{36} square centimeters. Even if all the matter were in the form of fine dust, at least 0.1 milligram per square centimeter would be required (§ 895) to produce heavy obscuration. This would make the whole mass 7×10^{32} grams, or about one third of the sun's mass. The actual mass is probably much greater, and it is likely that the nebula contains enough matter to form a number of stars or perhaps a small star-cluster.

902. The planetary nebulæ number between a hundred and a hundred and fifty; and since they are very easily recognized by their gaseous spectra, our lists are probably almost complete. They appear as roundish or oval masses of faint light, usually showing a good deal of internal detail, as is illustrated in Fig. 286; and there is almost always a faint central star in the very middle. They frequently exhibit two or three concentric shells of luminosity, and a few appear as rings, the central portion being much fainter, although not dark. It is probable that these ringlike forms are due to transparent shells of luminous matter shining most strongly at their outer boundaries.

A few of them are from 3′ to 12′ in diameter, but most of them are less than 1′ across, and a good many are so small as to be indistinguishable from stars except with high powers. Most of these have been picked up by their spectra. The central stars are usually very faint, most of them being below the eleventh magnitude photographically and still fainter visually, since they are

bluer than any other known stars. Their spectra are of Class O, with dark lines of hydrogen and ionized helium, when the nebulosity is extensive, and of the Wolf-Rayet type, with conspicuous bright lines, when it is of small diameter.

The larger planetaries are often found at considerable distances from the Galaxy in the sky, but the smaller ones are almost all seen close to it, and the minute "stellar" nebulæ are strongly concentrated in the region of Sagittarius. This suggests very strongly that these nebulæ are roughly of the same size, those which seem larger being nearer and hence showing less galactic concentration, while the very small ones lie at great distances.

(a) (b) (c)

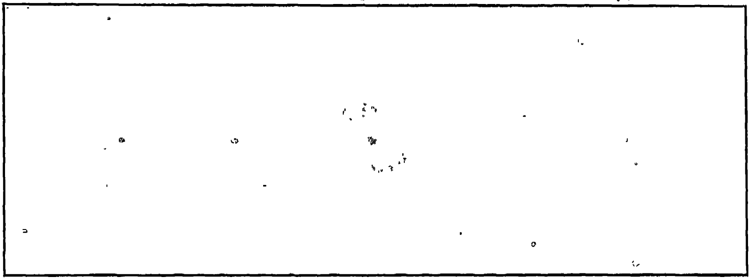


FIG. 286. Three Planetary Nebulæ

(c) N.G.C. 3587; the "Owl Nebula" in Ursa Major; 200″ in diameter; central star, 12^m (photographic). (b) N.G.C. 7662; in Andromeda; 33″ × 28″; central star, $11^m.5$. (a) N.G.C. 6720; the "Ring Nebula" in Lyra; 83″ × 59″; central star, 13^m (visually $15^m.4$). (Mt. Wilson Observatory)

903. Distances, Motions, and Dimensions. The parallaxes of the central stars of twenty-three of these nebulæ have been determined by van Maanen at Mt. Wilson. The individual values range from $+ 0''.040$ to $- 0''.010$. The largest parallax belongs to the "giant" planetary N.G.C. 7293 in Aquarius, 12′ in diameter, and probably the nearest of all.

For the rest the mean parallax comes out $0''.008$, the mean diameter 54″, and the mean photographic magnitude of the central star 12.5.

The radial velocities of about a hundred of these nebulæ have been determined at the Lick Observatory. They indicate a solar motion of 28 km./sec. and a mean peculiar radial velocity of the nebulæ of ± 37 km./sec. Six nebulæ, all of them very small, or

stellar," have velocities exceeding 100 km./sec. Even if these are xcluded, the average radial velocity for the rest is ± 30 km./sec., —considerably greater than for any of the ordinary classes of stars.

The proper motions of planetaries are very small, and accurate leterminations are few. Recent work by van Maanen gives, for ıine nebulæ, a mean P.M. of 0″.022 (relative to the faint comɔarison stars used as standards). From the observed radial velocities we might expect a tangential velocity about 1.6 times ıs great, or 10 astronomical units, per year, which indicates a mean parallax of about 0″.002 and suggests that the direct observations are affected by some small systematic error, as might well happen for objects of such abnormal color, enveloped in nebulosity.

In a few years more enough should be known about the proper motions to settle the question. Meanwhile it is evident that the planetary nebulæ must be bodies of very large size. With the observed parallax the diameter of a typical planetary comes out about 7000 astronomical units, and with the rough value derived from the proper motions, about 25,000. Consequently the minor details of these nebulæ must be larger than the whole solar system.

The average absolute magnitude of the central stars comes out about + 7 with the larger parallax and + 4 with the smaller, while that of isolated stars of Class O is probably about − 4. The reason for this difference is still unknown.

904. Internal Motions and Probable Masses and Densities. A majority of the planetary nebulæ are distinctly elliptical in outline. Spectrograms of these, with the slit set along the major axis of the ellipse, show in most cases a decided difference in radial velocity between the opposite edges, which exhibits itself in an inclination of the spectral lines, like that observed on spectrograms of a rotating planet (Fig. 188, p. 489). This inclination disappears when the slit is set along the minor axis, as it should do if it arose from the rotation of the nebula.

Campbell and Moore have observed this effect in twenty-three nebulæ. In nineteen of these the motion appears to be one of simple rotation, while in the other four it is more complicated. The rotational velocity of the outer parts is usually slower than that of the parts at an intermediate distance, as it should be if

the central parts rotated like a solid mass and the outer portions like a swarm of satellites (cf. § 460). Those planetaries which are nearly circular show no such evidences of rotation, presumably because we see them nearly from the direction of their poles.

The observed velocities of rotation range from 1.4 to 18 km./sec. The average value is 5.3 km./sec. at an apparent distance of 5″.7 from the center.

The chief nebular lines (on which these observations were made) are usually wider in the spectrum of the central parts of the nebulæ and often dark at the center, as if a narrow absorption, arising in the outer layers, were superposed on a broader emission line originating in the interior.

On the assumption that the centrifugal force due to rotation balances gravity at the point where the rotational velocity is most rapid, the mass of a nebula can be found if its parallax is known.

If v is the velocity (in astronomical units per year) at a distance of a astronomical units from the center, the period of rotation, P, is $2\,\pi a/v$, and the mass, m, is $a^3/P^2 = av^2/4\,\pi^2$. If d is the distance in seconds of arc, p the parallax, and V the velocity in km./sec., this equation becomes $m = V^2d/890\,p$, as is easily verified.

For a representative nebula, with $d = 5''.7$, $V = 5.3$ (the mean values), $m = 0.18/p$.

The mean observed parallax, 0″.008, gives a mass twenty-two times that of the sun; the parallax derived from proper motions, one four times as great. The values computed for individual nebulæ, with the same parallax, are of the same order of magnitude. For example, for N.G.C. 7026, in which the rotational motion is most rapid, $d = 3''.6$, $V = 18$ km./sec., and $m = 160$, if $p = 0''.008$; for N.G.C. 6720, in which it is slowest, $d = 25''$, $V = 1.4$, and $m = 7$, with the same parallax. Allowance for the fact that only a component of the rotational velocity lies along the line of sight would increase these values, on the average, by something less than 50 per cent.

In spite of the present uncertainty of their parallaxes it is fairly certain that the planetary nebulæ have large masses, comparable with those of the brightest stars and perhaps exceeding them. Much, if not most, of this mass is probably concentrated close to the central nucleus.

The periods of rotation must be very long; for the typical example considered above, the period comes out 4000 years with the larger parallax and 15,000 for the smaller. The mean density is amazingly low; on the first hypothesis (which gives the

larger value) it comes out 6×10^{-18} times that of the sun, which is only a little greater than would be obtained if a cubic inch of ordinary air were expanded to occupy a cubic mile,

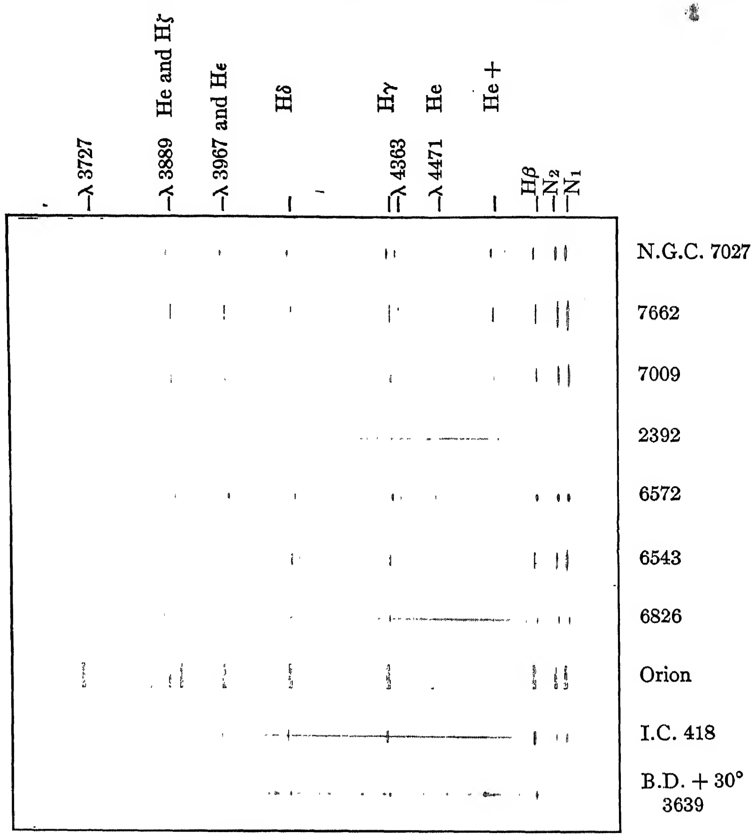


FIG. 287. Spectra of Gaseous Nebulæ

The hydrogen lines and the principal nebular lines (N_1 and N_2) are conspicuous in all the spectra. The helium lines, and some of the other nebular lines, differ considerably in intensity in different nebulæ. (From photographs by W. H. Wright, Lick Observatory)

but is nevertheless greater than the maximum value estimated above for the Orion nebula.

905. The spectra of gaseous nebulæ are very similar to one another, with minor differences (Fig. 287). All the lines are sharp, showing that they come from a gas of low density. The hydrogen series is conspicuous, and there is a continuous emission

beyond the series limit, which is probably also from hydrogen (cf. § 636). The lines of helium are usually present, though fainter. Ionized helium (λ 4686) often appears, and occasionally ionized carbon and doubly ionized nitrogen. But the strongest lines of all have never yet been reproduced in any earthly laboratory.

The most conspicuous of these are at $\lambda\lambda$ 5006.84 and 4958.91 (I.A.), to which the gaseous nebulæ owe their conspicuous green color. These are commonly denoted by N_1 and N_2. There is a strong close pair in the ultra-violet $\lambda\lambda$ 3726.16, 3728.91, to which much of the photographic activity is due, and other prominent lines at $\lambda\lambda$ 6583.6, 6568.1, 4363.21, 3967.51, and 3868.74, as well as many fainter ones, some far in the ultra-violet.

The suggestion is tempting that the nebular lines may be emitted only in gas of very low density. (This would happen, for example, if it took a relatively long time (as atomic events go) for an atom to get into the right state to emit them, and if a collision with another atom in this interval prevented the completion of the process.) In such a case it might require a great thickness of the very rarefied gas to emit these lines strongly enough to be visible, and ordinary vacuum tubes might be far too small to show them.

906. Identification of "Nebulium." The last paragraph (written in 1926) was justified within a year by the discovery by Bowen that most of these lines are due to oxygen and nitrogen. The singly and doubly ionized atoms of these elements possess *metastable states* (§ 626) in which they are stored with energy, but have very little tendency to unload it by radiation, so that, except in an exceedingly rarefied gas, this is discharged by collisions of the second kind (§ 627). Transitions between these states, with emission of radiation, are, however, not strictly "forbidden"; and given free time enough, they will occur. The relative levels of these metastable states are accurately known from the study of the lines in the spectra (cf. § 634), and the positions of the lines which would be produced by transitions between them can be calculated (§ 628). These agree perfectly with the strongest nebular lines and make their origin certain. The lines at $\lambda\lambda$ 5006, 4958, 4363 are due to transitions in doubly ionized oxygen (O III); those at 3726, 3728, and one in the red at 7325, to O II; and $\lambda\lambda$ 6583, 6568, to N II. Besides this, several lines in the far ultra-violet are due to ordinary transitions

in O III, and two others, at 3346 and 3426, probably belong to O IV and N IV.

One more hypothetical element is thus eliminated. "Nebulium" has literally "vanished into thin air."

In a vacuum-tube, collisions of molecules with the walls alone must be frequent enough to prevent the emission of the nebular lines.

907. Excitation of the Gaseous Spectrum. All the nebular lines of known origin belong to elements of high ionization potential and demand a great deal of energy for their excitation. The atoms which emit them must previously have been loaded up with energy amounting to about 13 "volts" for the hydrogen lines, 23 volts for those of neutral, and 51 volts for those of ionized helium. In the atmospheres of the hottest stars, energy of this amount is fed into the atoms by wholesale, all to about the same degree; but such high temperatures are out of the question in the nebulæ. Many diffuse nebulæ are opaque, and if they were as hot as this they would shine more brightly than the sun.

Moreover, there is good physical evidence that an atom remains in an excited state for much less than a millionth of a second before it unloads its store of energy by radiation.

To account for the nebular spectrum, therefore, we must find some process which is capable of feeding large amounts of energy into individual atoms, here and there, in a cold and exceedingly rarefied gas. The only known ways in which this happens are by collisions with electrons of high speed and by the absorption of radiation of very short wave-length, and the only source of either of these which comes at all into consideration from the astrophysical standpoint is a very hot body. Hot bodies give out electrons by "thermionic" emission (as does the filament in a radio tube), and also emit thermal radiation; and the hotter the body the faster the emitted electrons move, and the shorter are the wave-lengths present in the radiation. The hottest bodies known in nature are the O-stars and B-stars. It might therefore have been predicted, on general physical considerations, that gaseous nebulæ would be associated with stars of these spectral classes.

908. Relation between Gaseous Nebulæ and Hot Stars. It has been known for many years that the stars of Class B are con-

nected in some way with the nebulæ. The relation was fully elucidated in 1922 by Hubble, who found that when the star whose radiation set a diffuse nebula shining is of Class B2 or later, the nebula has a continuous spectrum; if the star is of Class B0 or earlier, the nebular spectrum is gaseous with very little continuous spectrum; if it is of Class B1, the continuous and bright-line spectra are blended in the nebula. The only exception is the "North America" nebula (so called because of its shape), which has a continuous spectrum with faint emission lines and appears to be excited by α Cygni, a super-giant of spectrum cA2.

The planetary nebulæ conform to the same rule. The nuclei of the larger ones have spectra of Class O with dark lines only; spectra of the smaller planetaries are often of the Wolf-Rayet type with bright bands.

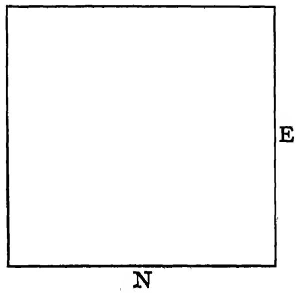


FIG. 288. Faint Nebulosity around R Aquarii

From photograph at Lowell Observatory

In one exceptional case a faint nebulosity (Fig. 288) is associated with a long-period variable, R Aquarii. The nebular lines appear bright in the spectrum of this star and do not change in intensity as its light varies. They probably originate from inner portions of the visible envelope. The reasons for this remarkable association of star and nebula are still unknown.

These relations afford practically conclusive evidence that the luminosity of the gaseous nebulæ arises from the excitation of the atoms of gases within them by radiation of some sort (corpuscular or ethereal) emitted from neighboring stars of very high temperature. It does not follow at all that the gas constitutes the major part of the matter in the nebula, — only that it has a selective capacity for shining under the influence of the radiation.

The rapid transition, for a relatively small change in the spectrum of the star, from a continuous nebular spectrum without bright lines to a bright-line spectrum practically without a continuous background, is strange. If the continuous spectrum arises from the reflection of starlight by particles of dust or fog, it should still appear in the gaseous nebulæ, and its suppression

is puzzling. It may be, as Hubble suggests, that both the continuous and the bright-line spectra arise, in different ways, from the excitation of the nebular gases.

In the planetary nebulæ too the extent and brightness of the nebulosity increase, in general, with the brightness of the nucleus; but there are many exceptions. The outermost nebulosity is, on the average, about forty times as bright as a white surface illuminated by the light of the nucleus would be, indicating that the invisible radiation from the nuclei carries much more energy than the visible light.

909. Localization of Emission of Different Kinds. If a planetary nebula is photographed with a slitless spectroscope, the spectrum shows a series of monochromatic images of the nebula,

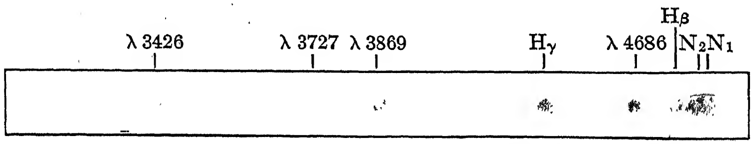


FIG. 289. "Slitless" Spectra of the Planetary Nebula N.G.C. 7662

Different sizes of images are formed by different wave-lengths. (From photograph by W. H. Wright, Lick Observatory)

threaded like beads upon the narrow continuous spectrum of the nucleus (Fig. 289). These images are of very different sizes; and after allowance is made for the loss of the outer portions of faint images, there can be no doubt that some radiations are emitted from much smaller parts of the nebulæ than others.

The line at λ 3727 gives the largest images and usually also the most detail. The lines in the green, N_1 and N_2, come next, closely followed by hydrogen. Neutral helium gives, on the average, somewhat smaller images, and ionized helium (λ 4686) much smaller, and some of the unknown ultra-violet lines (for example, λ 3426) the smallest of all.

So far as the origin of these lines is known, the larger images belong to the radiations of lower excitation energy, as if the influence proceeding from the nucleus, whatever its nature, grew feebler at a distance and was able to influence only the more easily excited atoms. Corroborative evidence is found in the diffuse gaseous nebulæ. In the Orion nebula N_1 is stronger than $H\beta$ in

the central region, as it is in the planetaries; but the outer portions show the hydrogen lines only, and the outer peripheral regions, according to Slipher, give a continuous spectrum. In most other diffuse nebulæ Hβ is as strong as the nebular lines, or stronger.

Bowen has explained all this on the assumption that the central star is excessively hot, perhaps 100,000° C. It will then emit light of very short wave-length, capable of removing the first three electrons from oxygen by photo-electric ionization (§ 636). Near the star, then, only the spectrum of O IV, and other spectra requiring similar excitation, will be emitted. But this ionization process gradually uses up the light of shortest wave-length; so farther out there will be a region containing doubly ionized oxygen and emitting the lines of O III, and so on. The photo-electrons, after a free path lasting perhaps for weeks, will be recaptured by ionized atoms, and cause emission of the bright-line spectrum.

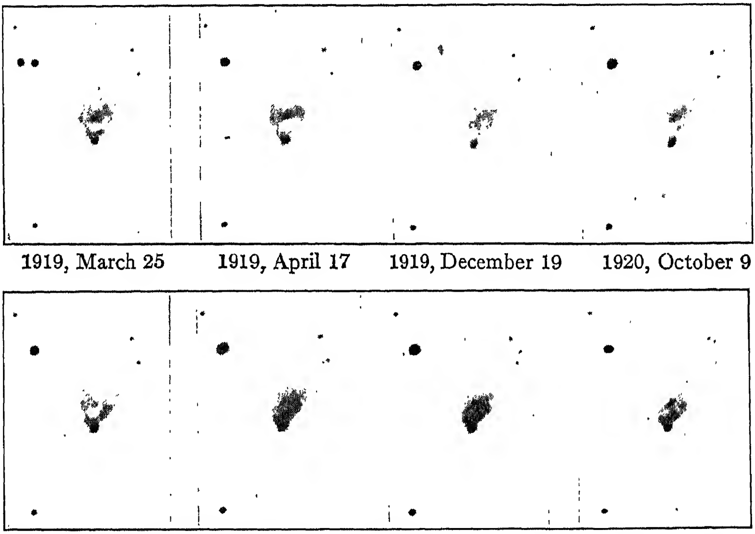
A comet, when at a great distance from the sun, shows a continuous spectrum; nearer perihelion most of its light is in the bright bands of nitrogen and carbon compounds; if it is within twenty or thirty million miles of the sun, the sodium lines come out, and the bright bands weaken.

The sun's radiation has to be very intense before it excites this gaseous luminosity; the hottest stars can do so more easily. It is perhaps not going too far to suggest that from a hypothetical planet revolving about an O-star numerous brilliant green comets might be seen, showing the nebular spectrum.

910. Summary. Leaving speculation aside, it seems now to be fairly well established that the galactic nebulæ are not truly self-luminous bodies, but are set shining by neighboring stars. When these stars are of moderate temperature, the nebulæ probably shine by reflected light; when the stars are very hot, their radiation in some way stirs up monochromatic emission by the atoms of gases which are present in the nebulæ but need not form a large proportion of its mass. If no bright star is close by, the nebulous cloud is dark and can be seen only if it hides the Milky Way behind it.

The planetary nebulæ are hundreds of times as large as the whole solar system, and their nuclei appear to be stars of the very hottest types. Their total masses are probably equal to those of

the greatest stars, but the density of the visible nebulosity must be excessively low. They appear to be permanent configurations, in slow rotation about their centers. To maintain their ellipsoidal forms and keep the gas near the poles from falling into the nucleus some force additional to gravitation must be at work (possibly radiation pressure), but the details are obscure.



1919, March 25 — 1919, April 17 — 1919, December 19 — 1920, October 9

1921, December 29 — 1922, October 24 — 1923, January 27 — 1923, April 5

FIG. 290. The Variable Nebula N.G.C. 2261, in Monoceros

Photographed by Lampland with the 42-inch Lowell reflector

The expanding nebula around Nova Aquilæ may be regarded as a special type of planetary nebula, excited to luminosity by the radiation of the star.

The diffuse galactic nebulæ appear to consist of clouds and wisps of matter in chaotic motion (if the Orion nebula may be taken as an example). The average rate of radial motion in this nebula would carry a particular wisp clear across it in a few hundred thousand years, unless the attraction of the nebulous mass is sufficient to hold it back, which is doubtful. The whole region of Orion is full of obscuring clouds of dark nebulosity, and it may well be that the great nebula is merely a superficial fluorescence of the gaseous portion of such of these clouds as happen to drift

into the region where they are stimulated to shine by the influence of certain very hot stars.

911. Variable Nebulæ. A few nebulæ are unquestionably variable in form and brightness. The most remarkable of these, N.G.C. 2261, in Monoceros and N.G.C. 6729, in Corona Australis, have a very distinctive "cometary" form (Fig. 290), with a fairly bright nucleus at the head, which is itself an irregularly variable star. Both these stars have remarkable spectra, — one, R Monocerotis, resembling a nova, and the other, R Coronæ Australis, being of a peculiar G type with bright lines. The spectra of both nebulæ are continuous, with faint superposed bright lines, and are apparently similar to those of the nuclei.

The changes in the nebulosity are extremely rapid, being often perceptible in a single day's interval, and suggest alterations in brightness of various parts of a cloudy mass of fairly permanent form, details disappearing, and reappearing later in the same place, as if they had been obscured and reappeared, while the nebulosity close to the variable star at the apex also changes greatly in brightness. The cause of these remarkable changes is not known; but unless they proceed at a greater speed than the velocity of light (which seems very improbable), these nebulæ must be relatively near to us. Hubble estimates the distance of N.G.C. 6729 as about 100 parsecs. It lies in a region of heavy obscuring clouds, which are probably at about the same distance.

EXTRA-GALACTIC NEBULÆ

912. General Characteristics. The extra-galactic nebulæ are by far the most numerous; their number is counted in hundreds of thousands. They show a marked avoidance of the whole galactic region (very few of them being within 20° of its central line), and they are most numerous near the galactic poles. Curtis has found more than three hundred of them in a single plate, in an area 50′ × 40′ in Coma Berenices. These very numerous objects are small and faint; but the Great Nebula in Andromeda (Fig. 291), the largest and brightest of all, is conspicuous to the naked eye, appearing as a hazy patch of about the fourth magnitude, and photographs show it to be about 3° in diameter.

Visual observations, though they have led to the discovery of thousands of these nebulæ, reveal little regarding their real nature. They appear as faint patches of light, often brightening up to a definite, but not stellar, nucleus, and are sometimes crossed by faint dark markings; and two or three show a spiral structure.

FIG. 291. The Great Nebula in Andromeda (Messier 31 = N.G.C. 223)

The central portion of this nebula is conspicuous to the naked eye, and very much brighter in comparison with the outlying regions than is suggested by the photograph, on which the central part is greatly overexposed. This nebula is evidently an extensive flat spiral, nearly edgewise to the line of sight. The numerous stars scattered over it are "foreground" objects, much nearer than the nebula. The ovate nebula directly above the center (N.G.C. 221) has almost the same radial velocity as the Great Nebula, and is probably really a companion to it. On the margin of this figure are roughly indicated the boundaries of the region which is shown in Fig. 296 on a much larger scale. (See also the frontispiece of Volume II). (Photographed by Ritchey, at Yerkes Observatory, with the 24-inch reflector)

Photography (which has nowhere else quite such great advantages over the eye as here) brings out a wealth of detail in many cases, and makes classification possible.

913. Classification. We will here follow the scheme proposed by Hubble, who divides extra-galactic nebulæ as follows:

(1) *Regular nebulæ*, showing rotational symmetry about dominating non-stellar nuclei. Of the extra-galactic nebulæ approximately 97 per cent are "regular" in this sense.

The various forms of the regular nebulæ fall into a progressive sequence ranging from globular masses of unresolved nebulosity (N.G.C. 3379, Fig. 293) to open spirals whose arms are swarming with stars (M. 101, Fig. 292). The sequence comprises two sections, elliptical nebulæ and spirals, which merge into one another.

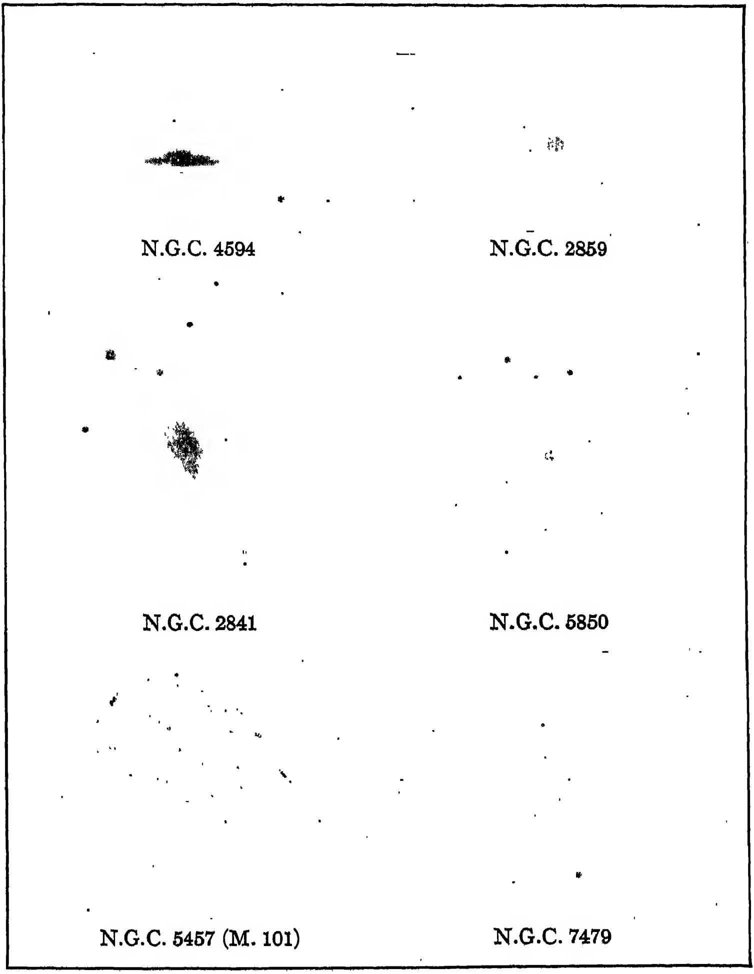


FIG. 292. Six Spiral Nebulæ

The three nebulæ on the left are typical forms, — the first nearly edgewise toward us and showing a strong band of obscuration across it, the second inclined at a large angle, and the third much less. They also represent different degrees of the "curdling" of the nebulosity into starlike details. The nebulæ on the right are less usual forms, N.G.C. 5850 being of the "bar" type and the photograph of N.G.C. 2859 suggesting heavy, short appendages on each side, instead of long, spiral arms. (Photographed by Hubble, Mt. Wilson Observatory)

(*a*) The elliptical nebulæ show no signs of resolution, but brighten up smoothly toward the center and fade out gradually at the edges, according to a definite law, so that their apparent diameter increases with the photographic exposure. They show forms ranging from circular disks to spindle-shaped figures which are evidently lens-shaped bodies seen edgewise (N.G.C. 3115, Fig. 293). Some of the nearly circular forms are probably lenticular bodies seen face on, but a statistical study shows that many others must really be nearly spherical in shape and that all intermediate forms occur.

(*b*) The spiral nebulæ are evidently flat objects, some seen nearly edgewise, others almost in plan (Fig. 292). The regions near the nuclei project out of the plane in which the spiral arms lie, and have much the shape of the most flattened elliptical nebulæ. The large majority of the spirals may be arranged in a sequence, beginning with forms in which the nuclear region is large and the arms are closely coiled and unresolved (N.G.C. 4594, Fig. 292), and ending with forms in which the arms are widely open and resolved into very numerous condensations while the nucleus is relatively inconspicuous (M. 101, Fig. 292). Here again all intermediate forms may be observed. The resolution of the arms begins at the outer extremities and works in toward the center in the more open forms. In the larger spirals, where critical observations are possible, these condensations are known to be actual stars, or groups of stars.

In a minority of cases the spiral arms, instead of springing directly from the edge of the nuclear region, start at the ends of a straight "bar" which extends across the nucleus (N.G.C. 5850, Fig. 292). These *barred spirals*, too, range from compact to open spiral forms. Their relation to the main series of forms is not yet clear.

(2) *Irregular nebulæ*, relatively few in number and very similar in appearance to the Magellanic Clouds and N.G.C. 6822 (which was catalogued for years as a nebula). Some of the brighter ones are partially resolved into stars on the best photographs, and there can be little doubt that they too are star-clouds, at even greater distances. Indeed, Hubble now reckons the Magellanic Clouds as the nearest of the irregular extra-galactic nebulæ.

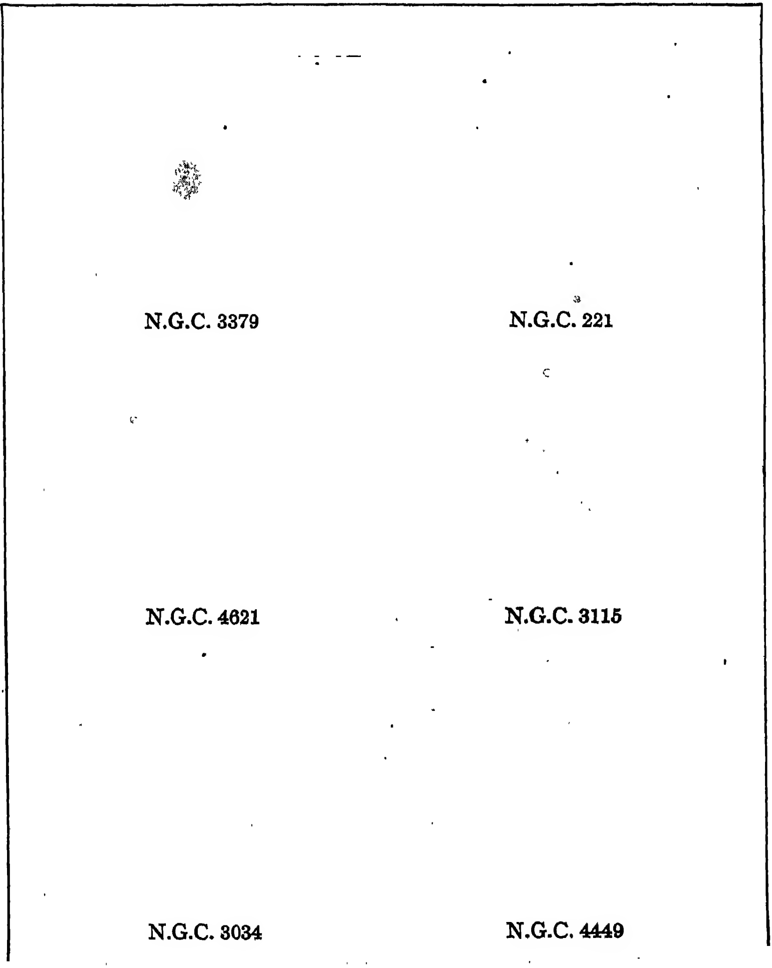


FIG. 293. Elliptical and Irregular Extra-Galactic Nebulæ

N.G.C. 3379 is a globular form, 221 and 4621 are ovate, and 3115 is of the "spindle" type. All four are wholly unresolved, with no trace of structure. The two lower photographs illustrate irregular forms. (Photographed by Hubble, at Mt. Wilson Observatory)

914. Spectra. It has been known since the earliest days of astrophysics that the extra-galactic nebulæ give continuous spectra; but these are too faint to be studied to any advantage visually or even with ordinary photographic spectroscopes. With instruments especially designed for the purpose, and with long exposures, good photographs can be obtained, and these show that the continuous spectra are crossed by a multitude of dark lines and resemble very closely those of stars of the solar type. This very remarkable conclusion (first announced by Slipher for the Andromeda nebula (Fig. 294)) has been fully confirmed for this and many others. Spiral, globular, and irregular nebulæ all show spectra of substantially the same sort. The observations

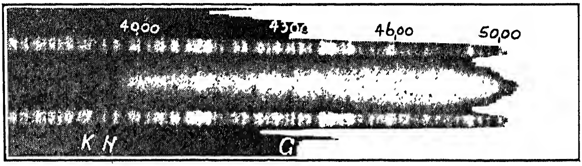


FIG. 294. Spectrum of the Andromeda Nebula

Photographed by V. M. Slipher, Lowell Observatory, December 3–4, 1912. Exposure, fourteen hours

refer to the brighter central regions of the nebulæ. Photographs taken through color screens show that the color of these regions is similar to that of stars of the solar type. The outer portions of the spiral nebulæ are much bluer, which helps to explain why they can be studied so much better by photography than with the eye. These regions are unfortunately too faint for spectroscopic study with present apparatus.

915. Gaseous and Dark Portions of the Nebulæ. In a few cases bright nebular lines are also present, but photographs with slitless spectroscopes show that these are localized in small patches, — gaseous nebulæ within the larger masses and forming parts of them. Obscuring clouds too (dark nebulæ) are recognizable as constituents of many spiral nebulæ. When such a nebula lies nearly edgewise, it is usually crossed by a dark band (as in N.G.C. 4594, Fig. 292) which is evidently due to the obscuration of the bright nebulosity by a ring of dark matter lying in the plane of the spiral, and beyond its visible extensions. Many of

the conspicuous dark lanes and patches in such objects as the Andromeda nebula also suggest strongly the obscured regions of the Galaxy.

916. Significance of these Spectra. The character of these spectra is highly significant. It shows that the light of the nebulæ must come originally from incandescent bodies surrounded by absorbing atmospheres, — in other words, from *stars*. Two hypotheses immediately suggest themselves: (1) that the nebulæ are vast clouds of stars, not visible separately because of their great distance; (2) that they consist of clouds of dust or fog of some sort, illuminated by great stars which lie within them. The first of these views dates back to the first study of the nebulæ in Herschel's time. It was almost abandoned in the latter part of the nineteenth century, when successive increases of telescopic power failed to "resolve" the white nebulæ into stars; but photographs with the great reflectors (especially with the 100-inch) under good conditions are beginning to resolve the outer parts of a number of the brighter nebulæ into swarms of exceedingly faint stellar points, and have brought back a widespread adoption of this interpretation.

917. The radial velocities of extra-galactic nebulæ are extraordinarily great. Slipher's first result (which was 300 km./sec. for the Andromeda nebula) was at that time unprecedentedly large, but most of the others are moving faster. The observations are very tedious, since the nebulæ are so faint that the exposure of a single spectrogram has to be continued for several nights; but radial velocities have been secured for more than forty nebulæ, mainly by Slipher. The mean radial velocity found for these is 620 km./sec. The greatest so far observed (for N.G.C. 584 in Cetus) is $+1800$ km./sec. There is a great preponderance of positive velocities. Seven eighths of the nebulæ appear to be receding, and all the most rapid motions are away from us.

These large radial velocities indicate a solar motion of about 400 km./sec. (with a probable error of at least 100 km./sec.) toward an apex in about $21^h, +60°$, agreeing, within the very considerable uncertainty of the data, with the direction found in the case of other swiftly moving bodies. The mean residual radial velocity of the nebulæ, after correction for this solar motion,

comes out nearly 700 km./sec.; and every one of the nebulæ appears to be receding from this common center of gravity.

Whether this represents a real scattering of the nebulæ away from this region where the sun happens to be is very doubtful. It may arise from some other cause. Certain forms of the generalized theory of relativity, which postulate that space ultimately returns into itself, as the surface of a sphere does, indicate that very distant luminous bodies should appear to have large velocities of recession, and it may ultimately turn out that these observations give us a clue to the extent of space itself.

The proper motions of the spirals are obviously difficult to determine with accuracy. The existing rather rough data indicate that they average, at most, a few hundredths of a second a year, — which, with these enormous velocities, means that their distances must be many thousands of parsecs at the least, and may be much greater if, as is probable, most of the observed proper motions arise from the errors of the older observations.

918. Internal Motions. The forms of the spiral nebulæ, and of the spindles and ovate nebulæ as well, suggest that they are in rotation; and spectroscopic observations have fully confirmed the suspicion. Rotation was first detected by Wolf in the spiral nebula Messier 81, and by Slipher in some others; the motions in two cases have been accurately measured by Pease. When the slit of the spectrograph is set along the major axis of the nebula (that is, parallel to the equatorial plane of the body), one side is found to be approaching, relative to the central nucleus, and the other side receding; but when the slit lies along the minor axis of the nebula, the velocity is the same along its length. The rate of motion, in the equatorial plane, appears to be nearly proportional to the distance from the center, showing that the inner portions of the nebula rotate like a solid body. Very long exposures (as much as eighty hours) are required to obtain spectra of the central regions of the nebulæ, and the outer portions are too faint to be observed in this way.

The velocities are remarkably great. In the Great Nebula of Andromeda a point 150″ from the center has a rotational velocity of 72 km./sec. (which would be increased to 75 km./sec. by allowance for the inclination of the plane of the nebula to the line of sight). The region covered by the observations extends only half-

way to the edge of the central portion which appears "burnt out" and uniformly white in Fig. 291. The velocity at the base of the spiral arms must be greater. In N.G.C. 4594 (Fig. 292) the velocity at 120″ from the center (rather more than halfway to the edge) has the extraordinary value of 330 km./sec.

To detect the corresponding proper motions is more difficult, for the faint "condensations" in the outer parts of the nebulæ, though numerous, are usually not perfectly sharp on the photographs and cannot be measured as accurately as star-images nearer the center of the field. Only photographs taken with the same instrument, and with about the same exposure, can safely be compared. Measures by van Maanen on Mt. Wilson and on Lick plates of seven different spirals show, in all cases, motions outward along the spiral arms, at average rates ranging from 0″.02 to 0″.04 per year in different nebulæ. For Messier 33, however, Lundmark, from measures of the very same plates, with the same measuring instrument, and by the same method, finds a motion in the same direction but at the rate of only 0″.0018 per year, — one tenth of van Maanen's value. The difference indicates the extreme difficulty of the measurements.

919. Variable Stars in Extra-Galactic Nebulæ. A temporary star of the seventh magnitude at maximum appeared in 1885 close to the center of the Andromeda nebula, and another in 1895 in the nebula N.G.C. 5253, in Centaurus. Since the study has been extended to fainter stars, large numbers of novæ have been found, especially in the Andromeda nebula, in which sixty-seven have been observed between 1909 and 1926. These occur mainly in the inner regions of unresolved nebulosity, and they are decidedly similar in brightness, the average at maximum being close to the seventeenth magnitude (photographic) (cf. Fig. 295).

In the outer parts of the Andromeda nebula (M. 31) the nebulosity, when photographed with the most powerful instruments and under good seeing, breaks up into a dense cloud of star-like points (Fig. 296). Here, with the aid of the blink microscope, numerous periodic variables have been found, many of which are certainly typical Cepheids with periods ranging from 18 to 50 days, and light curves of the characteristic form, with a rapid rise and slower fall.

More than forty variables have also been found in the spiral Messier 33 (which comes next to M. 31 in apparent size and brightness), and these also, in many cases, are clearly Cepheids. Variables have also been found in three other spirals, but their periods and types are not yet worked out.

920. Distances of the Nebulæ. The Cepheids, in both these nebulæ, show conspicuously the characteristic relation between period and luminosity. They appear to be confined to the nebulæ, for no such variables have been found in other regions of the sky equally far from the Milky Way, in spite of a special search. There can be little doubt, then, that they are really in the nebulæ and owe their faintness to great distance.

FIG. 295. Temporary Star in the Spiral Nebula N.G.C. 5236

This star, of the fourteenth magnitude, found on photographs taken early in May, 1923, was invisible on numerous photographs made between 1915 and 1922. It began to fade after about ten days, and decreased in brightness by nearly two magnitudes in the next six weeks. (From photograph at Lowell Observatory)

The brightest of them, with periods of forty days and more, are fainter than the eighteenth magnitude even at maximum, and the modulus found by comparing their apparent and absolute magnitudes is 22.15, both for the Andromeda nebula and for M. 33. This corresponds to the gigantic distance of 270,000 parsecs, or 870,000 light-years, and these two are the largest, brightest, and presumably the *nearest* of the multitude of spiral nebulæ! (See frontispiece, Vol. II.)

So amazing a conclusion demands confirmation, and there is much additional evidence to support it.

Hubble, to whom these investigations are due, finds that in Messier 33 (which is better resolvable than M. 31) the brightest

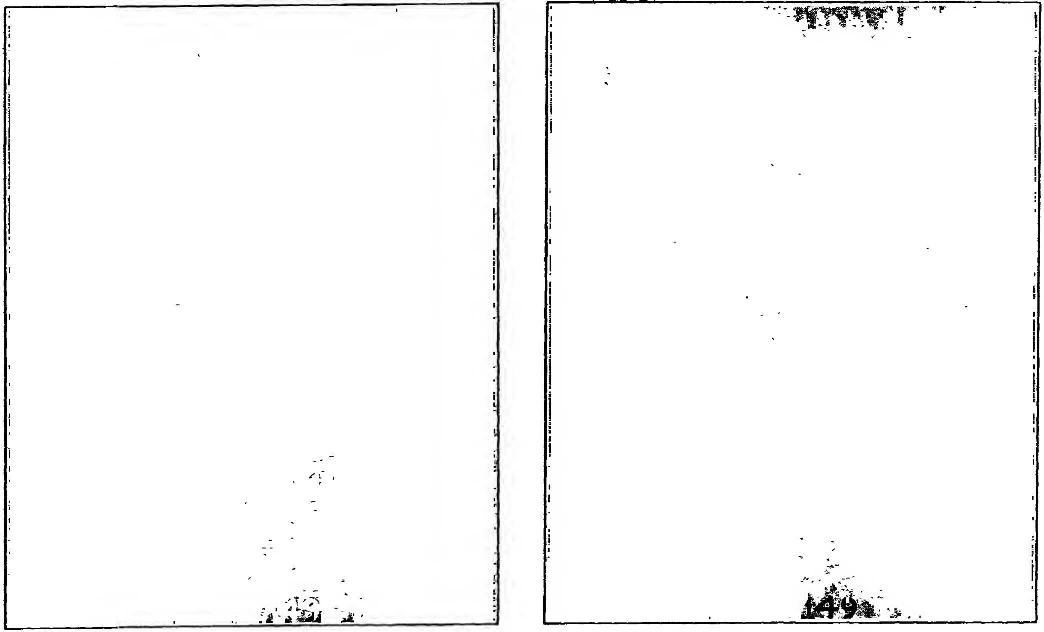


FIG. 296. Variable Stars in the Andromeda Nebula

Two photographs of a small region in the outer part of the Great Nebula, made with the 100-inch reflector. The nebulosity which appears on photographs with smaller instruments is resolved into a multitude of minute star-images. The variable stars are marked with short lines near the corresponding numbers. The changes in Nos. 35, 47, and 49 are conspicuous. (From photograph by E. P. Hubble, Mt. Wilson Observatory)

stars run up to about the absolute magnitude − 7, — a value quite comparable with that found in the Magellanic Clouds and in the galactic system. Color-indices have been determined, and show that the long-period Cepheids are red and that the stars involved in the patches of gaseous nebulosity are blue, while the faintest stars accessible are both red and blue, agreeing again in detail with what is known to be true elsewhere. The average absolute magnitude of the novæ in the Andromeda nebula comes out − 5.3, which is quite in accordance with what is known of galactic novæ. One variable in this nebula, which has been growing slowly brighter since 1906 and is now of the fifteenth magnitude, shows a spectrum of Class B with bright lines. Its absolute magnitude is now brighter than − 7, and it appears to resemble S Doradus (§ 872).

The only difficulty is with the observed internal motions, which, with van Maanen's average value of $0''.020$ per year, in M. 33, would correspond to 28,000 km./sec. Lundmark's measures give 2500 km./sec., — still very great. Until the actual amount of these motions is settled by further observations they can hardly be taken as very strong evidence against the distance supported by all the other data.

Within a few years the distances of other spiral and irregular nebulæ will probably be determined in the same fashion, and it may then be possible to apply statistical methods to estimate the distances of a larger number. It can only be said at the moment that the distances of the remoter ones probably run far into the millions of light-years.

921. Dimensions, Brightness, Masses, etc. The Andromeda nebula, with long exposures, is 3° in apparent length, and its real diameter must be of the order of 14,000 parsecs. Messier 33 is much smaller, but its apparent diameter is about 1°, corresponding to 5000 parsecs.

The total light of the former impresses the eye like that of a star of the fourth magnitude. The real luminosity therefore corresponds roughly to absolute magnitude − 18; that is, to about a billion and a half times the light of the sun.

If it is assumed that in the inner portion of this nebula the centrifugal force balances the gravitational attraction of the mass, the period of rotation comes out 17,000,000 years, and the mass

within a central sphere 5′ in apparent diameter is found to be 270,000,000 times that of the sun. The mass of the whole nebula is probably much greater.

922. The elliptical nebulæ — both round and elongated — resist resolution by the most powerful instruments, as do also the central regions of the great Andromeda spiral. In the latter case, and also for N.G.C. 221, which is probably a physical companion of the Andromeda nebula (as it has the same radial velocity), the failure to photograph the separate stars cannot be attributed to excessive distance. Their spectra are exactly like those of the spirals and seem to prove that the light comes originally from stars. Either these stars are of small luminosity (giants being absent) or else they are enveloped in some sort of haze, which hides the individual stars but is illuminated by their light. The latest observations favor the former explanation.

Hubble's measurements show that the surface brightness of these bodies falls off inversely as the distance from the center (except for a small region in the middle). This might be explained by an appropriate distribution of stars or haze, or both combined.

The forms of these nebulæ suggest strongly that they are in rotation, that of the most elongated forms being the most rapid in proportion to the density.

923. Relations between Form and Brightness. Hubble finds that for nebulæ of the same form the brightest are the largest in apparent diameter and that the surface brightness is much the same for them all. For nebulæ of different forms, but the same total apparent brightness, the angular diameter is least, and the surface brightness greatest, for the globular objects. The diameter is greater for those of increasing ellipticity, and greater still for the spirals, especially for those in which the arms are widely spread and resolvable.

This suggests strongly that the various forms represent stages in some process of dissipation of an originally nearly spherical rotating mass outward in the equatorial plane, but there is at present no proof that this interpretation is true.

It is supported by the theoretical investigations of Jeans. A mass of rarefied gas, much denser near the center than at the surface, and having a slow rotation, would be very nearly ellipsoidal in form. With more rapid rotation it would become lens-shaped, and, finally, when the centrifugal force became equal to gravity at the equator, matter would begin to fly off from it and form a thin external sheet in the equatorial plane. Such a

body, seen edgewise, would look very like a "spindle" nebula. If the mass were absolutely alone in space, this sheet would be uniform; but if it were disturbed ever so little by the tidal influence of other bodies, the ejection of material would take place from the two opposite regions of "high tide," and the escaping material would flow out in large streams which might then segregate into numerous separate masses.

All this is strikingly similar to the sequence observed among the nebulæ from globular, ovate, and spindle forms, through spirals with a prominent central amorphous mass and faint arms, to those in which the arms are more prominent than the nucleus. It is hard to escape the belief that this sequence represents successive evolutionary stages, yet difficulties remain. It now seems very probable that these nebulæ are not continuous masses of rarefied gas but vast aggregates of separate stars. It may be that such a huge star-cloud, if rotating and in equilibrium under its own attraction, might take on a series of forms very similar to those of a gaseous mass, but this has not yet been proved. Moreover, the arms of the spiral nebulæ follow nearly the course of a logarithmic, or equiangular, spiral. If the particles which form them are moving permanently along these curves, then, as Jeans has shown, unknown forces must be at work to make them do so. It is also possible (as Brown has suggested) that the arms may be regions in which bodies, moving under the influence of gravitation, tend to congregate; but this requires a regular arrangement of their orbits which demands explanation.

924. Island Universes. Those extra-galactic nebulæ which have been studied in detail are found to be gigantic systems far beyond the confines of our galactic "universe," even when this is extended to include the globular clusters. The irregular nebulæ, like the Magellanic Clouds, appear to be comparable with the "local system" in the Galaxy, — or even larger, — and give out ten million or a hundred million times as much light as the sun.

The spirals — at least those conspicuous ones which have been investigated — are huger still. The Great Nebula in Andromeda is of about three quarters the diameter of the Kapteyn universe and gives out an amount of light comparable with that of all the stars within it (taking its limits at the distance where the star density, as computed by Kapteyn, falls to 1 per cent of the central value). There are other points of similarity; for example, the dark obscuring nebulosity in the galactic plane (which is probably responsible for the division of the Milky Way into two streams and the absence of globular clusters in low galactic

latitudes) suggests the obscuring matter which forms dark lanes across those spiral nebulæ which are seen edgewise.

It may be, indeed, that the scarcity of nebulæ of the extra-galactic type within 20° or so of the galactic equator may be accounted for by the existence of still vaster obscuring clouds, surrounding the whole galactic system in its own plane.

There are, however, notable points of difference between the spiral nebulæ and the galactic system. The former have a nucleus much denser and more luminous than the outer portions; the latter has none (unless dark nebulæ hide it). The surface brightness of the inner parts of the spirals, too, according to Seares, is about a hundred times greater than that which the Galaxy (or at least the part of it near the sun) would exhibit if seen from a very distant point in the direction of its pole. Luminous matter (that is, presumably, the stars) must be far more thickly scattered in space in the central regions of these nebulæ than it is in our part of the Galaxy. Again, the galactic system, taken as a whole, as revealed by the globular clusters and by observations of faint galactic variables, must be much greater in diameter than even the Andromeda nebula.

The term "island universes" (which has often been used to describe the spiral nebulæ) appears, therefore, to be well chosen. In Shapley's happy phrase, however, "if we call them islands, the Galaxy is a continent."

In several instances there appear to be *clusters* of extra-galactic nebulæ. The most conspicuous of these is in Virgo and Coma Berenices with its center at $12^h\ 20^m$, $+13°$, and contains about a hundred nebulæ, fairly comparable in apparent brightness, in a region a little more than 10° in diameter. Shapley and Hubble have independently estimated its distance as about 10,000,000 light-years. Other smaller clusters of fainter nebulæ may be much more distant.

Whether the many smaller spirals and the other little roundish or oval nebulæ, which throng the sky, are comparable in magnitude with the great systems which have so far been studied is still unknown. The Andromeda nebula, even at a hundred times its present distance, would still be easily recognizable as a nebula on

modern photographs; and it may be (though we cannot prove it now) that some of the faintest objects on our plates are revealed to us by light which has been traveling, not for the petty space of human existence, even as a race, but for a great part of the time during which life itself has been upon the earth.

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Note. The authors are much indebted to Dr. Hubble for suggestions and unpublished data contributed while this chapter was in proof.

CHAPTER XXV

THE CONSTITUTION OF THE STARS

STELLAR ATMOSPHERES · EFFECTS OF TEMPERATURE ON THE SPECTRUM · ARC AND SPARK LINES · SIMILARITY OF COMPOSITION OF THE STARS · COMPOUNDS IN THE COOLER STARS · PRESSURES IN STELLAR ATMOSPHERES · RELATIVE ABUNDANCE OF THE ELEMENTS · SPECTRA OF GIANTS AND DWARFS · PHYSICAL MEANING OF SPECTROSCOPIC PARALLAXES · BRIGHT LINES IN STELLAR SPECTRA · THE INTERIOR OF THE STARS · CONDITIONS OF EQUILIBRIUM · LANE'S LAWS · RADIATION PRESSURE · EDDINGTON'S EQUATION · RADIATION THROUGH A GAS · THE OPACITY COEFFICIENT · THEORETICAL CALCULATION OF A STAR'S BRIGHTNESS · DEPENDENCE ON MASS AND RADIUS · AGREEMENT WITH OBSERVATION · MATHEMATICAL DETAILS

I. THE ATMOSPHERE OF A STAR

925. Photosphere and Atmosphere. In interpreting the physical constitution of the stars, modern atomic theory is no less valuable than in the case of the sun. A star also, being a mass of intensely hot gas, should have a nearly transparent atmosphere, shading gradually into a photosphere in which the gases are opaque enough to obstruct the radiation from the deeper layers and to emit a continuous spectrum.

The effective temperature of the photosphere is controlled by the supply of heat which it receives from the interior of the star, and hence by deep-seated conditions. Conditions in the atmosphere, on the contrary, depend almost entirely upon the photospheric temperature and the force of gravity at the star's surface, and mainly upon the former. The temperature of the atmosphere at its base is naturally that of the outer boundary of the photosphere, and it falls slowly outward according to the laws of radiative equilibrium (§ 663).

The density in the photosphere must be low (if the theory of the opacity of ionized gases (§ 661) is sound), not merely in the sun but in all the stars; and the density in the atmosphere must be much lower. Under these conditions the gases must be

very considerably ionized, especially in the upper parts of the atmosphere where the pressure is lowest (§ 645).

926. Effects of Temperature on the Spectra of the Stars: Arc Lines and Bands. We may now follow in detail the influences which the temperature and density in stellar atmospheres should have upon the spectra of the stars. Consider first a series of stars of increasing photospheric temperature. The intensity of any spectral line increases with the number of atoms (per square centimeter) above the photosphere which are in a condition to "absorb" it, that is, which are ionized to the right degree and are in the appropriate energy state (§ 635).

The great majority of the neutral atoms of a given kind are, at any given moment, in the state of lowest energy, and capable of absorbing the ultimate lines. The fraction in the higher energy states, which absorb the subordinate lines, is small, but increases with the temperature. As the temperature rises, however, more and more of the atoms become ionized and cease to absorb the arc lines at all. At the lowest stellar temperatures, therefore, we should expect the prominent features of the spectrum to be the ultimate arc lines of the elements, together with bands due to compounds. With rising temperature, the bands should weaken and disappear, — those due to the more easily dissociated compounds going first. The ultimate arc lines of the elements should at first nearly hold their own, and might even increase in intensity, if the atoms which absorb them were set free by the decomposition of compounds containing them. At higher temperatures, as ionization sets in, these lines should weaken.

The subordinate arc lines should at first grow stronger, owing to the increase in the proportion of the neutral atoms which are in excited states. Later, when the decrease of the total number of neutral atoms by ionization becomes serious, these lines too should weaken. At still higher temperatures the arc lines should disappear, the ultimate lines holding out the longest. For elements of easy ionization all the stages of this process should occur at lower temperatures than for those harder to ionize.

All these predictions are exactly verified among the stars. The conspicuous bands disappear early in the sequence, — those

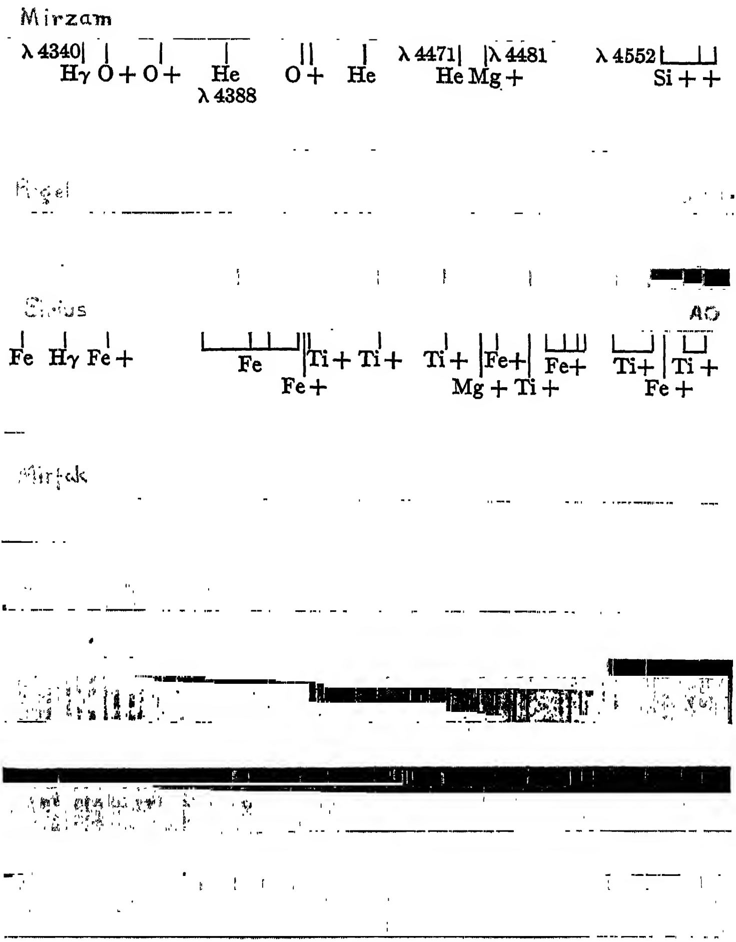


FIG. 297. Spectra of Stars of Various Classes

A small region in the blue, λλ 4320–4580. In the spectrum of Mirzam (β Canis Majoris), lines of H, He, O +, and Si + + are conspicuous, while one of Mg + is faint. In that of Rigel, O + and Si + + have disappeared, owing to the lessened ionization, while He weakens and H and Mg + grow stronger. In Sirius the hydrogen lines are very strong, helium has disappeared, Mg + is weakening, while many lines of Ti + and Fe + appear, and a few of the strongest of neutral iron. In Mirfak (α Persei) the enhanced lines of Fe and Ti are near their maximum intensity, and the hydrogen lines are fainter. In the later types the lines of neutral iron grow very heavy, and numerous arc lines of other metals appear. The apparent bright lines in the spectrum of Betelgeuse are clear spaces between regions crowded with dark lines. (Photographed at Yerkes Observatory with a three-prism spectrograph, with titanium spark comparison)

of titanium oxide, for example, in Class K2. The ultimate arc lines, in general, are strongest in the coolest stars, — especially in the redder dwarfs, where they are extraordinarily intense. Those of titanium, however, are weaker in the coolest M-stars, — where the bands of the oxide are very strong (Fig. 263).

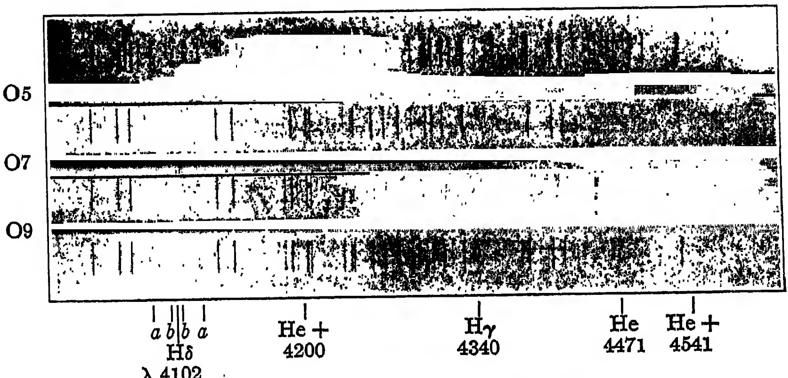
Passing to stars of higher temperature, we find that the ultimate arc lines weaken. For very easily ionized elements, such as rubidium and barium, they disappear in Class G, although bands of some refractory compounds, CN, CH, OH, still survive. For elements which are harder to ionize, such as iron, the ultimate lines persist as high as Class A. The subordinate lines show definite maxima (in Class M for the easily ionized calcium, in Class K for magnesium and iron, and in Class G for silicon), while for hydrogen, with its high ionization and excitation potentials, the maximum is at A0, and for helium, the hardest of all to excite, at B3. At yet higher temperatures the intensities of all the lines of the neutral atoms decrease; the ultimate lines (when accessible to observation) are the last to disappear.

927. Effects of Temperature on Enhanced Lines. Singly ionized atoms should be very few at low temperatures, should increase in number as the neutral atoms decrease, and should come to form the great majority. When but a minute fraction of the neutral atoms is left, a second ionization should begin, and the number of singly ionized atoms should decrease again (doubly ionized atoms replacing them), and so on.

The ultimate lines of the ionized atoms should therefore appear as soon as ionization sets in, and before those of the neutral atoms weaken perceptibly. They should reach a maximum when the arc lines are almost disappearing, and then gradually fade. This maximum should be flat, since, over a considerable range of temperature, the singly ionized atoms form a very large percentage of the whole. The subordinate enhanced lines should rise to a maximum at a higher temperature, especially if the excitation potential is high, and this maximum should be much sharper. Here again every stage of the process should occur at a lower temperature, the lower the energy required for a second ionization. Multiply ionized atoms and their lines should show a similar behavior, at still higher temperatures.

The predictions also are verified in stellar spectra. Only a few ultimate enhanced lines are observable (most of them lying too far in the ultra-violet), but these show flat maxima at low temperatures, in Class K0 for Ca + (the (H) and (K) lines) and in K5 for Sr +, as they ought to do, since these elements are easy to ionize.

For the subordinate lines the maxima come higher: F8 for Ti +, F5 for Fe +. As the ionization and excitation potentials



a, Si + + + 4089, 4116; *b*, N + + 4097, 4103

FIG. 298. Spectra of the Hottest Stars

(Negatives of slit-spectra, with iron comparison.) Upper spectrum, H.D. 46223, Class O5; middle, 9 Sagittæ (O7); lower, 10 Lacertæ (O9). The hydrogen lines (blended with those of ionized helium) are conspicuous in all three. The lines of ionized helium are strongest in O5, and very faint in O9; those of doubly ionized nitrogen are strongest in O7, and of trebly ionized silicon in O9. The lines of neutral helium vanish in O5, the helium being practically all ionized. (From photograph at Dominion Astrophysical Observatory, Victoria)

increase, the maxima move to higher temperatures: A3 for Mg +, A0 for Si +, B3 for C +, while for He + the maximum is barely reached in the hottest stars of Class O.

Lines of doubly and trebly ionized atoms rarely occur in the accessible part of the spectrum (although more may be found in future, for the spectra of multiply ionized atoms have been as yet incompletely studied). Those which are known conform to prediction. Doubly ionized silicon, for example, shows a sharp maximum at B1, while trebly ionized silicon only begins to appear at B2, and reaches its greatest intensity in Class O (Fig. 298).

If the temperature were increased indefinitely, all the elements would get into so high a state of ionization that they would have no lines in the visible region strong enough to be observed, and the spectrum would simply be continuous. This theoretical limiting stage has not yet been observed. The nearest approach to it is a star studied by H. H. Plaskett, for which all lines have disappeared except those of ionized helium and the ubiquitous hydrogen. He calls the spectral class of this O5, and suggests for the limiting stage the rather appropriate symbol O0.

928. Similarity of Composition of the Stars. From the discussion thus far, it appears that the observed sequence of stellar spectra is, not merely in general but in detail, just what might be expected from absorption by an atmosphere of fixed composition but of varying temperature.

It was once supposed that the vanishing of the metallic lines in the hotter stars, and the appearance of those of the permanent gases, indicated that the atoms of the metals were decomposed at these temperatures, with production of the gases. We know now that the disappearance of the metals is due to their getting into multiply ionized states, in which they have no lines in the accessible region; or at least no lines strong enough to be recognized in the stars. If only the earth's atmosphere were not opaque to light of shorter wave-length than λ 2900, we could probably follow the metals much farther up the spectral sequence.

As for the permanent gases, their ultimate lines are hopelessly inaccessible, — beyond λ 1500, — and their observable lines have such high excitation potentials that it takes a good deal more energy to get their neutral atoms ready to absorb these lines than to ionize the atoms of the metals. It is natural, therefore, that in the cooler stars, where the arc lines of the metals are strong, the lines of these gases should be weak or absent.

929. Compounds in the Cooler Stars. Branching of the Spectral Sequence. The sequence of spectra, which extends down in a single line from Class O to Class K, forks somewhere near K2. The majority of the cooler stars show bands of titanium oxide, faint in K5 but steadily increasing through M0 to M8, where they are exceedingly strong. A second set of stars shows bands of carbon (or some carbon compound not yet identified) and of cyanogen, increasing in strength from R0 to R5, N0, and N3 (cf. Figs. 215, 216).

In each case the sequence is continuous, and clearly leads back, at the point where the bands first appear, to a spectrum of Class K2 or thereabouts; but not a single star is known in which both the titanium and the carbon bands appear. They seem to be mutually exclusive.

Class S, with the bands of zirconium oxide, appears to form a third branch of the sequence, though its relations are not fully worked out. The stars of all these branches are red, — redder indeed than those of any other spectral classes, — and all are undoubtedly of low temperature.

A possible cause of this branching of the linear spectral sequence among the red stars has been suggested by R. H. Curtiss. At these low temperatures, chemical compounds can form, as the band spectra prove. If the carbon bands are due to some unoxidized carbon compound (as now appears to be very probable), they should disappear in an oxidizing atmosphere, while titanium oxide should be formed. In a reducing atmosphere titanium oxide should disappear and the carbon bands appear. The difference between the two main branches of the sequence may therefore correspond to this familiar chemical distinction. With an increasing amount of oxygen the carbon bands would be weakened; but only after they had practically vanished would there be any oxygen left over to combine with titanium and give the oxide bands. This explains why the two sets of bands are never found together, and also why the strength of the bands is very different in stars of the same color and temperature. In stars of higher temperature, where all compounds are dissociated, and only the lines of the elements appear, the changes in intensity of these lines, due to such relatively small differences in composition, would be inconspicuous.

The bands of zirconium oxide and titanium oxide are found together in a few stars of Class S. In this case there should be no chemical incompatibility. The zirconium bands appear to hold out to a higher temperature than those of titanium, indicating that the oxide of the former is harder to decompose.

The appearance of bands due to different compounds in different stars is of philosophical interest. It marks the first appearance, with falling temperature, of the integration of the simple into the complex, which may be followed in nature, through the multiplicity of forms which can exist when solid bodies appear, to the still greater complexity which culminates in living organisms.

930. Effects of Pressure on the Spectrum. The densities of stellar atmospheres are so low that such differences of pressure as exist among them probably have very little direct influence on the spectra in the way of widening or shifting lines, but the

pressure differences produce a very important indirect effect by altering the amount of ionization.

Low pressure always favors ionization, whether of the first or of a subsequent stage. Hence, if we could compare two sets of stars ascending through the same ranges of photospheric temperature, but one with high and the other with low atmospheric pressure, we should find that all phenomena which depend on

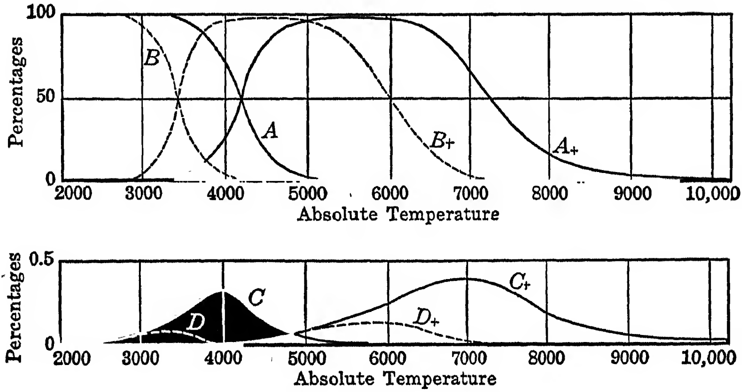


FIG. 299. Ionization of Calcium

The curve marked A shows the percentage of calcium atoms which will be in the neutral state (absorbing λ 4227), and the curve $A+$ shows the percentage in the singly ionized state (absorbing $\lambda\lambda$ 3933, 3968), in an atmosphere at various temperatures and at an electron pressure of 10^{-5} atmospheres. Ionization sets in strongly at about 3500° K. Curve A falls, and $A+$ rises. At about 6000° double ionization begins to be important, and $A+$ falls. The dotted curves B and $B+$ correspond to a pressure a hundred times less (10^{-7} atmospheres), and ionization sets in at a lower temperature. Curve C shows the percentage of all the atoms present which are neutral, but in an excited state, and which can absorb λ 6162 etc.; and curve $C+$ that of the atoms which are ionized and excited, absorbing λ 3737 etc. (both at the higher pressure), while D and $D+$ correspond to the lower pressure. The vertical scale here is different, as indicated; and the number of atoms in these states never reaches 1 per cent of the total. All these curves were computed from the theory of ionization and excitation described in sections 645 and 930

ionization would occur at lower temperatures in the stars where the pressure was low. The ultimate arc lines would fade out sooner; the subordinate arc lines would reach their maximum intensity at a lower temperature; and both ultimate and subordinate enhanced lines would appear, reach their maxima, and disappear, at lower temperatures.

These effects are illustrated in Fig. 299, which shows how the number of calcium atoms, in a state to absorb the most promi-

nent ultimate and subordinate arc and enhanced lines, should vary theoretically with the temperature at electron pressures of 10^{-5} and 10^{-7} atmospheres. The shift of the maxima is conspicuous.

931. Determination of Pressure in Stellar Atmospheres. Since the temperature at which a spectral line reaches its maximum intensity depends on the pressure, and the former can be found from observation, the latter should be calculable.

This was first pointed out by Fowler and Milne, whose calculations showed that the pressures were in all cases very low. Their study has been repeated with better observational data by Menzel, and by Miss Payne.

The pressure found from a study of any given line will be some sort of average of the pressures at the depths in the stellar atmosphere to which it is possible to see down with light of the corresponding wave-length. It is not surprising, therefore, that different lines indicate very different pressures. Ultimate enhanced lines (which are absorbed, when near their maximum intensity, by almost all the atoms of the corresponding element) give very low pressures, of the order of 10^{-9} or 10^{-10} atmospheres. Subordinate lines, whether of neutral or ionized atoms, lead to much higher pressures; but even at their strongest these lines are absorbed by only a small fraction of all the atoms of the element. If, for example, only one atom in a thousand is active in producing a line, we might expect to be able to "see down" in it through about a thousand times as much material, and to find the corresponding pressure about a thousand times as great as for an ultimate line.

It appears from these studies that the pressures in the reversing layers of the stars range from about 10^{-10} atmospheres at the top to 10^{-4} atmospheres at the bottom (this being the greatest value found for subordinate lines). These values are in excellent agreement with those found from different considerations in the case of the sun (§§ 590, 660–662). All the arguments previously employed — from the sharpness and narrowness of the spectral lines, the opacity of ionized gases, and the like — apply to the stars as well as to the sun, and all give consistent results. The method sketched above, however, appears to be best adapted to quantitative calculation.

In applying it the assumption is made that the average pressure in the atmospheres of the stars of different spectral classes is the same. This is probably at best very roughly correct. The results may be taken to represent average values for the giant stars, since almost all those whose spectra were studied were giants.

932. Levels of Origin of Different Lines. Throughout the spectral sequence the lines of neutral and ionized atoms overlap much more than is predicted by the elementary theory of section 930. For example, in Class G0, where the ultimate lines of Ca + are weakening, which indicates that much of the calcium is already doubly ionized, the lines of neutral calcium are still strong. In an atmosphere all at one pressure this would be hard to explain, but in the actual atmosphere the pressure is probably a million times greater at the bottom than at the top. Neutral calcium may therefore remain in considerable quantity at the bottom, while the calcium at the top is doubly ionized to a high degree. This localization of enhanced lines at high levels, and of arc lines at low levels, is well known in the sun.

933. Temperatures of the Hottest Stars. For the hottest stars, of Class B0 and earlier, the determination of effective temperature from color-index becomes very inaccurate (§ 812). Indeed,

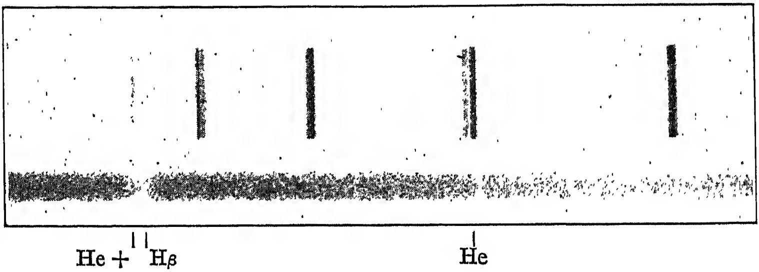


FIG. 300. Spectrum of 10 Lacertæ (Class O9)

The line of ionized helium is close to Hβ. The hazy lines can be best seen by viewing the figure at arm's length. (From photograph by Dominion Astrophysical Observatory)

the O-stars are observed to be slightly yellower than those of Class B0, which would indicate a lower temperature. The spectroscopic evidence is, however, conclusive that their temperatures must really be much higher.

The best way of getting at them is by reversing the procedure of section 931, estimating first the pressure under which the lines of various elements are produced, and then finding at what temperatures, with these pressures, the lines should be of maximum intensity. For stars of Class B3 Miss Payne thus finds a temperature (for the atmospheres) of 16,000° (from the maxima of the lines of He and C +); for the later O-stars, 25,000° (maxima of Si + + +); and for the earlier O's, 35,000° (maximum of He +). The temperatures of the photospheres should be 20 per cent higher.

The reason for the yellowish color of the O-stars is unknown.

934. Abundance of Different Elements. Uniformity of Composition of the Stars. It is evident that the lines of an abundant element should appear earlier as the temperature rises (except for the ultimate arc lines, which are there at the start), should be stronger at maximum, and should hold out longer at high temperatures than comparable lines of a rare element. From the conditions under which the lines are at the point of appearance or disappearance it is possible to calculate the relative abundance of the elements which produce them, — on the reasonable assumption that the number of atoms concerned in producing a just visible line is always nearly the same. It is found in this way that very much the same elements are abundant in the stars as in the sun or in the earth's crust — silicon, sodium, magnesium, calcium, aluminum, iron. The relative strength of the lines of different elements in stars of the same spectral class is remarkably similar, and the differences in different classes are fully explicable by the theory of ionization and temperature excitation. From these considerations Miss Payne concludes that "the uniformity of composition of stellar atmospheres appears to be an established fact."

There must be a reason for this uniformity (possibly one connected with the stability of the atomic nuclei), but its discovery is a matter for the future. In a very few stars the lines of some one element (strontium, silicon, or occasionally barium) are so much stronger than normal as to suggest that the element is unusually abundant; but some other explanation may be the true one, and these cases are so rare as to emphasize the general similarity of composition.

935. Behavior of Hydrogen and Helium. The hydrogen lines are puzzling. They appear even in the coolest M-stars and are still visible in the hottest O-stars, where nothing else but ionized helium can be detected. This indicates that hydrogen must be very abundant in stellar atmospheres.

Rosseland has shown that hydrogen ought to be concentrated at the surface of a star, since the minute electric field in the interior, which is necessary to keep the heavy metallic ions from settling down away from the very light electrons (§ 655), will overbalance gravity in the case of the relatively light hydrogen nuclei and pull them toward the surface.

But the excitation potential for the Balmer series is so high that, according to the theory outlined in section 643, only a very small fraction of all the hydrogen atoms in the atmosphere, even of an A-star, should be in a condition to absorb these lines. Unless some unrecognized influence is at work,

it is not easy to see how so small a proportion of excited atoms can produce the strong lines which are observed.

Helium, which has still higher ionization and excitation potentials, behaves in the same way, showing much stronger lines than the elementary theory predicts, even for an atmosphere composed largely of helium. Oxygen and nitrogen appear to do so, too, to a less degree.

936. Differences of Pressure in Different Stars. There can be no doubt that the pressures in the atmospheres of individual stars must differ widely from the rough averages mentioned in section 931. The principal controlling factor is the surface gravity; and this, as we have seen in Chapter XXI, varies over a wide range, being probably ten thousand times greater in the red dwarfs than in some red giants, and a hundred times greater still in the white dwarfs.

The average depth to which we can see with a given wave-length not corresponding to a dark line defines the position of the photosphere. If this depth depended only upon the quantity of material per unit area through which the outgoing light passed, this quantity would be the same in all stars; but there is good reason to believe that when the density is low we can see down through a greater quantity of material. For giant and dwarf stars of the same temperature, but of different surface gravity, we may therefore expect that the quantity of matter above the photosphere will be greater in the giant. The pressure in the photosphere will, however, be less, for the increase in the amount of material does not compensate for the diminished force of gravity. The depth of the atmosphere in miles will of course be much greater for the giant, but even so it will be but a negligible fraction of the diameter of the star.

The giant star should therefore have an atmosphere containing more material, but at lower pressure and density than a dwarf of the same photospheric temperature (cf. § 938).

937. Differences in Color between Giants and Dwarfs. In working out the effects of these differences upon the spectra it must be borne in mind that the characteristics by which the various spectral classes are distinguished depend almost entirely upon ionization and dissociation, — the weakening and disappearance of band spectra, the fading out of arc lines and the appearance of enhanced lines, the maximum intensity of subordinate lines, and so forth. In an atmosphere of low pressure all these phenomena should occur in the same order, with rising temperature, as in one of higher pressure; but every stage

should be reached at a lower temperature. We should therefore expect to find the spectra of giant stars very similar in general appearance to those of dwarfs, but not to those of dwarfs of the same temperature, the giants, for the same stage of excitation, being cooler and redder.

Exactly such a difference is shown by the observed color-indices of giants and dwarfs of the same spectral classes. These diverge at Class F5 (where the two kinds of stars begin to be separable) and differ more and more for the redder stars until the problem is complicated by the appearance of band absorption in Class M. The extreme difference in the computed temperatures (according to Seares's data) amounts to 20 per cent, which accords with the amount predicted theoretically.

When we classify stars by means of their spectra, we are then primarily classifying them by the *degree of ionization in their atmospheres*; and the differences in temperature among those so grouped together almost compensate for the differences in pressure. The effects of temperature, however, are so much more important than those of pressure, when the whole extent of the spectral sequence is considered, that this is quite legitimately described as a temperature sequence.

938. Differences in Spectra between Giants and Dwarfs. Although, in the main, the effects of differences of pressure and temperature between giants and dwarfs cancel one another, they do not do so in detail; this is a very fortunate circumstance, for on it depends the possibility of the spectroscopic determination of parallax. Two kinds of such residual differences may be anticipated.

(1) The compensation of a decrease in pressure by a decrease in temperature cannot be exact for all elements at the same time.

From the fundamental equation of the ionization theory (§ 645),

$$\log \frac{x}{1-x} = -\frac{5048\, I}{T} + \frac{5}{2} \log T - 6.5 - \log P_e,$$

it follows that a decrease in the electron pressure P_e increases the degree of ionization (that is, the ratio $\frac{x}{1-x}$ of the number of ionized atoms to that of neutral atoms) in the same proportion for all elements, while a decrease in the temperature T decreases this quantity in different proportions, the change being greater when the ionization potential I is high and less when it is low.

For elements of easy ionization the compensation by the temperature difference will be inadequate, while for those of difficult ionization it will be overdone. In passing from a dwarf to a giant star we should expect, therefore, that easily ionized elements should be more ionized in the giant, and difficultly ionized ones less ionized, while those of intermediate properties should be little affected (Fig. 301). This accords, in general, with the observed behavior of the "absolute magnitude lines." The lines of neutral calcium and of strontium, for example, are strong in the dwarfs and weaker in the giants, while the reverse is true of their enhanced lines. These elements are easy to ionize, so that this is what might be expected. To remove a second electron from an ionized calcium atom takes much more energy, and this process is somewhat inhibited in the giants. Most of the other cases are similarly explicable.

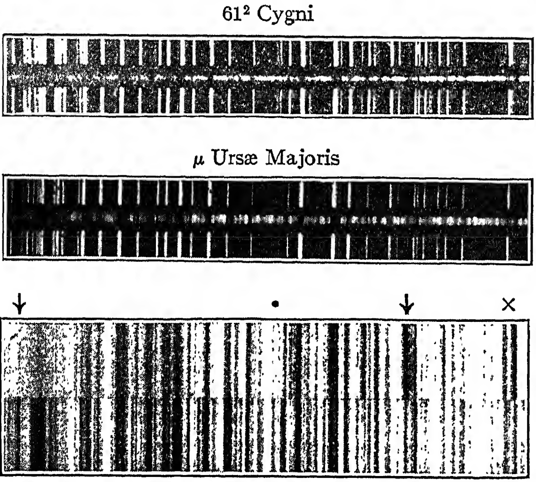


FIG. 301. Spectra of Giant and Dwarf Stars

These stars are both of spectral Class K5; the absolute magnitude of 61² Cygni (the dwarf) is 8.7, and of μ Ursæ Majoris 0.7. The two spectra as originally photographed are shown above. Those below are mechanically widened. The line marked with the arrow at the left is λ 4215 (Sr +), which is very much stronger in the giant than in the dwarf. The arrow toward the right indicates λ 4454, a low-temperature line of neutral Ca, which is greatly strengthened in the dwarf. The cross at the right marks a close group of low-temperature titanium lines at λ 4535, which are strengthened in the dwarfs. In the middle of the group is an enhanced line of titanium, which is stronger in the giant. The dot marks a line of ionized yttrium at λ 4376, greatly strengthened in the giants. (From photographs at Mt. Wilson Observatory)

(2) We have seen that the atmosphere of a giant star probably contains more material per unit surface than that of a dwarf, and also that increase in the number of absorbing atoms above the photosphere increases the

strength of the spectral lines. We should therefore expect the spectral lines to be stronger in the giants than in the dwarfs, and this is actually the case. The lines in the spectra of giants are also sharper, which is not surprising in view of the lower density.

The cyanogen bands in Class G and thereabouts are much strengthened in the giants; indeed, this is the most conspicuous of all the differences, and may be seen with low dispersion (Fig. 251). This has recently been explained (1926). These bands are the "ultimate lines" of the fragmentary molecule CN which is produced by the dissociation of the ordinary cyanogen molecule C_2N_2. Hence, among the cooler stars, the bands increase in strength with rising temperature, and with low pressure. At temperatures higher than the sun's the bands fade out, showing that the CN molecules must either dissociate into the elements or become ionized.

939. Spectra of c-Stars. The c-stars, or super-giants, are the brightest of all (§ 807). Their spectra exhibit the giant charac-

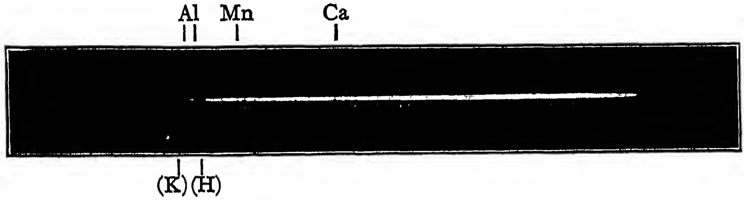


Fig. 302. Spectrum of 61 Cygni

The ultimate lines, such as those of Al, Mn, and Ca, which are marked on the photograph, are very strong. The (H) and (K) lines of Ca + (enhanced lines) are relatively weak, and reversed, having conspicuous bright centers. (From photograph at Mt. Wilson Observatory)

teristics in an exaggerated degree, and are unmistakable in appearance. The enhanced lines of the metals, especially of iron and titanium, are much stronger, and the arc lines weaker; the hydrogen lines are strengthened in the redder stars, and those of helium (which usually vanish at B9) appear in α Cygni, which is of Class A2. All the lines are unusually intense and sharply defined. These stars are decidedly redder than the average for their spectral classes.

940. Spectra of Dwarf Stars. The redder dwarfs, which are so faint that they can be observed only with the most powerful instruments, show spectra characterized by great strength of the ultimate arc lines and by faintness of the enhanced lines and even of subordinate lines of high excitation potential. Thus, in the M3 star, Lalande 21185 (Fig. 303), the ultimate lines of

aluminum (λλ 3944–61) are stronger than the (H) and (K) lines of Ca + (λλ 3933–68), and the ultimate titanium lines (λλ 5210, 5193, 5174) are stronger than the subordinate triplet of magnesium (λλ 5183–73–67), so that the whole appearance of the spectrum is altered. The lines are also broad and hazy. Similar, but less marked, peculiarities appear in 61 Cygni (Fig. 302).

No dwarf stars are known with spectra more advanced than M6, doubtless because of the extreme faintness of still cooler stars; nor have any dwarfs of classes R, N, or S been found. This may be because stars of these classes are very few in number, or perhaps because the higher atmospheric pressure in the

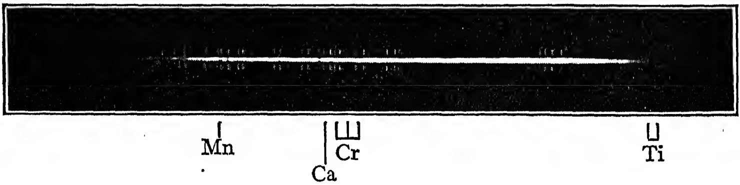


FIG. 303. Spectrum of the Dwarf Star Lalande 21185

The ultimate lines of Mn, Ca, Cr, and Ti are exceptionally strong. (From photograph at Mt. Wilson Observatory)

dwarfs changes the conditions of chemical equilibrium to the advantage of the titanium bands. (Compare section 929).

The colors of dwarf stars, as Hertzsprung has shown, grow steadily redder with decreasing luminosity, down to about the eighth absolute magnitude. Beyond this they change very little. This similarity in color is probably due to increasing band absorption in the red among the cooler stars (§§ 814, 848).

It has also been suggested that, in these faint stars, clouds of condensed material cover part of the surface, and that almost all the light comes from the gaps between them; but it is doubtful whether the pressure is high enough, even in these stars, to permit condensation.

941. Physical Meaning of Spectroscopic Parallaxes. Those differences in spectra which depend upon the pressure in stellar atmospheres are successfully employed to give "spectroscopic parallaxes" (§ 805); that is, to determine the absolute magnitudes of the stars, and consequently their distances. The reason why this method is practicable deserves close consideration.

The characteristics of a star's spectrum, we have seen, depend upon two physical quantities: first, the rate at which heat is supplied to, and escapes

from, its surface (this determines the temperature of the photosphere and atmosphere, and the color-index); second, the force of gravity at the surface (this determines the pressure and density in the atmosphere, and the spectral characteristics in which giants differ from dwarfs). The spectral class is determined by both jointly, but mainly by the former.

Any two stars for which the outgoing flux of energy, J, and the surface gravity, g, are the same should, then, have identical spectra, even in detail (barring possible effects of differences of chemical composition); and consequently the "spectroscopic absolute magnitude" should be the same for both. But if M is the mass, E the total energy radiation, and r the radius, $E = 4\pi J r^2$, and $g = GM/r^2$, or $M = gr^2/G$, where G is the gravitational constant.

A star of *any* given total radiation, E, should show the spectrum under consideration, provided that its radius and mass were determined from these two equations, with the appropriate values of J and g.

Why, then, do we find that stars selected in the way described have actually almost the same absolute magnitude, that is, the same value of E? The reason is that there exists a relation between the mass and absolute magnitude of actual stars, clearly shown by observation (§ 772) and capable of theoretical explanation (§ 959). According to this relation M increases with increasing E, but much more slowly; so that the ratio E/M depends upon E and is greatest when E is large. Now, from the above equations

$$E/M = 4\pi GJ/g,$$

so that, for the group of stars under consideration, E/M must have a fixed value. This value will correspond to some definite value of E, that is, to a definite absolute magnitude.

If the relation between E and M were exact, all stars which have identical spectra would have exactly the same luminosity, and also the same mass. But it is probably only statistically true, so that while most of the stars with a given E will have values of M close to the average, a few will differ more widely. For these exceptional stars the spectroscopic method, which is adjusted to give correct results in the average case, will yield an erroneous absolute magnitude and parallax. The excellent general agreement between the spectroscopic and trigonometric values shows that such cases are decidedly rare.

A few of these stars, however, undoubtedly exist, the most notable being Arcturus, for which the spectroscopic parallax determined at Mt. Wilson is $0''.16$, at Victoria $0.''10$, and at Harvard $0''.21$, while the accurate trigonometric parallax is $0''.080 \pm 0.005$. The latter makes the actual absolute magnitude -0.2, while the spectroscopic value is $+1.1$ (from the mean of the three determinations). Arcturus is therefore about three times as bright as the average of the stars showing the same details of spectrum; and since for such stars E/M is constant, it must be three times as massive. Since from the observed relation between absolute magnitude and mass the mean

mass of stars of absolute magnitude + 1.1 is about 2.7 times that of the sun, the mass of Arcturus may be estimated at 8 times the sun's. The mean mass of stars of absolute magnitude − 0.2, however, is 4.2, so that Arcturus is only about twice as massive as the average star of the same actual brightness.

As the discrepancy of the spectroscopic parallax of Arcturus is conspicuous, it would appear that the similarity in mass among stars of the same absolute magnitude must be close.

942. Continuous Absorption in Stellar Spectra. In spectra of Class A the intensity of the continuous background in the ultra-violet drops abruptly near the limit of the hydrogen series (λ 3646), and continues low toward shorter wave-lengths. Comparison with spectra of classes F and B shows that this effect rises or falls with the intensity of the hydrogen lines, and makes it clear that it is actually due to hydrogen. Its appearance is that of a general continuous absorption, beginning at the last visible hydrogen line and extending with gradually diminishing intensity beyond the ultra-violet limit of ordinary observations (about λ 3300).

This absorption is clearly due to photo-electric ionization (§ 636) of excited hydrogen atoms, an electron being lifted from the two-quantum state and sent flying away from the atom with kinetic energy to spare. This energy may be of any amount, though more usually small than large; and the absorption is therefore continuous but decreases toward the ultra-violet.

The absorbed energy may be transformed back to light by reversal of the process, a hydrogen ion picking up a free electron and emitting radiation of shorter wave-length than the series limit. Such continuous emission spectra have been observed by Evershed in the solar chromosphere, and by Wright in the gaseous nebulæ.

Continuous absorption of this sort may be strong beyond the limits of the principal series of all the elements, but these all lie at too short wave-lengths to be observed in the stars. No other subordinate lines are anything like as strong as the Balmer series, so that it is not surprising that no other case has yet been observed. Milne, however, has calculated that the overlapping effects of absorption of this sort by atoms of many kinds and in many states may account for most of the opacity of the solar photosphere (§ 661).

The observed effect in the case of hydrogen is so strong, and extends over such a wide range of wave-length, that it probably involves a larger depletion of the outgoing radiant energy than the sum total of all the Balmer lines; that is, more energy is spent in the photo-electric ionization of the atoms than in changing them from one quantum state to another.

943. Bright Lines in Stellar Spectra. *The Hotter Stars.* A few stars (less than one in two hundred, on the average) show bright lines in their spectra, more intense than the neighboring continuous background. In almost all cases only a few lines are bright, the rest being dark, as usual, and permitting classifica-

tion of the spectrum in the ordinary manner. More than 95 per cent of the bright-line spectra belong either to classes O and B or to classes M, S, and N, that is, to the hottest or to the coolest stars. In classes A, F, G, and K bright lines are very rarely found, though not entirely unknown.

The most conspicuous bright lines are almost always those of hydrogen, although lines of helium and of various ionized atoms sometimes show bright in the hotter stars, and arc lines of the metals in the cooler stars. A remarkable feature of such spectra is that in many cases the bright lines vary in intensity; in some the variation is periodic, in others apparently irregular. These

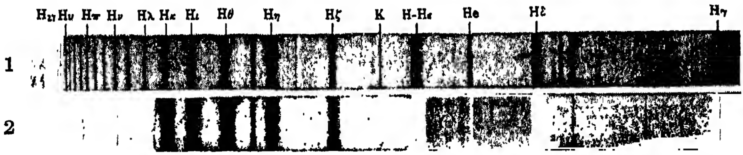


FIG. 304. Spectra of ζ Tauri and 11 Camelopardalis

From photograph at the University of Michigan

changes are often, though not always, associated with variability of the star's light. Indeed, bright lines are nearly always present in the spectra of variable stars of certain types (§ 846).

Spectra which contain bright lines ("emission lines") are denoted by the letter e, following the usual symbol. Thus, η Tauri is described as B5e. Cases in which the bright lines are variable are denoted by ev. Thus, *o* Ceti is M6ev.

The hydrogen bright lines are much more frequent in the early subdivisions of Class B (B0, B1) than in the later (B8, B9). The first line, Hα, is always the brightest, and successive lines grow fainter, so that the ultraviolet ones well on in the series are always dark (Fig. 304). Sometimes, as in η Tauri (Alcyone), only Hα is bright; again, as in γ Cassiopeiæ or 11 Camelopardalis, several lines are bright.

The bright lines are nearly all superposed on wider dark lines, which grow stronger in the later members of the series as the bright lines fade. Sometimes the bright lines themselves are double, as if reversed at the center. These complex structures are usually symmetrical about the normal position of the line; but in a few cases (the type being a fourth-magnitude star, P Cygni) the bright line is entirely on the long-wave side of the dark line, and both are displaced in opposite directions from the normal wavelength. This peculiar type of spectrum, which shows some resemblance

to those of the temporary stars, is known as Beq. The most luminous known star, S Doradus (§ 872) possesses it.

Bright lines other than those of hydrogen (usually of helium or ionized iron), sometimes appear, and are occasionally prominent. The bright lines sometimes fade out, after being visible for years, and in other cases appear where the lines were previously dark. These changes are usually irregular.

In Class O a majority of the spectra contain bright lines, though some show dark lines only, and the bright lines are often faint, so that the absorption lines predominate. In the Wolf-Rayet stars, however, the bright lines are far more prominent than the dark lines, and in a few such stars no dark lines have been observed. The emission lines are often enormously widened, and appear as broad bright bands, the most conspicuous of which are at λ 4686 (He +) and λ 4650 (C + +). All the lines, bright or dark, indicate very high excitation, O + +, N + +, and Si + + + being present, along with helium and the ubiquitous hydrogen. Why the emission bands should be so wide and so bright is still unknown.

From the rather scanty data available the O-stars appear to be much more luminous and more massive than even those of Class B. Like the latter they do not show any great tendency toward conspicuous variability.

The temporary stars, or novæ, show very remarkable spectra, full of very wide bright lines or bands. These spectra are put in a class by themselves (Class Q). They can best be discussed when dealing with the novæ (Chapter XXII).

944. Bright Lines on the Cooler Stars. In the spectra of red stars of classes M, N, and S bright lines often appear, — and are nearly always associated with conspicuous variability of light. Great numbers of variable stars have indeed been discovered in this way.

When such a star is near its maximum brightness, the hydrogen lines in its spectrum are very bright; those in the blue and violet ($H\beta$, $H\gamma$, $H\delta$) are much more prominent than $H\alpha$. Several of the ultra-violet hydrogen lines are usually also bright; but $H\epsilon$, which falls inside the great winged (H) line of Ca +, is absent or very faint. As the star's brightness diminishes, the hydrogen emission fades out, to be replaced near minimum (at least in *o* Ceti, the brightest and best-studied star of this sort) by emission lines belonging to the arc spectra of the metals. These are ultimate lines of metals of rather high ionization potential, such as Mg and Fe. Lines of the easily ionized metals remain dark (Fig. 263). These changes are reversed as the star brightens again.

Various other variable stars show bright lines in their spectra, especially at minimum. Stars of the later spectral types which do not vary in light rarely show bright lines. In a few stars, as in σ Geminorum (Class K0), the (H) and (K) lines show narrow bright central reversals. Deslandres has found that fainter reversals appear in Arcturus and Aldebaran, and Adams has found strong ones in 61 Cygni, a typical dwarf of Class K8. A few M dwarfs show bright hydrogen lines. The remaining scattered cases in which

bright lines appear in spectra of classes A to K can hardly as yet be brought under any rule, — perhaps because their numbers are so few.

945. Physical Interpretation of Bright-Line Spectra. The explanation of the bright lines presents very difficult problems. The existence of a line brighter than the continuous background means that the star's atmosphere emits in this wave-length more strongly than the photosphere. There is good evidence that the latter radiates substantially like a black body of the same effective temperature, and that the atmosphere does not extend so far above the photosphere as to present a significantly greater area. For the wave-lengths involved in a bright line, then, the atmosphere acts as if it were hotter than the photosphere; but that it is actually hotter is most improbable, as its outer boundary is exposed to the cold of space. Moreover, not all the spectral lines are bright, but usually only a selected few. There must therefore be some special process by which energy is fed into the particular atomic transitions which produce the emission lines. The observed variability of emission lines suggests that the process is complex and rather delicately balanced.

The bright hydrogen lines in Class B may be provisionally explained as follows: Numerous hydrogen atoms are photo-electrically ionized by the absorption of radiation in the extreme ultra-violet, — beyond the limit of the Lyman series at λ 912, — or are raised to higher energy states by absorption of lines in this series. These ionized or excited atoms may return to normal in many ways, the electrons sometimes dropping first into a higher quantum state, and then back by successive steps to the lowest, emitting the corresponding spectral lines. Every such transition from a higher state to the second will result in the emission of some line in the Balmer series. This emission will represent but a small fraction of the energy absorbed in the ultra-violet, but it may nevertheless exceed the emission by the photosphere in neighboring wave-lengths, which, in a very hot star, is much smaller in the visible than in the ultra-violet.

The line emission should be stronger for $H\alpha$ than for $H\beta$ and so on, while the continuous background is fainter in the red. The progressive weakening of the emission lines from $H\alpha$ onward is thus explicable.

Such "fluorescent" emission will not occur unless there is considerable photospheric radiation to be absorbed in wave-lengths less than 1000 angstroms, which is true only in very hot stars. The fact that bright lines of this type are confined to classes O and B, and are most numerous in the earlier and hotter subdivisions of the latter, is thus also explained.

The atmospheres of the stars which show bright lines are probably of great extent. The origin of the emission lines in the spectra of the long-period variables and other stars of low temperature is not yet understood.

946. "Stationary" Lines and their Explanation. The (H) and (K) lines of ionized calcium and the (D) lines of sodium behave in a peculiar manner in the spectra of certain stars of Class B.

They appear in these spectra, although the star's temperature is so high that one might expect them to have vanished altogether, owing to increasing ionization; they are sharp and narrow, even though all the other lines are wide and diffuse, and, when the star is a spectroscopic binary, they do not share in the periodic changes of wave-length arising from the radial velocity, but remain stationary, showing a radial velocity which sometimes agrees with that of the binary system, but in other cases differs considerably from it. These lines can be observed only in spectra of Class B, since in the later types they are overwhelmed by the heavy lines which are normally present in the spectrum.

Their interpretation involves some difficulty. All their characteristics suggest, — indeed, almost compel, — the belief that these lines originate, not in the star's atmosphere, but in a region far enough from it to be free from the disturbing effects of orbital motion, and in a very rarefied gas at a temperature much lower than that of the star's surface.

Such absorbing "clouds" may surround the star, or may be anywhere else in space on the line of sight. Their radial velocity, after correction for the sun's motion in space, is usually small, and tends to be similar for stars close together in the sky. There is also a tendency, as Struve has shown, for the stationary lines to be strongest in stars such as those of small proper motion, which are presumably most distant.

Eddington has recently suggested that these "clouds" may be composed of atoms of sodium, or of ionized calcium, which have been ejected from the stars by radiation pressure (§ 657) and are wandering in interstellar space, — so far apart that there is practically no chance that the ions may meet electrons and become neutralized. This bold hypothesis appears to give a good account of the facts.

II. THE INTERIOR OF A STAR

947. The portion of a star which is accessible to direct observation is but a very small fraction of the whole. The vast interior is beyond our reach. We know the size and bulk of a star, the total mass within it, and the rate at which heat is emitted from its surface. From these rather scanty data we must find out all

we can about the internal constitution. We can penetrate to these depths only by the use of what Eddington, in a whimsical mood, has called an "analytical boring machine," that is, by the application of general physical and mathematical principles to the problem.

Twenty years ago very little could be done toward a solution. It was easy enough to show that the pressure deep within a star must be counted in hundreds of millions of atmospheres, and that the internal temperature would be many millions of degrees — provided that the physical laws, deduced from terrestrial experiments, still held true under such extremely different conditions; but no one knew then whether it was safe to reason on this assumption.

With our present knowledge of atoms and their properties, confidence has replaced doubt, and well-founded conclusions are possible, — as has already been explained in discussing the constitution of the sun (§§ 666–668).

Though in a star's interior the atoms are stripped of all but their innermost electrons, they retain their identity, and the material must behave almost like the "perfect gas" of elementary theory, even at great densities.

The mean molecular weight (including the free electrons in the count) can be calculated by the theory of ionization if the pressure, density, and chemical composition (that is, the relative numbers of atoms of different kinds which are present) are given. Recent detailed calculations by Fowler and Guggenheim show that it is almost independent of the composition (unless the star is composed largely of hydrogen or helium), and that its value in the central region of the stars is usually a little greater than 2. It increases toward the surface, slowly at first and rapidly in the outer layers; but it is probably less than 3 throughout the main mass of the star. Radiation in the interior is enormously intense. It consists mainly of very short waves, such as would ordinarily be called soft X-rays, and exerts a pressure which cannot be neglected, and which may be very great.

Strange as some of these conditions seem, they combine to make the problem more tractable. The properties of matter are exhibited in their elementary forms, the laws which they follow are few and simple, and corresponding advances have been

made toward a solution, with the result that the remarkable relation which exists between the masses and the absolute magnitudes of the stars (§ 772) has been satisfactorily explained.

A general account of the character of the investigation, and of the results which have been reached, will first be given, followed, at the end of the chapter, by a more detailed discussion (§§ 962–966), which may be omitted by the elementary student.

948. Mechanical Equilibrium. The first principle to be applied is that the star must be in mechanical equilibrium, the inward pull of its own gravitation being balanced by the net outward pressure of the gas and of radiation. That is to say, the pressure at every point must be just enough to support the weight of the overlying layers.

Because of the symmetry of the problem, a star which is not rotating must be spherical, and the density must be the same at equal distances from the center. (Rotation introduces ellipticity of figure, and various other complications not considered here.) The pressure must increase inward, and, in general, the density will do so too. If we know the way in which the density varies with the distance from the center, the pressure can be calculated.

First, the mass inside a sphere of any given radius is found (starting at the center and working outward step by step), then the force of gravity at different distances from the center, then the weights of successive concentric layers, and finally (adding the latter, and working inward from the surface), the pressure at any depth.

Such a calculation of pressure is valid inside any spherical body, — solid, liquid, or gaseous, — provided only that it is in internal equilibrium. It would not, of course, apply to a body which was undergoing changes of shape or size, — such as probably occur in the case of the Cepheid variables. The theory of such oscillations is much more complicated.

949. The Gas Laws. If the mass is gaseous, it is possible to calculate the internal temperature, as well as the internal pressure, when the law of density is given. There is good reason to believe that the properties of the ionized gas, under the conditions which prevail inside almost all the stars, deviate very little from those of a "perfect" or ideal gas. For such a gas the "equation of state" is

$$p = \frac{R}{m} \rho T,$$

where p is the gas pressure, ρ the density, T the absolute temperature, m the mean molecular weight, and R a constant, the same for all gases. Now at any given point within the gaseous mass the pressure and density are already known, the molecular weight can be calculated from the ionization theory, and the temperature then follows.

The ionization, and hence the mean molecular weight, varies with the temperature, so that an exact solution of the equation must be obtained by trial and error; but it is none the less definite. On the other hand, the value of m under stellar conditions is almost independent of the chemical composition of the gas, — which greatly simplifies the problem.

For any arbitrarily assigned law of internal density, then, it is possible to calculate values of the pressure and temperature at all points such that, for these values and no others, the star will be in mechanical equilibrium throughout its interior.

950. Homologous Spheres of Gas. To determine what is the actual law of density within the stars is a very difficult problem, which is at present far from being solved.

It is, however, relatively easy to calculate many of the properties of gaseous spheres which, though of different sizes and masses, have their internal density distributed according to the same law; and the results are of much astrophysical importance. Such stars are said to be *homologous* or, sometimes, to be "built on the same model." Points within them whose distances from their centers are the same fractions of the radii are called *corresponding* points.

At any two corresponding points the densities will be the same fraction of the mean density of the star; thus, for example, in one model of considerable theoretical importance (§ 953) the density at the center is 54 times the mean density, one quarter of the way to the surface it is 15 times, and halfway out 1.2 times the mean density, while three quarters of the way to the surface it is only 1/20 of the mean density. The actual condensation toward the center in the stars, though perhaps not so great as in this particular model, is probably very considerable.

951. Lane's Laws. Homologous stars have certain important properties, which were first worked out by Lane in 1870. He showed that, at corresponding points within any number of such

stars, the densities vary as M/r^3, the pressures as M^2/r^4, and the temperatures as mM/r, where M represents the star's mass, r its radius, and m the molecular weight of the gas at the point considered within it.

If the radius only is altered, the density varies as $1/r^3$, the pressure as $1/r^4$, and the temperature as $1/r$. This last relation is commonly known as Lane's law, but the more general relations deserve equally to bear his name.

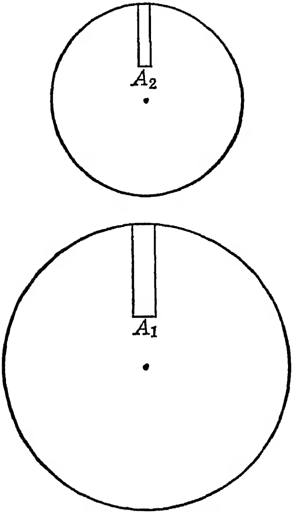


FIG. 305. Homologous Stars

The proof of Lane's laws is simple. Consider any two homologous stars of radii r_1 and r_2 and masses M_1 and M_2, and two corresponding points A_1 and A_2 within these stars, and let ρ_1 and ρ_2 be the densities at these points. These will, by the definition of homology, be in the ratio of the mean densities of the two stars; that is,

$$\rho_1 : \rho_2 = M_1/r_1^3 : M_2/r_2^3. \tag{1}$$

Now let P_1 and P_2 be the pressures at these two points. Each of these is equal to that which would be produced by the weight of a vertical column of material extending upward to the surface of the star. If w is the weight of such a column, and s its cross-section, then $P = w/s$. We may suppose the columns also to be homologous (Fig. 305), their dimensions being proportional to those of the stars. The masses m_1 and m_2 of these columns will then be proportional to M_1 and M_2. To find the weights, w_1 and w_2, of these columns, we should have to divide them into small portions, or "elements," calculate the attraction of the mass on each one of these, and add the results. These elements themselves, however, can also be made homologous, as can also the elements into which, in a general solution, the attracting body must be supposed to be divided.

If m_1, m_1', are the masses of any pair of elements in the first star, and d_1 their distance apart, their mutual attraction is $f_1 = G\,m_1m_1'/d_1^2$. For the homologous elements of the second star $f_2 = G\,m_2m_2'/d_2^2$. But $m_1 : m_2 = m_1' : m_2' = M_1 : M_2$, and $d_1 : d_2 = r_1 : r_2$, where $f_1 : f_2 = M_1^2/r_1^2 : M_2^2/r_2^2$.

Since this is true of each of the small elementary contributions which combine to form w_1 and w_2, it will be true of their resultant; that is,

$$w_1 : w_2 = M_1^2/r_1^2 : M_2^2/r_2^2.$$

Now if s_1 and s_2 be the cross-sections of the two columns, $s_1 : s_2 = r_1^2 : r_2^2$.

Hence $$P_1 : P_2 = M_1^2/r_1^4 : M_2^2/r_2^4. \tag{2}$$

That is, in homologous stars the pressures, as well as the densities, are homologous, and their ratio is given by equation (2).

As for the temperatures, writing the gas equation in the form $T = mp/R\rho$, and assuming that the gas pressure p in this equation is equal to the gravitational pressure P in the last, we find at once

$$T_1 : T_2 = m_1 M_1/r_1 : m_2 M_2/r_2. \tag{3}$$

If then we have calculated, for one single star, what values of the internal pressure and temperature correspond to equilibrium for a given law of density, and specified values of the molecular weight at each point, we can find at once the pressure and temperature within all other stars which are built on the same model, by applying the equations (2) and (3).

952. Radiation Pressure and its Effects. In the preceding discussion it has been assumed that the pressure of the hot gas inside the star carries the whole load of the weight of the overlying layers. Eddington, in 1916, showed that this is not the case. Radiation pressure (which under more familiar circumstances is very small) may attain very large values at the high temperatures which prevail inside the stars, and bear an important part of the load. The gas pressure arises from the motions of the atoms and electrons, which are continually colliding with one another; the radiation pressure, from that of the radiant energy which is continually flying about from atom to atom, being emitted from one and absorbed by another. The amount of this radiation increases as the fourth power of the temperature, so that at very high temperatures the radiation pressure becomes great.

Eddington has given a remarkable equation by which the share of the radiation pressure in holding up the load may be calculated. If the total gravitational pressure due to the weight of the overlying layers is P, we may assume that the gas pressure p balances a certain fraction β of it, and the radiation pressure q the remainder,[1] so that

$$p = \beta P, \quad q = (1 - \beta)P.$$

The quantity β at any point within the star is then given by his equation

$$\frac{1-\beta}{\beta^4} = 0.0325 \left(\frac{M}{\odot}\right)^2 m^4 \frac{Y^3}{X^4}, \tag{4}$$

[1] This neglects the influence of possible electrostatic forces between the charged particles, which is legitimate for all but a few exceptional cases.

in which m represents the mean molecular weight of the gas, M is the star's mass, and $\odot$ the sun's mass. The quantities X and Y depend upon the model on which the star is built, X being the ratio of the density at the point considered to the mean density of the star, and Y the ratio of the pressure at this point to the pressure which would be found at the center of a star of uniform density and of the same mass and radius.

A proof of this equation will be given in section 963. The numerical value of β may be calculated from it by a process of approximation, or by a table of $(1-\beta)/\beta^4$.

Allowance for radiation pressure does not affect at all the values of the density and total pressure in a star built on a given model, but it diminishes the computed gas pressures and temperatures by the factor β. Lane's law, for corresponding points in different stars, now becomes

$$T_1 : T_2 = \beta_1 m_1 M_1/r_1 : \beta_2 m_2 M_2/r_2. \tag{5}$$

953. Eddington's Model. The calculation of the internal temperature of a star, when radiation pressure is considered, is definite, but rather complicated. For one particular model, however, it becomes very simple. In this model the value of $m^4 Y^3/X^4$ is the same at all points inside the star. It then follows from (4) that β must be constant throughout the star's mass; that is, the radiation pressure is always the same fraction of the total pressure.

For a star built on this model, of gas of fixed molecular weight, Y^3/X^4 would be constant; that is, the pressure would vary as the 4/3 power of the density. This condition, together with those derived from gravitation and the gas laws, suffices to determine the law of density completely. It is found that the central density is 54 times the mean density (that is, $X = 54$ at the center), while at this point $Y = 91$ and $Y^3/X^4 = 0.091$, here and all through the star. Introducing this into (4), we have

$$\frac{1-\beta}{\beta^4} = 0.0031\, m^4 \left(\frac{M}{\odot}\right)^2.$$

The value of m depends upon the composition of the material. At the temperature and pressure prevailing at the sun's center

it would be about 2.0 for oxygen, 2.2 for iron, and 2.4 for silver. Eddington, considering the relative abundance of light and heavy atoms in the stars, adopts the value 2.11, so that

$$\frac{1-\beta}{\beta^4} = 0.061\left(\frac{M}{\odot}\right)^2. \tag{6}$$

It is probable that m decreases slowly toward the center of a star. If this is taken into account, the increase of density toward the center is slower, but the value of β given by (6) may still be adopted as sufficiently accurate. Eddington estimates that the central density is actually about 20 times the mean density. This would make the density at the sun's center 28 times that of water, the central pressure 36,000,000,000 atmospheres, and the temperature 29,000,000° K.

The density, temperature, and pressure at the center of any other star built on this model may be calculated by equations (1), (2), (5), and (6).

954. The Escape of Radiation from a Star's Interior. We have now seen how the temperature and pressure inside a star could be calculated, provided that we knew a still undetermined law of density. But we have as yet paid no attention to the most conspicuous of all properties of the stars, namely, that they *shine*. Enormous floods of radiation are continually escaping from their surfaces, yet they do not cool down; and it is clear that heat must in some way be generated within them (doubtless by transformation of energy of some other sort) and must work out to the surface, to supply the loss.

It is possible to calculate, on general physical principles, the rate at which this escape of heat from the interior should take place. The application of these principles to the stars is due mainly to Eddington. It is evident that heat will flow, inside the star, from the hotter to the cooler portions, that is, from the regions where the radiation pressure is greater to those where it is smaller.

Calculation shows that under stellar conditions this flow of heat must be carried almost entirely by radiation, in the manner already described (§ 668), the amount carried by conduction (that is, by the motion of the atoms and electrons) being but a

negligible fraction of the whole. If the intervening space were clear, the flow of heat from one part of the star to another would be exceedingly rapid; but it is actually filled with matter, and the radiation is continually being trapped by atoms, sent out again in a quite different direction, and so having a hard time to get through.

We may, in a sense, regard the flow of heat as being driven through the gas by the difference of radiation pressure, against a resistance due to the opacity of the gas. If this can be determined, the rate at which heat will escape from the star can then be calculated.

955. The Opacity Coefficient. The opacity of a layer of gas of thickness h and density ρ may be set equal to $k\rho h$, where the *opacity coefficient* k varies with the composition of the gas, its temperature, and its pressure. The theory of radiation and its relation to atomic processes has now advanced far enough to make an approximate (though not an exact) calculation of this coefficient possible. Several investigators have derived closely similar formulæ which may all be put in the form

$$k = \frac{K\rho}{mT^{\frac{7}{2}}}, \tag{7}$$

where K, if not an absolute constant for gas of a given composition, at least changes but little with its temperature and density.

These investigations (which are difficult) rest on the assumption that the absorption and re-emission of radiation are brought about primarily by processes involving interactions between ionized atoms and electrons. Such encounters will be more frequent, the more atoms there are in a given volume (that is, the greater is the value of ρ/m), but will be less likely to happen when the atoms and electrons are moving fast (T great) than when their motion is slow (T small). According to Milne, equation (7) should hold good right up to the boundary of the star. It accounts for the high opacity in the photosphere (§ 661), while in the reversing layer, where the temperature is but little less and the density much lower, the gas becomes almost transparent.

956. Theoretical Calculation of a Star's Luminosity. Our knowledge of the properties of a star built on a given model can now be extended by the calculation of the amount of heat which will

flow outward, in a unit of time, from any portion of its interior. Only the results will be given here, deferring the details of the calculation to section 964.

Suppose that a sphere S' of radius r' is drawn inside the star and concentric with it. Let the mass of the part of the star inside this sphere be M', the radiation pressure at its surface be q', the gravitational pressure P', and so on, while in corresponding quantities at a slightly greater distance r'' from the center are q'', P'', and so forth. It is then found that the whole quantity of heat E' which escapes in unit time through this sphere S' from the inner to the outer parts of the star is given by the equation

$$\frac{E'}{M'} = \frac{4\,\pi c G}{K}\,\frac{m' T'^{\frac{7}{2}}}{\rho'}\,\frac{q' - q''}{P' - P''} \tag{8}$$

(where c is the velocity of light and G the constant of gravitation).

This equation is of very general application, being valid for all stars which are in mechanical equilibrium (no matter on what model they are built), and for all parts of each star. If the sphere S' is taken just below the photosphere, substantially the whole star will be inside it, and M' may be set equal to the star's mass M, while E' represents the rate at which heat is supplied from below to the photosphere, and hence the rate E at which the star radiates heat into space.

It is thus possible, starting with pure theory, to calculate the absolute magnitude with which a star of given mass and radius, and built on a given model, ought to shine. The result will give what Eddington calls the "bolometric" magnitude (cf. § 814), which measures the whole radiation of energy and not that of visible light.

957. Application to Eddington's Model. The calculations are in general rather complicated, but in one case they are fairly simple, namely, for Eddington's model (§ 953). In this case the ratio $(1 - \beta)$ of q to P is constant throughout the star, and equation (8) becomes

$$E' = \frac{4\,\pi c G}{K}\,\frac{m' T'^{\frac{7}{2}}}{\rho'}\,M'(1 - \beta).$$

For stars of different masses and radii, but all built on this model, the temperatures and densities at corresponding points

will obey Lane's laws. It may then be shown (§ 965) that the rates of escape of heat through corresponding spheres inside the stars, and also from the surfaces of the stars into space, will be given by an equation of the form

$$E = C(1-\beta)^{\frac{15}{8}} M^{\frac{7}{4}} m' r^{-\frac{1}{2}}, \tag{9}$$

where C is a constant, the same for all stars built on this model, and m' is the molecular weight just below the photosphere.

With increasing mass $M^{\frac{7}{4}}$ increases rapidly, as does also $1-\beta$, by equation (6). Changes in m' or r affect E to a much smaller degree. The total radiation from a star should therefore depend mainly upon its mass, and relatively little on its size or composition (molecular weight). This conclusion is in excellent accordance with the observed facts. Among stars of a given mass those of smaller diameter should be slightly brighter and hence, on account of their smaller area, should have a considerably higher surface brightness and effective temperature.

958. Formula for the Absolute Magnitude. These differences also may easily be calculated. After a little more algebra (§ 966) it is found that if A is the absolute (bolometric) magnitude corresponding to the radiation E, and t the effective temperature of the star's surface, then

$$A = A' - 3.75 \log(1-\beta) - 3.5 \log M - 2 \log m - 2 \log t \tag{10}$$

when A' is another constant.

The value of this constant could be roughly calculated theoretically, but it is better in practice to determine it so that the formula shall give the correct absolute magnitude for some star of accurately known mass and effective temperature. At the same time the differences in molecular weight from star to star (which are probably not serious) may be disregarded.

For this standard star Eddington chooses the brighter component of Capella, for which he takes $A = -0.3$, $t = 5200°$, $M = 4.2$, whence by (6) $1-\beta = 0.282$, and from (10) $A' = 7.2$, giving the equation

$$A = 7.2 - 3.75 \log(1-\beta) - 3.5 \log M - 2 \log t,$$

which, if our assumptions are sound, should enable us to compute the absolute magnitude of any star for which the mass

and spectral type are known. The effect of the latter is comparatively small. If a temperature of 5200° is taken as standard (as Eddington does), the term $-2 \log (t/5200)$ amounts to $+0^m.5$ for stars of Class M, $0^m.0$ for Class G (giants), -0.6 for Class A0, and -1.2 for Class B0.

It must be remembered, too, that A is what Eddington calls the "bolometric" absolute magnitude, corresponding to the total radiation of all wave-lengths, which was called M_r in section 814. To obtain the visual absolute magnitude M_v the correction for the luminous efficiency must be applied. Taking the standard temperature, as above, as 5200°, this correction is small for stars of classes F and G, about $+0^m.4$ for A0 and K0, and $+1^m.8$ for the extremes of Class B and Class M. The two corrections nearly cancel one another except for the K and M stars, for which they range from about $+0^m.5$ to $+2^m$ in extreme cases.

Setting $t = 5200°$ in the last equation, we obtain

$$A = -0.2 - 3.75 \log (1 - \beta) - 3.5 \log M. \qquad (11)$$

The value of $(1 - \beta)$ is to be found from equation (6).

This equation should give nearly correct values of the visual absolute magnitude for stars of classes A, F, and G. For classes B0 and K0 it should give results about $0^m.5$ too bright, and for Class M too bright by about $1^m.5$. For the coolest stars, such as the long-period variables, the correction is much greater.

959. Comparison with Observation. In this way Eddington has derived theoretical values for the masses of stars of different absolute magnitudes, from which Table XXXIII may be derived.

TABLE XXXIII. THEORETICAL RELATION BETWEEN MASS AND ABSOLUTE MAGNITUDE

Absolute Magnitude	$1-\beta$	Computed Mass	Observed Mass	Absolute Magnitude	$1-\beta$	Computed Mass	Observed Mass
17.5	0.00025	0.064	—	2.5	0.113	1.72	1.72
15.0	0.0007	0.11	0.14	0.0	0.257	3.70	4.0
12.5	0.0020	0.18	0.22	− 2.5	0.472	10.1	12.0
10.0	0.0058	0.31	0.34	− 5.0	0.687	35.	—
7.5	0.016	0.53	0.55	− 7.5	0.841	148.	—
5.0	0.045	0.92	0.91	− 10.0	0.924	675.	—

The values of the mean mass for stars of various absolute magnitudes, read from the curve derived from the observed data (§ 772), are given in the last column for comparison. The agreement is extraordinarily close except at the ends of the observed range, where the curve is somewhat uncertain. The deviation for the faintest stars is largely explained by the fact that these are mainly K5 and M dwarfs, for which the theoretical *visual* brightness for a given mass is about one magnitude fainter than equation (11) indicates, which means that the mass for a given brightness is about 25 per cent greater. The most massive stars which have been observed are of Class B, for which a similar, though smaller, correction is necessary. If these corrections were applied, the theoretical curve would represent the observed data just about as well as the one which was drawn free-hand in Fig. 241.

The only important exceptions among well-observed individual stars are the companions of Sirius and Procyon. The first of these is a white dwarf of very high density, — so high that it is probable that even the ionized fragments of the atoms are fairly close together. Under these conditions the stellar matter should be less compressible than a perfect gas, and the internal temperature lower than that indicated by the elementary theory. The star should therefore be less luminous for its mass, or more massive in proportion to its brightness, than others, and the observed deviation is in this direction. It appears probable that the companion of Procyon (which is far too faint to be observable spectroscopically) is a white dwarf too.

The observed mass-luminosity relation is therefore satisfactorily explained on general physical principles, and it appears quite safe to extend it beyond the range of observation, as has been done in Table XXXIII. It follows that there is little hope of being able to observe any star less than one tenth as massive as the sun, — it would be too faint, — and that the very brightest stars, like S Doradus, may be two or three hundred times as massive as the sun.

Eddington has described the results of this investigation in a striking fashion. Suppose that there were a series of spheres of gas, numbered 1, 2, 3, . . ., each in equilibrium under its own

gravitation, of masses 10 grams, 100 grams, 1000 grams, and so on indefinitely. It may be shown from equation (6) that the ratio $1-\beta$ of the radiation pressure inside the sphere to the total pressure would be exceedingly small for all the spheres up to number 32. For the next few spheres this ratio rises rapidly, and for number 36 and all that follow it the ratio is nearly equal to unity, the radiation pressure being much more important than the gas pressure.

On general principles we should expect something to happen in the region where this ratio changed rapidly, "and this time what happens is the stars."

960. Alternative Models. Striking as has been the success of Eddington's model, it would be wrong to conclude that it must be true in all its details. Indeed, there is considerable reason to believe that the ratio of the radiation pressure to the total pressure is not actually constant throughout the whole mass of a star.

It may be shown, however (§ 965), that for a set of homologous stars built on some different model the equation connecting the absolute magnitude with the mass and radius will still be very approximately of the form (10) except that the numerical constant C' will be different — as will also be the constant coefficient in the equation (6), which gives $1-\beta$.

The way in which the absolute magnitude changes from star to star will, however, be similar; and if the constant C' is adjusted to fit the observed data for some star, such as Capella, the representation of the data for all the other stars will be very much like that given in Table XXXIII.

Although we cannot therefore conclude that Eddington's *model*, that is, his suggested law of internal distribution of density, represents the facts, there can be no doubt of the success of his *theory* of the luminosity of the stars, for if applied to almost any model it gives results in accordance with observation.

The constant in the luminosity formula may be (rather roughly) calculated from general physical principles instead of being determined from observed data for a star. If this is done, the computed brightness of the stars, using Eddington's model, comes out about three times too great. Why this happens is not yet certain. It may be that the methods used in calculat-

ing the constant K in the opacity formula (which are admittedly only approximate) are at fault; or that the stars are actually built on a model considerably different from Eddington's; or, as Jeans suggests, that deep inside the stars there are to be found atoms of greater atomic weight than any known on the earth's surface (which would modify the theoretical value of K in the right direction). The true explanation may be quite different from any of these.

Finally, it may be noted that there is good observational evidence that stars of the same mass are not *all* of the same brightness. The case of Arcturus has already been mentioned (§ 941). Another notable instance is the eclipsing variable S Antliæ, whose brighter component (by rather insecure photometric data) is 1.66 times the sun's diameter and 0.7 times its mass. As the spectrum is of Class F0, the absolute magnitude of each component (by the formulæ of section 809) should be + 3.1, while that corresponding to the actual mass is + 5.7.

A more complete theory capable of dealing with these questions may come within a few years more.

961. Summary. The main outcome of these extensive investigations may be stated in a sentence: The characteristics of the stars depend upon the simplest and most fundamental laws of nature, and even with our present knowledge might have been predicted from general physical principles if we had never seen a star.

The laws of gravitation, of radiation, of atomic structure, and of simple gases suffice as a basis for such a prediction. From these it could have been shown that only great masses of matter, tens of thousands of times that of the earth, could be self-luminous, but that these *must* be so, if condensed into single bodies. Such bodies would be intensely hot inside and gaseous throughout. Smaller ones would not be hot enough inside, or would not let enough heat escape to the surface, to keep it incandescent.

For bodies massive enough to shine, the radiation would increase rapidly with the mass, but would change but little with the diameter, the smaller bodies (for equal mass) being hotter inside, but letting only a little more heat leak out to the surface, than the larger ones; this, however, would keep the surface much hotter, on account of its smaller area.

These luminaries would emit light of different colors, depending on the effective surface temperatures. The hotter ones would be very much whiter than any artificial light sources; the

coolest ones would be comparable with the hottest of these. Light would escape directly into space only from regions close to the surface, and the gas would be free from haziness only where the pressure and density were very low. This outer atmosphere would "absorb" a dark-line spectrum. When the light was red (surface temperature 3000° or less), band spectra due to compounds would appear. For successively higher temperatures the arc lines of the metals would be the main feature, then their enhanced lines, and finally the lines of the permanent gases and of multiply ionized atoms. Bodies for which the surface gravity was low would reach a given stage in the sequence at somewhat lower temperatures than the denser, smaller, and fainter bodies of high surface gravity. The enhanced lines of easily ionized elements would be stronger in the spectrum of the former, and all the lines darker at the centers.

These properties of the stars could be predicted, not merely in a qualitative fashion, but in most cases with considerable numerical accuracy. The only important discordance would be that the predicted luminosity of the stars of a given mass and size would be about a magnitude too bright. So detailed an interpretation of the properties of the greatest bodies known in nature directly from the properties of the smallest constitutes one of the most notable triumphs of modern physical science.

This chapter may be concluded by giving proofs of some of the formulæ which have been quoted earlier. The elementary student may omit them; the more advanced student may find it of advantage to have them here rather than to search for them in the original papers, some of which are hard reading.

962. Pressure and Temperature inside a Star. In this problem (as in all questions involving general physical relations) it pays best to adopt as units at the start the centimeter, gram, and second, with the units derived from them (erg, dyne, etc.), the absolute centigrade scale of temperature, and the chemical unit of molecular weight.

The law of gravitation is then

$$f = GM_1M_2/r^2; \qquad G = 6.66 \times 10^{-8},$$

and the gas law,

$$p = \frac{R}{m}\rho T,$$

where $R = 8.32 \times 10^7$ if p is expressed in dynes per square centimeter. (If p is in atmospheres, $R = 82.0$.)

Consider, first, a sphere of constant density ρ_0, of mass M, and of radius r. Since $M = \frac{4}{3}\pi\rho_0 r^3$, we have $\rho_0 = \frac{3}{4\pi}\frac{M}{r^3}$. The surface gravity g_1 is given by the equation $g_1 = GM/r^2$. The pressure P_0 at the center will equal the weight of a column of unit cross-section, height r, and density ρ. The mass of this column is evidently ρr, and the average value of gravity acting upon it will be $\frac{1}{2}g_1$ (since in this case gravity is proportional to the distance from the center). Hence

$$P_0 = \frac{1}{2}\rho_0 g_1 r = \frac{3G}{8\pi}\frac{M^2}{r^4}.$$

Finally, the central temperature T_0 (if $P = p$) will be given by

$$T_0 = \frac{m}{R}\frac{P_0}{\rho_0} = \frac{1}{2}\frac{m}{R}g_1 r = \frac{G}{2R}\frac{mM}{r}.$$

These equations satisfy Lane's laws, as of course they must. To make them applicable to any point within any star, we may suppose that the density at this point is $X\rho_0$, that is, X times the mean density of the star, and the pressure YP_0, or Y times what the central pressure would be if the star were homogeneous. Then we find

$$\rho = X\rho_0 = \frac{3}{4\pi}\frac{M}{r^3}X;\ P = YP_0 = \frac{3G}{8\pi}\frac{M^2}{r^4}Y;\ T = \frac{Y}{X}T_0 = \frac{G}{2R}\frac{mM}{r}\frac{Y}{X}. \quad (12)$$

When the law of density is given, X is known, and Y may then be determined (by the integral calculus).

If the density increases toward the interior, X is greater than 1 near the center and less than 1 near the surface. Y is also greater than 1 at the center (since the attracted material is, on the average, nearer the center than in a homogeneous star), and is therefore more strongly attracted. It would usually be somewhat greater than X.

For the sun, $M = 1.99 \times 10^{33}$ grams, $r = 6.95 \times 10^{10}$ centimeters, whence $\rho_0 = 1.41$ gm./cm.^{3}, $P_0 = 1.35 \times 10^{15}$ dynes/cm.^{2} $= 1.33 \times 10^9$ atmospheres, and $T_0 = 1.15 \times 10^7\,m$ in degrees centigrade.

Setting $m = 2.1$ (Eddington's estimate), $T_0 = 2.4 \times 10^7$ degrees. The actual density at the sun's center is doubtless greater than the mean density, and the temperature and pressure must exceed T_0 and P_0.

963. Proof of Eddington's Equation. The radiation pressure q in dynes/cm.^{2} at the temperature T is given by the equation

$$q = \tfrac{1}{3}aT^4, \quad (13)$$

where $a = 7.63 \times 10^{-15}$. It is very small when the temperature is measured in tens of thousands of degrees, but very great when it rises to tens of millions. Thus, if $T = 2.4 \times 10^7$, as in the example of the last section, $q = 8.4 \times 10^{14}$ dynes/cm.^{2}, or 830,000,000 atmospheres, — almost as great as

the gravitational pressure P_0. If, as in section 952, we call the gas pressure p, and the gravitational pressure P, we may assume that $p = \beta P$, $q = (1-\beta)P$ (neglecting electrostatic forces between the ions and the electrons).

Introducing the values of these quantities found in (12), we have

$$p = \frac{R}{m}\rho T = \frac{3}{4\pi}\frac{R}{m}\frac{M}{r^3}XT,$$

and

$$P = \frac{3}{8}\frac{G}{\pi}\frac{M^2}{r^4}Y;$$

whence

$$\beta = \frac{p}{P} = \frac{2R}{G}\frac{r}{mM}\frac{X}{Y}T, \tag{14}$$

$$1-\beta = \frac{q}{P} = \frac{8\pi a}{9G}\frac{r^4}{M^2}\frac{T^4}{Y}. \tag{15}$$

Dividing (15) by the fourth power of (14), we find

$$\frac{1-\beta}{\beta^4} = \frac{\pi}{18}\frac{aG^3}{R^4}\frac{Y^3}{X^4}m^4M^2. \tag{16}$$

If the law of density is known, as we have supposed, everything on the right-hand side can be calculated, and β may then be found by trial and error (or from a table of $\frac{1-\beta}{\beta^4}$ prepared once for all). The temperature follows from (14) and is β times the value computed in the last section, where radiation pressure was ignored.

The remarkable equation (16), which is due to Eddington, shows that importance of radiation pressure within a star does not depend on its radius (which cancels out during the calculations), but upon the mass, the density law, and the molecular weight. The numerical coefficient $\frac{\pi}{18}\frac{aG^3}{R^4}$ is extraordinarily small, — only 8.21×10^{-69}. (To verify this value is a good exercise for the student.)

The masses are here supposed to be measured in grams. The sun's mass is a handier unit for astronomical purposes. Denoting it by $\odot$, we have $\odot = 1.99 \times 10^{33}$ grams, $\odot^2 = 3.96 \times 10^{66}$, and

$$\frac{1-\beta}{\beta^4} = 0.0325\frac{Y^3}{X^4}m^4\left(\frac{M}{\odot}\right)^2.$$

The small coefficient has vanished, and we obtain equation (4) of section 952.

964. The Flow of Heat inside a Star. Suppose (as in section 956) that two neighboring spheres S' and S'' concentric with a star, and of radii r' and $r'' = r' + h$, are drawn within it. The opacity of the layer of gas contained between them will then be $k'\rho'h$ (§ 955), and it may be shown that the rate of flow of heat H' per unit area, through the layer, will be given by the equation

$$H' = \frac{c(q'-q'')}{k'\rho'h},$$

where c is the velocity of light, and q' and q'' are the radiation pressures at points on s' and s''.

This equation is true only when the layer is so thin that its opacity is small. Students familiar with the calculus will prefer to write it in the form $H' = \frac{c}{k\rho}\frac{dq}{dr}$, where dq denotes the change in radiation pressure for an infinitesimal increase dr in r. It was first given in this form by Jeans.

Now the gravitational pressure P' at the bottom of the layer exceeds the pressure P'' at the top by an amount equal to the weight of the layer per unit area. This latter equals the product of the mass per unit area $\rho' h$, by the acceleration of gravity, g'.

Hence $P' - P'' = \rho' g' h$, and we have

$$\frac{H'}{g'} = \frac{c(q' - q'')}{k'(P' - P'')}. \tag{17}$$

Now let M' be the whole mass included within the sphere S', and E' be the whole amount of heat which escapes through it in unit time. The area of S' is $4\,\pi r'^2$; hence $H' = E'/4\,\pi r'^2$. Also, by the law of gravitation, $g' = GM'/r'^2$ (since, as Newton proved, the attractions of the various parts of the star outside the sphere S' completely annul one another at a point on its surface).

Substituting in (17), we find

$$\frac{E'}{M'} = \frac{4\,\pi c G}{k'}\;\frac{q' - q''}{P' - P''}.$$

But by (7), section 955,

$$k' = \frac{K\rho'}{m' T'^{\frac{7}{2}}},$$

whence

$$\frac{E'}{M'} = \frac{4\,\pi c G}{K}\;\frac{m' T'^{\frac{7}{2}}}{\rho'}\;\frac{q' - q''}{P' - P''},$$

which is equation (8) of section 956.

Now $q' = (1 - \beta')P'$, and $q'' = (1 - \beta'')P''$. The values of β' and β'' will, in general, be different in the two cases, but we may always write

$$P'' = (1 - e')P';\; 1 - \beta'' = (1 - f')(1 - \beta'),$$

in which case e' and f' represent the *proportional* changes in P and $1 - \beta$.

Then $q'' = (1 - f')(1 - e')q'$, or $q' - q'' = (e' + f' - e'f')q'$. Since the changes e' and f' are small, we may neglect their product and write

$$\frac{q'' - q'}{P'' - P'} = \frac{(e' + f')q'}{e'P'} = \left(1 + \frac{f'}{e'}\right)(1 - \beta').$$

Equation (8) now becomes

$$E' = \frac{4\,\pi c G}{K}\;\frac{m' T'^{\frac{7}{2}}}{\rho'}\;M'(1 - \beta')\left(1 + \frac{f'}{e'}\right). \tag{18}$$

965. Comparison of Different Models. For any given model the values of e and f, corresponding to the change h in the distance from the center, can be found, and the radiation E may then be calculated.

In passing from the photosphere (or just below it) to the center the pressure P increases enormously, while it may be expected that the proportional change in $1-\beta$ will not be nearly as great. This means that, on the average, f will be much smaller than e. Laws of density could of course be devised for which this was not the case, but it would require extreme assumptions to alter the computed radiation by as much as one stellar magnitude. Hence the term f/e may be regarded a rather small correction to the computed brightness, depending on the model on which the star is built.

If now we introduce the values of T and ρ from (12) and (14), we find

$$\frac{T^{\frac{7}{2}}}{\rho} = \frac{4\pi}{3}\left(\frac{G}{2R}\right)^{\frac{7}{2}} \frac{Y^{\frac{7}{2}}}{X^{\frac{9}{2}}} \frac{M^{\frac{5}{2}}}{r^{\frac{1}{2}}} (\beta m)^{\frac{7}{2}}.$$

But by equation (16)

$$(\beta m)^4 = \frac{18R^4}{\pi a G^3}\frac{X^4}{Y^3}\frac{1-\beta}{M^2}, \text{ or } (\beta m)^{\frac{7}{2}} = \left(\frac{18}{\pi}\right)^{\frac{7}{8}} R^{\frac{7}{2}} G^{-\frac{21}{8}} a^{-\frac{7}{8}} X^{\frac{7}{2}} Y^{-\frac{21}{8}} M^{-\frac{7}{4}} \left(1-\beta\right)^{\frac{7}{8}}.$$

Substituting in the last equation, we find, after a little reckoning,

$$\frac{T^{\frac{7}{2}}}{\rho} = 2^{-\frac{5}{8}}\, 3^{\frac{3}{4}}\, \pi^{\frac{1}{8}}\, G^{\frac{7}{8}} a^{-\frac{7}{8}} Y^{\frac{7}{8}} X^{-1} M^{\frac{3}{4}} (1-\beta)^{\frac{7}{8}} r^{-\frac{1}{2}}. \tag{19}$$

We may apply this equation to the region in the star which is now under consideration by placing accents on X, Y, and β (which are the only quantities in it that vary from point to point within the star) to show that their values are to be taken for a point on the sphere S'. If Z' is the fraction of the whole mass which is inside the sphere (which will be the same for all homologous stars) we have $M' = Z'M$. Substituting in (18), we find

$$E' = 2^{\frac{11}{8}}\, 3^{\frac{3}{4}} \frac{cG^{\frac{15}{8}}\pi^{\frac{9}{8}}}{a^{\frac{7}{8}}K} \cdot \frac{Y'^{\frac{7}{8}}Z'}{X'} \cdot M^{\frac{7}{4}}(1-\beta')^{\frac{15}{8}} r^{-\frac{1}{2}} m' \left(1+\frac{f'}{e'}\right).$$

This expression for the amount of heat which escapes through the sphere S' is the product of three factors: the first is a constant, the second involves X', Y', Z', and depends only on the law of density and the part of the star under consideration, while the third alone involves the star's mass and radius.

For corresponding spheres in a series of homologous stars we can combine the first two factors into a single constant, C', and write

$$E' = C'M^{\frac{7}{4}}(1-\beta)^{\frac{15}{8}} r^{-\frac{1}{2}} m' \left(1+\frac{f'}{e'}\right). \tag{20}$$

For such corresponding spheres equation (4), page 885, may be written

$$\frac{1-\beta}{\beta^4} = B'm'^4\left(\frac{M}{\odot}\right)^2; \quad \text{where} \quad B' = 0.0325\,\frac{Y'^3}{X'^4}. \tag{21}$$

If the corresponding points are taken just below the photosphere, equation (20) will give the observable heat radiation from all stars built on the given model. The only differences for different models will be that the constants C' and B' will have different numerical values, as will also f'/e'. The latter will not be strictly constant for homologous stars of different masses and radii, but its variations will usually be unimportant. The variations of the radiation for homologous stars will therefore follow substantially the same laws of change with mass and radius in all cases.

966. Formula for the Absolute Magnitude. If t is the effective surface temperature of the star, the rate of radiation of energy from its surface will be, by Stefan's law,

$$E = 4\,\pi\sigma r^2 t^4.$$

Multiplying (20) by the fourth root of this, we find

$$E^{\frac{5}{4}} = C''M^{\frac{7}{4}}(1-\beta)^{\frac{15}{8}}m'\left(1+\frac{f'}{e'}\right)t,$$

whence

$$E = C'''M^{\frac{7}{5}}(1-\beta)^{\frac{3}{2}}m'^{\frac{4}{5}}\left(1+\frac{f'}{e'}\right)^{\frac{4}{5}}t^{\frac{4}{5}}, \tag{22}$$

where C'' and C''' are new constants whose values we do not need to write down.

If the bolometric absolute magnitude corresponding to this radiation is A, we must have

$$A = \tfrac{5}{2}\,(\log E_0 - \log E),$$

or, introducing the value (22) of E,

$$A = \tfrac{5}{2}(\log E_0 - \log C''') - \tfrac{15}{4}\log(1-\beta) - \tfrac{7}{2}\log M - 2\log t - 2\log\left\{m'\left(1+\frac{f'}{e'}\right)\right\}.$$

The variations in the molecular weight, and in f'/e', at homologous points, are probably small from star to star, and we may ignore them and set

$$A = A' - 3.75\log(1-\beta) - 3.5\log M - 2\log t, \tag{23}$$

which is similar in form to equation (10), page 890, but has now been proved to hold good (with the approximations stated) for stars built on any model, provided that the proper values of the constants C' and B' are used in equations (20) and (21).

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These original articles are for the most part hard reading for the novice.

EXERCISES

1. Lanthanum has many strong arc and spark lines in the visible spectrum. Only the enhanced lines appear in the solar spectrum, — even in sun-spots. What may be concluded regarding the lanthanum atom?

2. Manganese and calcium both have very strong *raies ultimes* in the violet belonging to their arc spectra. With rising stellar temperatures the arc lines of manganese disappear in Class A2, while those of calcium hold out to Class B9; yet the ionization potential is higher for manganese by 1.3 volts. How may this be accounted for?

3. How much would the constant K in the formula (7) for the opacity coefficient (§ 955) have to differ from the normal value to account for the peculiarities of S Antliæ (§ 960)?

Ans. By equation (8), page 889, the luminosity is inversely proportional to this constant (other things being equal). Hence K would have to be only $1/11$ as great as usual.

CHAPTER XXVI

THE EVOLUTION OF THE STARS

THE SOURCE OF STELLAR ENERGY · HYPOTHESES CONCERNING IT · THERMAL STABILITY · THEORIES OF STELLAR EVOLUTION · TENTATIVE SOLUTION OF THE PROBLEM · APPLICATION TO DOUBLE STARS AND CLUSTERS · POSSIBLE LIFE HISTORY OF A STAR · VARIABLES AND NOVAE · EVOLUTION OF DOUBLE STARS · THE PLURALITY OF WORLDS · THE DISSIPATION OF ENERGY, AND THE POSSIBLE FATE OF THE UNIVERSE

967. Phenomena still requiring Explanation. The theory of the constitution of the stars which has just been described accounts fully for the close correlation between their absolute magnitudes and their masses, but it gives no explanation at all for the equally conspicuous, though more complicated, relations which connect absolute magnitude and spectral class. So far as the general theory goes, a star of a given absolute magnitude, though it must have approximately a certain mass, may be of any diameter whatever, and hence of any surface temperature and spectral class. There is nothing in it to indicate why one temperature should be preferred above another.

Yet we know, as a matter of observation, that this indiscriminate distribution of stars of a given absolute magnitude among the various spectral classes occurs only among those of great luminosity — the giants and c-stars — and is imperfect even there. Among stars of about the sun's luminosity there is a very strong concentration of the spectra in the vicinity of Class G0; among those a couple of magnitudes brighter, near F0; among those as much fainter, near K0; and so on.

Study of the way in which the stars which appear in our lists were selected shows that this concentration must be real, and not a spurious effect due to selection such as has been discussed in section 802. Indeed, stars are occasionally found (such as white dwarfs) which fall outside the concentrations.

These are not the only characteristics of the stars which cannot be accounted for merely by the laws of their internal equilibrium under gravitation and radiation. Stars of large mass and great luminosity are less numerous than those which are less massive and luminous. The number per unit volume of space increases steadily down to the faintest absolute magnitudes for which we can obtain reliable results. Why we do not see still less massive stars is clear, — they do not, and cannot, send out enough radiation; but stars of great mass (say one hundred times the sun's) should be very luminous, and visible at enormous distances, yet we find hardly a single one.

The explanations of such phenomena as these must depend upon other factors not yet considered, such as the way in which heat is generated within the stars, and perhaps their past history as well.

968. The Source of Stellar Energy. It has already been pointed out, in connection with the maintenance of the sun's radiation, that no known stock of potential energy is sufficient to supply the gigantic amount of heat which the sun has radiated away during geological time, and that this heat must have been furnished by the gradual transformation of some vast and otherwise unsuspected store of energy, probably tied up in the nuclei of the atoms. The sun is apparently a typical star, so that we may extend this conclusion to the stars in general. Further evidence of the existence of this internal store of energy is found in the Cepheid variables, and especially in δ Cephei. The luminosity, mass, and diameter of this star are all pretty well known, and the rate of contraction which would be necessary to supply the radiation from gravitational sources can be computed. The resulting increase in the density should shorten the period at a rate which can be calculated, and comes out, according to Eddington, as 40 seconds per year. The observations cover more than a century and show a very slight decrease of the period, at the rate of only about $0^s.08$ per year, or 500 times as slow as it would be if the energy were supplied by contraction.

No known physical process — not even radioactivity — will account for the liberation of heat at so great a rate per gram of mass as would supply the radiation of the giant stars. The process which keeps the stars shining must therefore be one as

yet unknown in the laboratory, and it can be studied at present only by the effects which it produces. At this point we lose the great aid of atomic physics, and can hope at best for only a sketchy account. When more is known about the structure of atomic nuclei, and possibly something of the reasons why electrons and protons have the charges and masses which they do, it may be possible to go farther. For the moment this great store of energy may be called simply the "unknown source." Concerning its nature, little can here be added to what was said in Chapter XVII. Some process involving the diminution or disappearance of the mass of some form of active matter, and the liberation of the enormous corresponding amount of energy, appears to be the only thing which can provide heat enough. The energy may be liberated initially in some other form, — for example, radiation of very short wave-length, — but inside the stars it will be transformed into heat so soon that we may regard it as if it were directly liberated as heat.

To learn what else we can about this process, we must study its effects, beginning with the effects of liberation of energy inside a star.

969. The Effects of Liberation of Energy inside a Star. Consider a star of any given mass, and suppose it to decrease gradually in diameter, remaining built upon the same law of internal density. The temperature at homologous points will rise in accordance with Lane's law (§ 951), but it does not follow that the star, as a whole, will contain more energy than before, for the gravitational potential energy is greater in the state of larger diameter, and has decreased. Numerical calculation shows that the latter effect is more important than the former, so that the star, though hotter, contains less energy after contraction, more work being done by the gravitational forces than is required to heat the gas.

For a star composed of monatomic gas it can be shown that half the gravitational work goes into heat. In an actual star some of the remainder would be used in increasing the ionization of the gas, but an unexpended balance would still remain.

A star, then, cannot contract except by *losing* energy. If no other sources of energy were present, the loss by radiation from

the surface would be met by such contraction, the star gradually shrinking (as Helmholtz suggested many years ago) and growing hotter inside. The whole amount of available gravitational energy, however, as has already been said, is much too small to account for the time during which the sun has been shining.

If, by any means, heat should be liberated inside such a star, the process just described would be reversed, — the star would be forced to expand, and the expansion would lower the internal temperature at all points. This apparent paradox, — that putting heat into a star makes it cooler, — resembles the case in which a resisting medium makes a comet move faster (§ 488). In both instances the resultant changes compel the transfer of so much gravitational potential energy into energy of other sorts (or vice versa) that the effect which might have been expected is reversed.

970. Alternative Hypotheses. The first question which arises regarding the unknown source concerns the rate at which heat is liberated from it.

Two hypotheses suggest themselves:

(1) The rate of production of heat (per gram of active material) is independent of the temperature and pressure.

(2) The rate depends on the temperature and pressure.

The first alternative would appear preferable if the process of liberation depended entirely upon something which happened inside the atomic nuclei (as seems to be the case in radioactivity); the second, if this process depended also upon any external influences (such as collisions with other nuclei, probably under very special conditions). In the latter case the rate of transformation would probably increase with the temperature, and might vary also with the density.

The first hypothesis is open to objection. The rate of generation of heat per gram within the earth and the other planets must be very much less than in the sun; otherwise the planets would all be brilliantly incandescent. For the earth, judging from the rate at which heat escapes upward through the crust, it must be much less than a millionth part as great as in the sun (where it amounts to 1.4 calories per gram per year). Now there is good reason to believe (§ 541) that the planets were

produced by an eruption from the sun a few billions of years ago. The sun is still shining, and the planets should still be shining, too, unless they were produced from a part of the sun which contained practically none of the active material. This is imaginable if the latter were confined to the core of the sun, too deep for the planet-forming disturbance to reach; but in this case the active substance would have to be different from all known elements and of very great atomic weight. This seems to be rather a far-fetched hypothesis; yet Jeans has recently pointed out that, by accepting it, the discordance between the computed and observed absolute magnitudes of the stars (§ 960) may be removed.

The second alternative (that the rate of liberation of heat increases with the temperature, and perhaps varies with the density, of the stellar material) escapes this difficulty. The temperatures and pressures inside the planets must be so much lower than in the stars that it might be expected that in the former the "unknown process" would not work at all.

971. Thermal Stability. Another question, however, arises. If a great store of energy, transformable into heat at a rate increasing with the temperature, exists inside a star, and some of it is liberated, would not this make the star's substance hotter, and liberate heat still faster, till the process ended in an explosion of almost inconceivable violence? In a star inclosed by a rigid envelope, and prevented from expanding, this would doubtless happen; but in an actual star things would be very different. If the interior were so hot that more heat was generated than could escape from the surface, expansion would be forced, and this would cool the interior and shut off the oversupply. If the interior were too cool, contraction would ensue, and the rate of production of heat would be increased.

It follows from this reasoning that a star will be *thermally stable*, provided that, as it contracts, the rate of generation of heat within it *gains* upon the rate of loss by radiation from the surface, — increasing more rapidly or, it may be, decreasing more slowly. If this condition is satisfied, any disturbance of the internal generation of heat will gradually die out, the star returning toward an equilibrium condition in which heat is generated

in the interior just as fast as it escapes from the surface. If it is not satisfied, any disturbance will steadily increase.

It should be realized, however, how slow these thermal changes are. A gaseous star, devoid of any store of sub-atomic energy, would be thermally unstable under this definition, since, in order to keep shining, it would have to eat up its gravitational potential energy by continuous contraction and would never be in a state of exact equilibrium at all; but, as Helmholtz and Kelvin showed, the process would last for millions of years.

The argument which has just been made would not be valid if the rate of generation of heat were so rapid that the cooling by expansion could not keep up with it, or if the star were so transparent that heat, liberated inside it, could escape to the surface so fast that it would be gone before much expansion could occur. In an actual star, however, neither of these things will happen. In the sun, for example, the store of actual heat in the interior (ignoring altogether the potential energy of all sorts) must amount on the average to many millions of calories per gram of matter. If all heat-generating processes could be stopped, and the sun were prevented from contracting and simply allowed to cool by radiation, this supply would keep it shining for millions of years; and if the present rate of internal generation of energy should suddenly be doubled, the rise in temperature would be correspondingly slow. The gravitational adjustment to a change of internal pressure, on the contrary, would require something like half the time of a free oscillation, or pulsation (§ 843), of the mass; and for the sun it would take only about an hour. In the sun, therefore, the gravitational adjustment would be practically instantaneous compared with the thermal changes, and the argument given above is valid. For a giant star of great luminosity and low internal temperature the factor of safety is smaller, but it is always great.

972. Choice of Hypotheses. The first alternative, with a constant rate of generation of heat, leads to stability only when the rate of escape of heat from the surface diminishes as the star contracts. A star of such a kind would automatically adjust its radius so that the outflow of heat from the surface would balance the fixed income from internal sources. This is physically possible (provided that the "model" on which the star is built changes as it contracts), but the hypothesis meets with grave observational difficulties.

Among the stars whose masses do not much exceed the sun's, those of a given mass are very similar in spectrum and size and absolute magnitude. It would therefore be necessary to assume that the amount of active material is substantially the same in them all. Stars of a smaller mass radiate much less, and for these the quantity of active material would again be the same, but a much smaller fraction of the whole mass than in the first case. How such an exact apportionment of the percentage of active material in stars of different masses could come about is very hard to see.

The second hypothesis escapes these difficulties, for according to this a star automatically adjusts its size so that the rate of generation of energy within it (which is now supposed to be variable) balances the loss from the surface. It meets, however, with troubles of its own. For instance, general principles of thermodynamics make it improbable that a process which liberates such enormous amounts of energy should be seriously accelerated by a temperature of only a few tens of millions of degrees. In spite of this it appears so much better than the other hypothesis that it has been adopted in one form or another by all workers on the subject.

How much of the mass is consumed in this way can be determined, if at all, only by following the history of a star.

973. Stellar Evolution. To observe the successive changes that may happen during the life of a single star is hopeless. Some progressive alteration might be perceptible in an interval comparable with geological time, but it would not be conspicuous (at least for stars like the sun). The whole period of recorded human history would be absurdly inadequate for appreciable change. But among the millions of visible stars there are to be found, in all probability, representatives of every stage of a star's history in which it shines at all; and if we can but pick these out and arrange them in the right order, we need not wait millions of years to see one stage actually change into the next.

Following an analogy suggested by Herschel, suppose that an intelligent observer, who had never before seen a tree, were permitted an hour's walk in a forest. During that space he would not see a single leaf unfold; yet he could find sprouting seeds, small saplings, young, full-grown, and decrepit trees, and fallen trunks moldering back into earth, and in that brief hour he might form a correct idea of the life history of a tree.

In the same manner our task is to take the various types and classes of stars with whose properties we have become familiar and arrange them in some rational scheme of evolution, — some orderly sequence of development.

974. The Older Theories. The first rudimentary scheme of stellar evolution was based on the simple notion that the stars are losing heat and so must be growing colder; in this case the hottest stars (that is, the whitest) would be the youngest, and the reddest the oldest. This belief survived remarkably long, — well into the twentieth century, — and traces of it still remain in the terms "early" and "late" spectral types (though the latter

may be taken as describing merely the position of the spectral class in the Harvard sequence, and are likely to remain permanently useful in this sense).

The first rational theory was based on Helmholtz's conception of the maintenance of heat by contraction under gravitational force, and on Lane's law. On this view a star would contract and grow steadily hotter inside until the density became so high that the simple gas laws were no longer applicable, the molecules of the gas being nearly in contact. It is easy to show that the central temperature would then reach a maximum and begin to fall, till finally the star, like a solid or liquid body, would cool off, contracting very slowly.

It was not then possible to calculate just how the surface temperature would behave, but there were good reasons for believing that it would follow the general course of the central temperature, though with a smaller proportional range.

On this hypothesis a star, beginning as a body of large size and low surface temperature, should contract very considerably and grow hotter till the gas in its interior reached the critical density at which it became hard to compress further. It should then reach a maximum surface temperature and cool down, still contracting slightly.

This scheme was suggested by Sir Norman Lockyer, who called attention to the importance of distinguishing between the young stars of rising temperature and the older ones which were cooling. It was advocated and extended, in 1913, by Russell, who pointed out that the giant and dwarf stars (which had then recently become known) presented exactly the characteristics which should be exhibited by stars of rising and falling temperature, the former being large and of low density, and the latter small and dense. According to this theory a star begins its visible life as a giant of Class M, rises to a maximum temperature, perhaps that of Class A or even of Class B, and then cools, ending its life as an M-dwarf.

This theory represented almost all the observed facts, not only in a general way but in detail. For example, it could be shown that stars of large mass would reach the highest maximum surface temperature; and the B-stars are actually the most massive. Again, the mean density of the A-stars (as found from eclipsing variables) was such that the gas in the interior, if similar to ordinary known gases, would actually be just in the stage of difficult compressibility where the maximum temperature would be reached.

We know now, however, that this assumption about the properties of the gas is certainly wrong, though no one would have supposed this in 1913. The theory, moreover, fails entirely to account for the white dwarfs.

975. Present State of the Problem. Recent advances (the recognition that stellar energy must come not from contraction but from some unknown source, the development of the theory of stellar luminosity discussed above, and, over all, Eddington's

recognition, in 1924, that the ionized gas inside a star must be almost indefinitely compressible) have greatly changed the aspect of the problem. Progress is still rapid, and the best theory that can be advanced at present may be radically altered within a very few years. With this warning it may be appropriate to proceed to a discussion of the situation as it presents itself in 1926.

If the law according to which the heat is liberated from the unknown source of energy could be derived (as can those dealing with ionization and opacity) from atomic and radiation theory, this problem, like that of the luminosity of the stars, could be handled deductively, and it would be possible to predict the history of a star from general considerations; but this is quite out of the question in the present state of physical knowledge, and we are faced with the more difficult problem of finding out what we can about the process by observation of its results.

One proposition appears to be demonstrable. Suppose that heat is generated at the rate of U ergs per second from each gram of material. U may be supposed to vary with the temperature, density, and composition of the gas, and will probably be very different at different points in the star's interior.

For a given composition U may be supposed to depend only on the temperature and density. In this case, given the model on which a star is built, the amount of heat produced inside any sphere S can be calculated. If this equals the outflow E, the star will be in a state of *radiative equilibrium* and will not change in temperature or size; otherwise changes will occur. If the laws determining the ionization, opacity, and generation of heat are given, the model on which the star must be built in order to be in radiative equilibrium can be determined by long but practicable calculations. It is found that a star of given mass and composition will usually be in equilibrium for only one value of the radius, and hence for definite values of the luminosity and surface temperature. For stars of different masses these values will be different; but so long as the composition is the same, all the stars of a given luminosity will have to be of some one definite size, surface temperature, and spectral type.

If these values are plotted on a diagram like that showing the giant and dwarf stars (Fig. 250), they will all lie along a *line*, —

a curve of some sort, like AA in Fig. 306. (In more complicated cases there might be two or more different configurations of equilibrium for a given mass, but all the plotted points would still lie on a curve, of more complex shape.)

If, then, the stars were actually all of the same composition, a diagram such as Fig. 250 would show all the points lying on one line except for such scattering as arises from the errors of observation. This is emphatically not the case, and from this fact the important conclusion may be drawn that *the rate of generation of heat, at points where the pressure and temperature are the same, is different in different stars.* The stars cannot therefore all contain the same proportion of "active matter"—or of the various kinds of it which may exist. They must differ in *composition.*

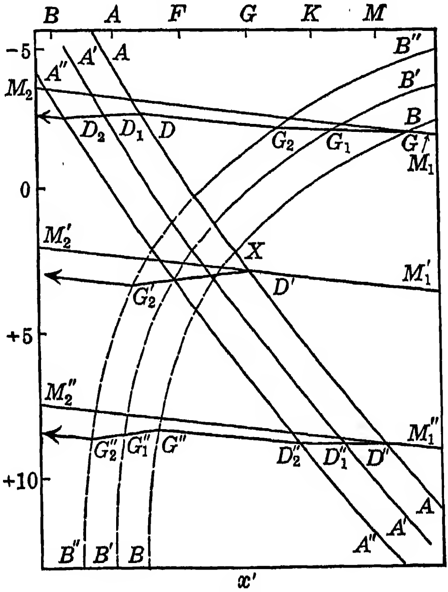

FIG. 306. Stellar Evolution

This conclusion is accepted by all investigators of the subject; but with regard to the properties of the active matter, present theories differ widely, which is not surprising, since different theories may predict almost the same observable effects, and these alone are available for a test.

The discussion here given follows recent suggestions by Russell and Eddington. It should be regarded as a tentative interpretation of the phenomena in a field where definitive work is not yet possible.

976. Two Suggested Processes of Liberation of Heat. For stars in which heat is generated by the same process, but in which the quantities of active material are different, we might expect to find curves $A'A'$, $A''A''$, etc. similar in general course to AA and

roughly parallel to it, and occupying a certain belt upon the diagram. The stars of the main sequence (§ 799), when plotted on such a diagram, fall in a belt of this sort, and the inference is obvious that in the stars of the main sequence the supply of heat is maintained by some one definite process, though the amount of active material is probably not the same in all cases. The curves AA etc. on the diagram are drawn to represent this case.

The giant stars, though more scattered, show evidence of belonging to a sequence of quite a different sort, in which the brightest stars are the reddest, as indicated roughly by the curves BB, $B'B'$, etc. on the diagram. The downward slope of these curves from M to F is clearly indicated by the data for the components of double stars and for the stars in globular clusters. It is therefore reasonable to suppose that a second process of generation of heat, at the expense of a different kind of active matter, and at a rate varying in a different manner with temperature and density, takes place in these stars.

We have already seen (§ 971) that in order to assure thermal stability we must assume that the rate of generation of heat by each of these processes increases as the star contracts and the temperature rises. For a star rich in active material a sufficient supply of heat will be generated at a lower internal temperature, and hence a larger radius and lower surface temperature, than for one of the same mass but poor in active material; hence AA, $A'A'$, $A''A''$, represent the equilibrium states for stars successively poorer in the first sort of active material, and BB, $B'B'$, $B''B''$, for those successively poorer in the second.

This second kind of active matter, being responsible for the radiation of the giant stars, may, for the sake of a name, be called "giant-stuff"; the first, which keeps the dwarf stars going, may be called "dwarf-stuff."

The central temperatures of the stars all along the main sequence, as calculated by Eddington's latest formulæ (on the assumption that they are built on the same model), come out nearly the same, and about 40,000,000° K. The temperatures at other corresponding points, although lower, must be nearly equal in all these stars.

It appears, therefore, that the first process is one which becomes effective at about the same temperature under very different conditions.

It would be wrong to conclude, however, that the rate of production of heat by this process depends on the temperature alone. As a star contracts, T increases; but T^4/p remains nearly constant, being equal to $3(1-\beta)/a\beta$, which depends primarily upon the mass and increases rapidly with it. Since much more heat is lost per gram per second from a star of large mass than from a small one, we must assume that the rate of generation of heat per gram per second from the dwarf-stuff at the same temperature is greater when T^4/p is large, that is, when the pressure and density are low.

The second process becomes effective at low temperatures when T^4/p is large, and at high temperatures when it is small. It is necessary to assume, therefore, that this process of transformation of the giant-stuff is much hindered by high pressure.

It would be unprofitable, in the present state of knowledge, to speculate upon the nature of the atomic changes involved in these processes, otherwise than to assume, as has already been done, that they involve the gradual exhaustion of some forms of active material, and a diminution of the star's mass by an amount corresponding to the radiated energy.

977. Penetrating Radiation. We have no independent means of determining whether such processes of transformation of mass into energy are physically possible, except perhaps in one way. The investigations of several observers — notably Kohlhörster and Millikan — have recently given strong evidence of the existence of a radiation, similar in nature to γ-rays (§ 549), but far more penetrating, which is stronger at great heights above sea-level and which appears to come from outside the earth. Although it is absorbed in the atmosphere, a part of it gets through even to the surface. The extraordinary penetrating power of this "cosmic radiation" shows that it has a very short wave-length. This indicates that it originates in some atomic or sub-atomic transition in which an exceptional amount of energy is liberated (roughly, the amount which would be liberated by the formation of an atom of helium out of hydrogen (§ 672)).

It is a remarkable but well-established fact that the intensity of this radiation is almost the same by day and by night, proving that it cannot come from the sun. Unless it originates in the outer regions of the earth's atmosphere, in some way that is utterly unknown, it must come from the region of the stars, possibly from the hotter stars, or perhaps from the nebulæ of one or another sort. It cannot come directly from the hot interior of the stars, — it has not penetrating power enough for that, — but its existence appears to support the belief that changes involving the transformation of mass into energy occur somewhere in the universe. When further investigated, this penetrating radiation may yield information of great value in testing and redesigning theories of the liberation of energy

within the stars. For the present, however, the only available test is to see how well the deductions from a given theory will account for the observed properties of the stars.

978. Resulting History of a Star. Returning to the assumptions of section 976, we find that the simplest case is that in which the active material constitutes only a small fraction of the mass of the star, so that all through its history its mass, and therefore its absolute magnitude, will be nearly the same. A star of large mass will therefore be represented on our diagram by a point lying initially on a line such as M_1M_2 (Fig. 306), but dropping slightly below it as the mass is diminished by the radiation of energy.

Suppose that at first both giant-stuff and dwarf-stuff are present. If the star is of large diameter, it will be represented by a point such as M_1. Its internal temperature will be low; little or no energy will be liberated by either process of transformation, and the loss by radiation will have to be made up by contraction under the influence of gravity. The representative point will move to the left, toward M_2. If the initial state (that is, the first state of the star which we choose to consider, without asking how it got there) was of small diameter, and represented by M_2, the internal generation of heat would be rapid and would force expansion of the star, so that the representative point would move toward M_1. In either case, equilibrium would be reached at a point G corresponding to the state in which just enough heat was generated at the expense of the giant-stuff to supply the superficial radiation.

Such a process of contraction or expansion would occupy a time comparable to that assigned by the contraction theory for the past life of a star, — that is, a few millions of years for a star like the sun and much less for a bright giant. Once arrived at the state G, the star would slowly exhaust its stock of giant-stuff, and the representative point would move from G to G_1 and G_2, dropping downward a little as the mass diminished. A loss of 1 per cent of its mass would keep the sun shining for 150,000,000,000 years. For a giant star, of 10,000 times the sun's brightness and about 35 times its mass, a corresponding percentage loss of mass would maintain this radiation for about 500,000,000 years, — very much longer than the process

previously discussed would occupy, so that the star would linger in these stages as a typical giant. When the giant-stuff became exhausted, gravitational energy would once more be drawn upon, and the star would once more contract rapidly (as time goes in the history of a star) till it reached the state, represented by the point D, in which the dwarf-stuff began to be transformed. Here again it would remain for a long time, gradually consuming this active material through stages represented by the points D_1 and D_2, and having all the properties of a star at the upper end of the main sequence, and of spectral class B or even O, if the mass were great enough.

At length, as the dwarf-stuff too became exhausted, contraction would become more rapid, the diameter would diminish to a very small value, and finally the density would become so great that even the greatly ionized atoms would be almost in contact and no further contraction would be possible. The temperature would then fall, internally and externally, and the star's career, so far as we can trace it in imagination, would be at an end.

For a star of small mass all the observable states would be represented by points near the line $M_1''M_2''$. Contracting at first rapidly, the star would be stabilized at D'', drawing first upon its content of dwarf-stuff and passing slowly to D_1'' and D_2'', during which interval it would appear as a typical dwarf of late spectral class. When the dwarf-stuff became exhausted, contraction would carry it to G'', where it would consume the giant-stuff and pass to G_1'' and G_2''. During these stages it would be a white dwarf. As before, the final stage would come when the density precluded further contraction, which might perhaps happen in this case before all the active material was used up.

A star of intermediate mass would follow a course like $M_1'D'G_2'$, using up the giant-stuff and the dwarf-stuff simultaneously, in a manner obvious from the figure.

979. Alternative Histories. If the active material constitutes most if not all of the mass of the star, the course of its history may be a little more complicated. Starting with a large mass at M^1 (Fig. 307), the representative point moves to G as before.

But the loss of mass in passing to G_1 is considerable; and before all the giant-stuff is exhausted, D is reached and the dwarf-stuff comes into action. It is clear from the figure how the consumption of this material may shift the point to D_1 and D_2, the loss of mass now being supposed to be large, and that of brightness correspondingly great. When the dwarf-stuff is practically exhausted, gravitational contraction will set in again, and the representative point will move rapidly toward G_2, which marks the beginning of another stage of lingering, during which the rest of the giant-stuff is used up.

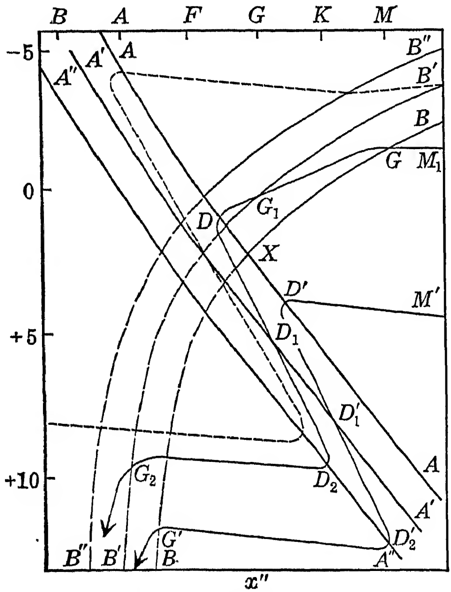

FIG. 307. Stellar Evolution

The evolution of a star of smaller initial mass is shown by the line $M'D'D_1'D_2'G'$. . . . Here the dwarf-stuff is used up first. That of a star of large initial mass, poor in giant-stuff and rich in dwarf-stuff, is shown by the dotted line. In this case all the giant-stuff is consumed first; the star then contracts and becomes a B-star, passes down the main sequence, and ultimately contracts to extinction. Various other cases can easily be worked out.

980. Comparison with Observation. Though the detailed history of a star, according to the theory here sketched, varies considerably, according to the amount of active material which is assumed to be present, the general statistical results, regarding the number of stars of various kinds which should be found among a great number chosen at random, are very nearly the same in all cases. The stars should be found in large numbers in the long-lived states (in which the giant-stuff or dwarf-stuff is

being consumed), — some along the main sequence, others as typical giants, and still others as white dwarfs. Stars which would be represented by points on other parts of the diagram should be rare (perhaps very rare), but there is no reason why some should not be found. All this is in excellent agreement with the observed facts.

Moreover, a giant star lives at a much more rapid rate (so to speak) than a dwarf, its radiation being much greater in proportion to its mass. Giants of all classes should therefore be relatively less numerous than dwarfs, and the brightest giants should be conspicuously so; this is again just what is found when we consider the stars in a given volume of space (and not merely those visible to the eye and affected by observational selection).

Again, a star cannot (on this theory) become a B-star unless it has used up substantially all its giant-stuff before drawing upon its dwarf-stuff. This will not always happen, so that the B-stars should be fewer than the red giants, as is actually the case. To become a white dwarf, a star must exhaust its dwarf-stuff while retaining mass enough to enable it to shine perceptibly, so that these stars might be expected to be less numerous than the red dwarfs; and in fact they are, though they are probably more numerous than the stars of any of the brighter varieties.

981. Clusters and Double Stars. Suppose, now, that a great quantity of matter begins to coalesce into numerous stars of different mass. Whatever its original density, each star will soon (that is, within a few millions of years) find itself in the appropriate state of stable radiative equilibrium. The larger masses will be in states represented by G (Fig. 306) or by other points on the line BX; the smaller ones, in states represented by D', D'', or other points on the line XA. That is, we shall have a star cluster in which the brightest stars are red giants, those about two magnitudes fainter are A-stars, and the fainter ones are dwarfs of the main sequence. This is exactly what is found in the globular clusters (where only the brighter part of the sequence is observable) and in many open clusters.

Suppose that this cluster should be left to itself long enough. The larger masses, which go through their careers more rapidly,

will exhaust their giant-stuff and will now be found on the main sequence (the most massive as B-stars, if they have not lost too much mass in previous changes), while the smaller masses, which live their lives more slowly, will still be using up their dwarf-stuff and be on the main sequence. Clusters like the Pleiades show all the characteristics predicted by theory for this stage. A similar result would be obtained almost at the start if the matter of which the cluster was formed were almost devoid of giant-stuff. If no very large masses were originally formed, there would be no giant stars at all in the cluster, and perhaps none earlier than Class F. Much the same situation would be found in a very old cluster, in which the originally massive stars had lost seriously by radiation. Clusters showing all these types of spectral distribution have been observed by Trumpler (§ 862).

The same principles apply to double stars. If a star should in any way divide into two masses, each would, after a few millions of years, come automatically into the stable state corresponding to its mass. The more massive star would always be the brighter. If both masses were large, the stars would be giants represented by points on the curve BX, and the brighter star would be the redder, as in ϵ Boötis; if both were of small mass, they would be dwarfs (on XA), and the fainter star would be the redder, as in η Cassiopeiæ or α Centauri; if one were above and the other below the critical mass corresponding to the point A, one would be a giant and the other a dwarf, the spectra would be similar, and the pair would resemble ϵ Hydræ. If the stars were of equal mass, they would be of similar brightness and temperature.

For massive stars, in which giant-stuff was initially absent or practically exhausted, one or both components would be of Class B, or even of Class O.

All this is in entire agreement with the observed facts.

It is therefore quite unnecessary to assume that the members of a binary system or of a cluster are of different ages. The observed distribution may be fully accounted for as a necessary result of the conditions which determine the radiative equilibrium of stars of different masses. This important suggestion was first made by Jeans, though in connection with a different scheme of evolution from that here outlined (§ 983).

Certain difficulties still remain. For example, the companion of Sirius is a white dwarf and should be very old (or at least should have used up all its dwarf-stuff), while Sirius itself is on the main sequence. In this case, and in a few others, we seem forced to assume that the components are of decidedly different composition, which is not at all satisfactory.

Again, it is hard to explain why the globular clusters should all contain giant red stars, which should theoretically be short-lived, and almost no B-stars as bright as the brighter ones in the Galaxy or the Magellanic Clouds.

982. The Course of Stellar Evolution. The theory here sketched is therefore evidently imperfect; but it may fairly be claimed that it accounts for a larger number of the properties of the stars than does any that has previously been proposed. It is, however, so flexible that it leaves us in some doubt as to the life history of a star.

If the active material, both giant-stuff and dwarf-stuff, constitutes but a small part of all the matter in a star, then the red stars are all in early life, and the white stars in the latter part of their careers, though the age in years of a slowly changing red dwarf may be much greater than that of a B-star which is far along in its evolution. On this hypothesis a star is of much the same brightness all through its life, and the sun, in particular, has probably not changed much in size and brightness since it began to draw on its internal energy, which may have been hundreds of billions of years ago.

But if the larger part of the material of the stars is expendable, they may once have been very much brighter than they are now. The red giants still represent an early stage, but the red dwarfs may be the shrunken remnants of what were once great stars far up the main sequence (and composed almost entirely of dwarf-stuff), or which, still earlier, were red giants. The white dwarfs are still to be regarded as at the end of their careers (the high density settles that). The B-stars do not represent a primitive stage (unless, indeed, they never contained any giant-stuff), but they may still have most of their lives before them.

A single star, on this scheme, might go through almost every known spectral type, starting as a very massive red super-giant, passing through various stages (as a c-star of rising temperature) to the main sequence at Class B, using up its dwarf-stuff along

the main sequence, perhaps as far as Class M, and finally contracting to the density and temperature of a white dwarf, though, on our present assumption, not lingering long in this stage.

The sun, on this view, may once have been as massive, bright, and hot as Sirius, and still earlier may have been a giant such as Capella; and in the remote future it may shrink to be a faint red dwarf like Krüger 60, and it may end as a white dwarf like van Maanen's star.

The time-scale opened up by such a conception is of impressive extent. Allowing for the more rapid radiation when the mass was greater, the time required for the sun to diminish to its existing mass from twice that value comes out about 5,000,000,000,000 years (5×10^{12}). To shrink to half its present mass would require 40,000,000,000,000 (4×10^{13}) years more, and to reach one fifth its present mass, more than twenty times as long.

Between these two hypotheses there is little evidence to decide, so far as we have gone. Certain facts, however, which have still to be mentioned (§ 990) appear to favor the longer time-scale and the belief that the stars, during their history, lose a large portion of their mass.

983. Jeans's Theory. A quite different hypothesis regarding the law of liberation of energy, which, however, leads to very similar conclusions, has been suggested by Jeans. According to this, the rate of liberation of energy from the active matter is constant, except for those atoms which have lost all their electrons by ionization, in which case the transformation ceases. Working on the assumption of conditions such that thermal stability occurs when, with increasing internal temperature, the internal supply of heat *decreases* relatively to the loss from the surface, he concludes that the stars will adjust themselves to such a degree of internal ionization that the output of heat from the remaining active material balances the radiation into space, and shows that this should cause a great concentration of the stars in the vicinity of the main sequence. As the mass of the star diminishes it becomes fainter, and the resulting course of evolution is roughly similar to that sketched above.

984. Stability of Pulsating Stars. One or two allied questions remain to be discussed. The treatment of the pulsations — or oscillatory changes in volume — of a star which was given in section 843 took no account of the internal generation of energy. The condition that pulsations of this kind shall be stable, in the short run, is exactly the same as that which permits a star to radiate heat upon contraction (§ 969), namely, that the total energy

of the star should decrease for the states of mechanical equilibrium of smaller radius. In this case, if the star should suddenly be compressed by some external force (allowing no time for heat to escape from the interior), it would have too much internal energy for equilibrium (being too hot inside) and would expand back to and beyond its original size, and oscillate about this.

In the long run, if there were no generation of energy inside the star, the "leakage" of heat from the hotter to the colder parts would act like friction, to diminish the amplitude of such a pulsation and in time to "damp" it out entirely.

Eddington has shown that this process, in a star like δ Cephei, would take but a few thousand years, so that the maintenance of Cepheid variation demanded explanation.

If, however, heat is generated inside the star from some "unknown" source, and most rapidly when the gas is hottest and the star smallest, this will be equivalent to a small impulse at every oscillation, tending to make the star expand a little faster than it contracted, which will help to maintain the pulsation against the dissipative tendency. Indeed, if great enough, it might reverse the action and cause a small pulsation to increase steadily in amplitude — a condition which Eddington calls "overstability." The tables are now turned, and the difficulty is to explain why all stars are not Cepheids, and why the actual Cepheids do not pulsate more and more wildly until they explode.

In considering the first point it is important to notice that the period of the pulsation and the magnitude of the oscillation which will be set up by a given small disturbing force depend upon the amount of surplus energy after a forced contraction. If no energy were expended in increasing the ionization of the gas at the higher temperature, the surplus of energy would be large; but if the change in ionization is considerable, this may eat up almost all the surplus, and the star will then be in a state in which a small force will set up a large oscillation. The period, in this case, will be longer, for stars of the same density, than in the previous instance.

It is under these sensitive conditions that pulsations of large amount would first be produced if the effects of the internal generation of heat were gradually increased.

Now Fowler and Guggenheim have shown by direct calculation that the "margin of safety" is least when both the temperature and the pressure of the gas are rather low (from the stellar standpoint); for under these conditions abundant atoms, such as those of iron, may lose or regain several electrons when the conditions are but a little altered. This situation, they find, will happen only in stars of large mass and low density, — in other words, in the super-giants. It is among just such stars that Cepheid variation is found. The theory is not yet sufficiently developed to make it possible to calculate at what density, for different values of the mass, this instability should set in. If this is the true explanation, a detailed theory should lead to an explanation of the period-luminosity curve.

Why other stars show no tendency toward pulsations is a harder question for the present theory to answer. Eddington suggests that an increased temperature of the stellar material may not immediately increase the rate of liberation of heat. This would happen, for example, if the primary transformation produced some sort of "self-destroying material" which afterwards broke down, with liberation of heat, at a rate independent of temperature and density (as radioactive substances do). The time-lag involved in this process must be a year or more, if the stars of lowest density are to be protected against overstability.

Not only the Cepheids but almost all the other periodic, or roughly periodic, variable stars are giant stars of low temperature and density, that is, stars which, on general considerations are probably close to the edge of mechanical instability. This suggests that the protection against overstability, from whatever source it arises, is effective except close to the border line.

When once the oscillation sets in, observation shows that it usually has about the same large amplitude. Why it does not go farther is not certain; but there are many cases in physics (such as a "howling" radio set) in which an automatically increasing oscillation, after it becomes considerable, meets with changing conditions which limit its amplitude to a finite value. The mathematical discussion of such cases is always difficult, and it cannot be attempted at present for pulsating stars.

985. Application to Novæ. The extraordinary energy exhibited by novæ is explicable without difficulty on the assumption that the rate of generation of heat from the unknown source increases with the temperature. Suppose (following Seeliger and W. H. Pickering) that a great swarm of meteorites or a body of planetary dimensions collides with a star. It will strike the surface with a very high velocity, owing to the star's attraction, and will penetrate deep below the rarefied photosphere before being seriously retarded. In the denser layers below, it will be stopped, its kinetic energy will be transformed into heat, and a "pocket" of excessively hot material will be formed, in which the temperature (as a simple calculation shows) may easily rise into the millions of degrees. If this temperature is high enough, it may bring about a rapid liberation of energy from the unknown source, making the pocket still hotter. The intensely heated gas will expand, blowing off the superficial layers of the star above it, and escaping with so much energy that the atoms fly clean away from the star, at a speed far too great for its attraction to hold them back. Radiation pressure will help to drive it.

This accounts for the expanding shell of matter, to which the most striking characteristics of the novæ appear to be due.

As the pressure is relieved by this expansion the temperature at the star's surface will drop, the abnormal liberation of energy will cease, and the star will settle back, covered with a surface layer of very hot gas, which accounts for the Wolf-Rayet spectrum. Being only superficial, this layer will soon cool, and in a decade or two the star will revert nearly to normal.

This sketch appears to account roughly for the principal characteristics of the novæ. The notion that the immediate cause of the outburst (which acts, so to speak, as a detonator) is a collision with some small external body finds confirmation in the existence of dark (reflecting) nebulosity near Nova Persei, and in the facts that many novæ appear near the edges of obscured regions, and that they are most numerous of all in the inner parts of the spiral nebulæ, where there is apparently a great deal of diffuse matter.

Why the novæ in the Andromeda nebula should be so nearly alike in maximum brightness is unexplained. The exceptional nova near the nucleus, which, if really in the nebula, must have been of absolute magnitude −15, appears to have been an affair of a different order, and suggests the possibility of a collision of two stars. The whole amount of light liberated, however, corresponds to the annihilation of a quantity of matter less than the earth in mass, so that collision with a body of planetary dimensions might suffice even in this case.

986. It should be emphasized once more, in conclusion, that the scheme of stellar evolution sketched in the preceding pages is *tentative*. It accounts for most of the observed facts, but it is far from resting on the solid basis of general physics that underlies the theories of stellar atmospheres and stellar luminosity. Other and quite different theories have been suggested; for example, Jeans has accounted for many of the facts on the hypothesis that the rate of generation of energy is independent of the temperature. The final theory, when it comes, may be very different; but if nothing should be set down here except what is firmly established, the subject of stellar evolution could not be discussed at all.

Certain other cosmogonic problems may next be mentioned.

987. Origin of Double Stars. The Fission Theory. The origin of double stars has long been a matter of discussion, and it is generally believed to be probable that the closer pairs, such as spectroscopic and eclipsing binaries, have originated by *fission*. If a slowly rotating mass of low density should lose energy and contract (retaining the same mass), its rotation would necessarily grow more rapid, it would become more and more flattened at the poles, and later it would take the form of an ellipsoid of three unequal axes. Beyond this point the mathematical complexities become so great that the process cannot be worked out in detail, but Jeans has shown that there is little doubt that in many cases the body would break up into two portions, of comparable but not necessarily equal masses, in orbital motion around the center of gravity. As these masses contracted further, tidal action would transfer angular momentum from the rotational to the orbital motion, and the components would recede from one another as the moon is still receding from the earth (§ 359).

The eclipsing binaries present such excellent examples of every stage of the latter part of the process, from stars almost in contact to those widely separated, that there can be little doubt that it has actually occurred. The earlier stages, before division has taken place, have not been recognized. It is possible, as Jeans suggests, that they may account for the variations of irregularly periodic stars like RV Tauri, and perhaps for the Cepheids, — affording an alternative to the pulsation theory.

The fission theory appears to be satisfactory for the close pairs, but it meets with serious difficulties in the case of those of longer period, such as the visual binaries. Under tidal action the mean distance of the components increases; but, when the masses are nearly equal, the increase cannot be nearly as great as in the case of the moon. The reason is that, for equal components, most of the angular momentum is in the orbital motion from the time of separation, so that the transference of the rotational momentum to this does not alter it very greatly. It can easily be shown that, if the larger mass is not more than three times the smaller, the distance (measured by the parameter of

the orbit) can at most be doubled by tidal action, so that spectroscopic binaries cannot grow into visual binaries in this way; and it is necessary to assume that, at the time of fission, the visual binaries were of exceedingly low density, — much less than that of even the most extreme giant stars. Moreover, during the process of tidal evolution it appears that the eccentricity of the orbit, though it may change somewhat, is not likely to undergo any great increase, so that the large eccentricities which are characteristic of visual binaries remain unexplained.

988. Effects of Encounters. Jeans has pointed out that these high eccentricities may have arisen in quite a different way. Suppose that a third independent star happens to pass near a binary system. It will move about it in a hyperbolic orbit (barring perturbations) and recede to an indefinite distance. During the close approach, or "encounter," its attractions upon the components of the binary will be different in magnitude and direction, and the net effect will be that, after it has passed on, the relative velocity of the two will have been altered, perhaps by a very considerable amount, and their relative orbit correspondingly changed. The detailed effect of such changes will be different in each case, but some of the average effects in a large number of cases may be foreseen.

If the original orbit was circular, any change in the velocity, whether in amount or direction, will make it eccentric (§ 315); hence encounters will tend to turn circular, or nearly circular, orbits into eccentric ellipses. Jeans shows that after a large number of encounters the average orbital eccentricity should be 0.66.

The distance within which a passing star must come in order that its disturbing force may be a given fraction of the mutual attraction of the pair is evidently much greater for wide than for close pairs. Hence wide pairs are much more likely to be "knocked about" by encounters than close pairs. The high eccentricity of visual pairs, especially those of long period, and the nearly circular orbits of systems of very short period, are thus simply explained. The chance that, by accident, one star should pass so near another is, however, exceedingly small.

989. Probability of Encounters. This may easily be shown in a simple case. Disregard the gravitational attractions and suppose that the stars are at rest in space and scattered at random, so that, on the average, there is one star in the volume S. Now consider a single star moving in a straight line with velocity v. How often will it pass within a distance d from some other star? If a cylindrical tube, of radius d, is described about the path of the star, all the stars inside this tube, and no others, will satisfy the conditions. On the average there will be one such star in a length l of this cylinder such that its volume is equal to S, which gives at once $l = S/\pi d^2$. The time T required to travel this distance will be given by $T = l/v = S/\pi\, d^2 v$. If the other stars are also in motion, this formula still holds, provided we substitute for v a properly defined average *relative* velocity of a pair of stars.

In the nearer parts of the galactic region there is perhaps one star for every 10 cubic parsecs, so that, in terms of astronomical units, $S = 10 \times (2.06 \times 10^5)^3 = 9 \times 10^{16}$ (closely enough). For the relative velocity, following Jeans, we may take 40 km./sec., or 8 astronomical units a year. If, finally, we take d as 30 astronomical units, — Neptune's distance from the sun, — we find $T = 4 \times 10^{12}$ years. Encounters between a given star and a binary pair, within a distance comparable with that of the components, should therefore be very rare. Unless the life of a normal star extends into the thousands of billions of years, such close encounters can have happened to only a few out of the thousands of known systems.

In the foregoing discussion the effect of the mutual attraction of the two encountering stars has been ignored. It draws their paths together and increases the chance of a close approach. If d' is the periastron distance in the hyperbolic orbit, and v' the velocity at this point, while d and v correspond as before to the motion if attraction were absent, it follows from the law of areas that $d' : d = v : v'$ and the chance of an encounter within the limiting distance d' is increased in the ratio $d^2 : d'^2$ or $v'^2 : v^2$. But if u is the parabolic velocity for the pair of stars concerned, at the distance d', $v'^2 = v^2 + u^2$, so that we must divide the value of T previously obtained by $1 + \frac{u^2}{v^2}$.

For a distance as great as Neptune u is much less than 40 km./sec. even for massive stars, and the correction is unimportant; but for an actual collision of two stars like the sun u exceeds 600 km./sec., and the probability is increased about 250 times. Even so, it is only 1/40,000 of the chance of an approach to within Neptune's distance. Collisions between two stars should therefore be excessively rare. An individual star like the sun would go something like 10^{17} years before colliding, and, even among ten billion stars, actual collisions should happen only at intervals averaging about ten million years.

990. Effects of Secular Decrease of Mass. The high orbital eccentricity of the average binary suggests that during its past history it has experienced not one, but several encounters. Even

the longest time-scale permitted by the hypothesis of decrease of mass (§ 982) hardly allows time enough for this, on the assumptions just made. The difficulty is resolved by another consequence of the slow decrease of mass. With constant masses the orbit of a binary star (barring tidal action and perturbations from without) should remain unaltered indefinitely; but with decreasing masses, as Jeans has shown, the mean distance should be inversely proportional to the mass of the system, and should slowly increase, while the eccentricity should vary somewhat, though not very greatly. If the effects of tidal action and encounters are added to this, the observed characteristics of the binaries of longer period become fully explicable.

Jeans has also pointed out that if, long ago, the stars were, on the average, of twice their present mass, the whole galactic system (provided that its dimensions are fixed by the motions of its stars under the gravitation of the whole) would, like any single orbit, have been of but half its present diameter, and encounters would then have been more numerous. This seems to allow leeway enough. With this long time-scale, also, the chances for a closer approach of two stars, such as might lead to the formation of a planetary system, though still small, become sufficient to permit the belief that such systems, though not common, may exist here and there among the stars, and that their total number may be great.

These arguments support, though they do not prove, the hypothesis that most of the matter in the stars is "active," that is, that it may disappear altogether as matter, with liberation of the energy corresponding to its mass, and hence that the actual annihilation of electrons and protons gradually takes place within the stars upon a very large scale.

Jeans has recently pointed out that the radiation escaping from a rotating star carries away angular momentum as well as mass. For the sun the changes in both since the probable date of origin of the planets are negligible, so that the arguments of section 540 are still valid; but for many double stars the loss of angular momentum may be important. He also concludes that the long-continued action of the stream of energy flowing out from the deeper parts of the star through the superficial

layers will compel the rotation of the surface to be much slower than that of the central core, and cause it to exhibit an equatorial acceleration like that observed on the sun. With such a rapid central rotation the formation of a binary pair may begin with a separation of the inner mass of the star into two nuclei before the surface is seriously affected. During the process of separation the star might be expected to be variable, and Jeans has shown that successive stages may exhibit the general characteristics of irregular, long-period, and Cepheid variation (§ 844), — the star ending perhaps as an eclipsing variable when the components have separated. A detailed mathematical treatment of the matter is likely to be of great difficulty.

We may conclude by discussing some topics of a general nature.

991. The Plurality of Worlds. The question is often asked whether, among the myriads of stars in the Galaxy, others, like the sun, are attended by planets fitted to be the abode of life and actually inhabited. No definite answer can be given, but from the reasons just discussed it appears at present to be quite unjustifiable to assert the negative. The same natural laws operate everywhere, and the same kinds of matter are present. Even if planetary systems are rare, there may be tens of thousands, or even millions, of them among the billions of stars.

Here and there, at least, among these there should be planets large enough to retain an atmosphere, with water upon their surfaces, and at such distances from the stars about which they revolve that their surfaces are maintained at a genial temperature. On such planets life may exist, as it does on the earth (and perhaps upon Mars), so that the habitable and inhabited worlds in the universe may be many. Whether there are actually many or few we cannot be sure, and what the forms of life upon them might be, our imagination is impotent to reveal.

992. The initial state of the stars is largely a matter of speculation. Of the stars known to observation the earliest in evolutionary condition appear to be the reddest giants, — the N-stars and especially the long-period variables. Bodies in still earlier stages would radiate much heat but very little light, owing to their low luminous efficiency (§ 814), and it would be very difficult to discover them even with great telescopes.

It is natural to suppose that at a still earlier stage the matter which later formed a star was cold and dark, and perhaps widely scattered through space. The chief difficulty here is to see how such a congeries of isolated particles should have possessed so little angular momentum as the stars (especially those which are not double) have at present. The loss of angular momentum by radiation modifies matters somewhat, but it is not improbable that only a small fraction of the scattered matter which was "originally" present had small enough relative motion to permit it to concentrate into stars. The remainder may still be in a diffused state. Matter of this sort appears to exist in great quantities in the dark nebulæ, and it is to these, if to any known objects in the heavens, that we may perhaps look for the birthplace of the stars. They alone may be permanent, just because of their chaotic nature. As far back in time as our reasoning carries us, they may even then have been substantially the same as now, — "without form and void."

An alternative theory, suggested by Jeans, looks to the spiral nebulæ for the origin of the stars. Gigantic rarefied masses of gas, if rotating fast enough, would shed material from their periphery in great sheets or streams. If such a stream had a relatively small mass, the molecules would escape from its attraction and fly off separately into space; but if it were massive enough, it would not dissipate, but would tend to break up into smaller masses, which might, under reasonable conditions, be great enough to condense into stars. The presence of individual stars in the outer regions of the greater spirals is favorable to this hypothesis; but it appears very improbable, on the existing evidence, that the spiral nebulæ are at present masses of gas (§§ 916, 923). It appears, rather, that these nebulæ are star-clouds, and this leaves obscure the origin of the stars themselves.

993. The final state of a star, so far as it can be followed by observation, is illustrated by the companion of Sirius, — small, exceedingly dense, and with the ionized atoms crowded almost into contact.

If all the matter of which a star is composed is transformable into energy (which, for all that we know so far, may be true, though it has not been assumed in the preceding discussion),

the star may continue to turn matter into energy until it is practically all gone. It seems rather more likely, however, that the star ultimately gets into a state where it can contract no more, and at last, with extreme slowness, cools down.

There is no reason to suppose that the matter in the interior ever returns to the state with which we are familiar. To do so would require an expansion against the force of gravity (in this case very great), for which no adequate source of energy can be suggested, and it is more likely that the free electrons slip by among the interstices of the ionized atoms; but here again we are on the edge of the purely speculative.

994. The Dissipation of Energy. One of the greatest questions of all remains. What becomes of the tremendous flood of energy which the stars are pouring out, and have for ages past poured out without intermission? A minute part of that radiated by the sun (about one part in 230,000,000) is caught by the planets. The stars themselves are so small in comparison with their distances that they intercept practically nothing. The dark nebulæ must absorb far more than all other known bodies together, but even they cover but a fraction of the celestial sphere, and most of the radiation goes out beyond them. Moreover, the radiation intercepted by the planets warms them, so that they too radiate heat into space. The same is doubtless true of the nebulæ. A small fraction of the intercepted heat may go to produce permanent physical or chemical changes, like those involved in the production of coal on the earth; but, so far as is known, almost the whole of it, sooner or later, is re-radiated and escapes.

All physical processes which involve changes in masses of matter are subject to the law of the "degradation of energy." In all bodies of sensible dimensions, which are necessarily composed of enormous numbers of atoms, all known mechanical, chemical, or electromagnetic changes are accompanied by the conversion of a certain portion of energy of other forms into heat. Such effects (mechanical friction, electrical resistance, magnetic hysteresis) appear to be inevitably present. Even in the motion of the planets in space there must be a minute tidal friction, due to the varying distortions of the bodies by the tidal forces, which operates steadily in the same fashion.

In the elementary theory small changes like this can usually be neglected, but in the very long run they become of great importance.

The process is *irreversible*; that is, reversing the change does not turn the heat back into the original forms of energy, but turns more of the energy into heat. Heat flows from the hotter to the cooler portions of any mass, and from its surface escapes by radiation into space. The available energy of any material system is thus steadily diminishing, more and more of it escaping in the form of radiation and flying away with the velocity of light.

In a few millions of years this has receded beyond the remotest known nebulæ. What becomes of it then? No one knows at all. It may simply keep on going, into the depths of infinite space, or it may be caught somewhere and turned again into some other form. It is imaginable that all radiant energy must ultimately be absorbed in matter, just as all known radiation is emitted from it; but if this be true, there must be a stupendous amount of cold matter far off in space, to hold, without becoming hot and sending it back again, the vast store of energy which the stars have radiated in the past. Others have suggested that some process, yet unknown to us, may be operative in the nebulæ, or elsewhere in the depths of space, which reverses that which happens in the stars, and rebuilds matter at the expense of radiant energy; but no one has suggested any means by which enough energy to reform an electron or a proton, or both at once, could be collected, approximately at one point, out of the feeble and diffused radiation which traverses interstellar space.

Once more, it may be that space itself is reëntrant, like the surface of a sphere, and that, after a very long flight, the radiant energy will return; but observation shows no signs that it is doing so. All that we know is that it goes out beyond our ken.

995. The fate of the physical universe, so far as the scientific imagination enables us to follow it, rests upon the answer to this unsolved riddle. If the radiant energy flies on forever, spreading out into infinite space, then all the available energy of the universe (including that liberated by the annihilation of most of the matter which was once present in it) must ultimately

be lost in this bottomless abyss, and the end will be darkness and stagnation. If, in some way still unknown to us, matter is formed again out of the radiant energy, a vast process of unending change may go on forever. What we know so far exhibits the processes of nature as irreversible, and makes the dissipation of energy appear inevitable; but our knowledge of nature is limited, and any answer to questions of such scope as this must at present be purely speculative.

It may be that at the last the dissipation will be complete, and the end will come with "darkness upon the face of the deep," or, instead, that in some way there may arise "a new heaven and a new earth," perpetually renewed; but which hypothesis is true our present science cannot tell.

APPENDIX

Table XXXIV. The Greek Alphabet

A knowledge of the Greek alphabet is essential in the identification of stars by means of a star map (§ 676).

Letters	Name	Letters	Name	Letters	Name
Α α	Alpha	Ι ι	Iota	Ρ ρ ϱ	Rho
Β β	Beta	Κ κ	Kappa	Σ σ ς	Sigma
Γ γ	Gamma	Λ λ	Lambda	Τ τ	Tau
Δ δ	Delta	Μ μ	Mu	Υ υ	Upsilon
Ε ε	Epsilon	Ν ν	Nu	Φ φ	Phi
Ζ ζ	Zeta	Ξ ξ	Xi	Χ χ	Chi
Η η	Eta	Ο ο	Omicron	Ψ ψ	Psi
Θ θ	Theta	Π π ϖ	Pi	Ω ω	Omega

Table XXXV. List of the Constellations

The Ptolemaic constellations are indicated by an asterisk; the zodiacal constellations by Z; constellations that are, for the most part, circumpolar in latitudes 40° North and 40° South are marked N and S, respectively.

Name		Genitive Case	Abbreviation
Andromeda*		Andromedæ	And
Antlia		Antliæ	Ant
Apus	S	Apodis	Aps
Aquarius*	Z	Aquarii	Aqr
Aquila*		Aquilæ	Aql
Ara*	S	Aræ	Ara
Aries*	Z	Arietis	Ari
Auriga*		Aurigæ	Aur
Boötes*		Boötis	Boo
Cælum		Cæli	Cæ
Camelopardalis	N	Camelopardalis	Cam
Cancer*	Z	Cancri	Cnc
Canes Venatici		Canum Venaticorum	CVn
Canis Major*		Canis Majoris	CMa
Canis Minor*		Canis Minoris	CMi
Capricornus*	Z	Capricorni	Cap
Carina	S	Carinæ	Car

TABLE XXXV (CONTINUED)

NAME		GENITIVE CASE	ABBREVIATION
Cassiopeia*	N	Cassiopeiæ	Cas
Centaurus*	S	Centauri	Cen
Cepheus*	N	Cephei	Cep
Cetus*		Ceti	Cet
Chamæleon	S	Chamæleontis	Cha
Circinus	S	Circini	Cir
Columba		Columbæ	Col
Coma Berenices		Comæ Berenices	Com
Corona Austrina*		Coronæ Austrinæ	CrA
Corona Borealis*		Coronæ Borealis	CrB
Corvus*		Corvi	Crv
Crater*		Crateris	Crt
Crux	S	Crucis	Cru
Cygnus*		Cygni	Cyg
Delphinus*		Delphini	Del
Dorado	S	Doradus	Dor
Draco*	N	Draconis	Dra
Equuleus*		Equulei	Equ
Eridanus*		Eridani	Eri
Fornax		Fornacis	For
Gemini*	Z	Geminorum	Gem
Grus	S	Gruis	Gru
Hercules*		Herculis	Her
Horologium	S	Horologii	Hor
Hydra*		Hydræ	Hya
Hydrus	S	Hydri	Hyi
Indus	S	Indi	Ind
Lacerta	N	Lacertæ	Lac
Leo*	Z	Leonis	Leo
Leo Minor		Leonis Minoris	LMi
Lepus*		Leporis	Lep
Libra*	Z	Libræ	Lib
Lupus*	S	Lupi	Lup
Lynx	N	Lyncis	Lyn
Lyra*		Lyræ	Lyr
Mensa	S	Mensæ	Men
Microscopium		Microscopii	Mic
Monoceros		Monocerotis	Mon
Musca	S	Muscæ	Mus
Norma	S	Normæ	Nor
Octans	S	Octantis	Oct
Ophiuchus*		Ophiuchi	Oph
Orion*		Orionis	Ori

TABLE XXXV (CONTINUED)

NAME		GENITIVE CASE	ABBREVIATION
Pavo	S	Pavonis	Pav
Pegasus*		Pegasi	Peg
Perseus*	N	Persei	Per
Phœnix	S	Phœnicis	Phe
Pictor	S	Pictoris	Pic
Pisces*	Z	Piscium	Psc
Piscis Austrinus*		Piscis Austrini	PsA
Puppis		Puppis	Pup
Pyxis		Pyxidis	Pyx
Reticulum	S	Reticuli	Ret
Sagitta*		Sagittæ	Sge
Sagittarius*	Z	Sagittarii	Sgr
Scorpius*	Z	Scorpii	Sco
Sculptor		Sculptoris	Scl
Scutum		Scuti	Sct
Serpens*		Serpentis	Ser
Sextans		Sextantis	Sex
Taurus*	Z	Tauri	Tau
Telescopium	S	Telescopii	Tel
Triangulum*		Trianguli	Tri
Triangulum Australe	S	Trianguli Australis	TrA
Tucana	S	Tucanæ	Tuc
Ursa Major*	N	Ursæ Majoris	UMa
Ursa Minor*	N	Ursæ Minoris	UMi
Vela	S	Velorum	Vel
Virgo*	Z	Virginis	Vir
Volans	S	Volantis	Vol
Vulpecula		Vulpeculæ	Vul

The genitive case is used with the letter or number designating a particular star. Thus, the brightest star in the constellation Ursa Minor is α Ursæ Minoris (α UMi). The three-letter abbreviation is that adopted in the 1922 meeting of the International Astronomical Union with the object of saving space in printing.

Puppis, Vela, and Carina, now regarded as separate constellations, were once embraced in the single constellation of Argo-Navis. Only one sequence of letters — α, β, etc. — is used for the three constellations.

TABLE XXXVI. THE CHEMICAL ELEMENTS

Element	Symbol	Atomic Number	Atomic Weight	Element	Symbol	Atomic Number	Atomic Weight
Hydrogen	H	1	1.008	Silver	Ag	47	107.88
Helium	He	2	4.000	Cadmium	Cd	48	112.41
Lithium	Li	3	6.94	Indium	In	49	114.8
Beryllium	Be	4	9.02	Tin	Sn	50	118.70
Boron	B	5	10.85	Antimony	Sb	51	121.73
Carbon	C	6	12.00	Tellurium	Te	52	127.5
Nitrogen	N	7	14.01	Iodine	I	53	126.93
Oxygen	O	8	16.000	Xenon	Xe	54	130.2
Fluorine	F	9	19.00	Caesium	Cs	55	132.81
Neon	Ne	10	20.2	Barium	Ba	56	137.37
Sodium	Na	11	23.00	Lanthanum	La	57	138.90
Magnesium	Mg	12	24.32	Cerium	Ce	58	140.25
Aluminum	Al	13	26.97	Praseodymium	Pr	59	140.92
Silicon	Si	14	28.06	Neodymium	Nd	60	144.27
Phosphorus	P	15	31.03	Illinium	Il	61	
Sulphur	S	16	32.06	Samarium	Sm	62	150.43
Chlorine	Cl	17	35.46	Europium	Eu	63	152.0
Argon	A	18	39.91	Gadolinium	Gd	64	157.26
Potassium	K	19	39.10	Terbium	Tb	65	159.2
Calcium	Ca	20	40.07	Dysprosium	Dy	66	162.52
Scandium	Sc	21	45.10	Holmium	Ho	67	163.47
Titanium	Ti	22	48.1	Erbium	Er	68	167.7
Vanadium	V	23	50.96	Thulium	Tm	69	169.4
Chromium	Cr	24	52.01	Ytterbium	Yb	70	173.6
Manganese	Mn	25	54.93	Lutecium	Lu	71	175.0
Iron	Fe	26	55.84	Hafnium	Hf	72	178.6
Cobalt	Co	27	58.94	Tantalum	Ta	73	181.5
Nickel	Ni	28	58.69	Tungsten	W	74	184.0
Copper	Cu	29	63.57	Rhenium	Re	75	
Zinc	Zn	30	65.38	Osmium	Os	76	190.8
Gallium	Ga	31	69.72	Iridium	Ir	77	193.1
Germanium	Ge	32	72.60	Platinum	Pt	78	195.23
Arsenic	As	33	74.96	Gold	Au	79	197.2
Selenium	Se	34	79.23	Mercury	Hg	80	200.61
Bromine	Br	35	79.92	Thallium	Tl	81	204.39
Krypton	Kr	36	82.9	Lead	Pb	82	207.18
Rubidium	Rb	37	85.44	Bismuth	Bi	83	209.00
Strontium	Sr	38	87.63	Polonium	Po	84	210.
Yttrium	Y	39	88.9	Unknown		85	
Zirconium	Zr	40	91.	Radon	Rn	86	222.
Columbium	Cb	41	93.1	Unknown		87	
Molybdenum	Mo	42	96.0	Radium	Ra	88	225.95
Masurium	Ma	43		Actinium	Ac	89	227.
Ruthenium	Ru	44	101.7	Thorium	Th	90	232.15
Rhodium	Rh	45	102.91	Protoactinium	Pa	91	234.
Palladium	Pd	46	106.7	Uranium	U	92	238.17

NOTE. These are the "international atomic weights" for 1925, to two places of decimals, where known. In a few cases more recent determinations are given.

Masurium, illinium, and rhenium have been reported very recently, leaving only "85" and "87" to be found, of the 92 elements no heavier than uranium.

In many cases, as for chlorine, the elements are known to be mixtures of "isotopes," whose atomic weights (for example, 35 and 37 for Cl) are practically integers.

TABLE XXXVII. THE REVISED PERIODIC SYSTEM OF THE ELEMENTS

																	H	**He**
																	Li	Be
											B	C	N	O	F	**Ne**	Na	Mg
											Al	Si	P	S	Cl	**A**	K	Ca
	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	**Kr**	Rb	Sr
	Y	Zr	Cb	Mo	Ma	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	**Xe**	Cs	Ba
La (rare earths)		Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	85	**Rn**	87	Ra

If the elements are arranged in the order of their atomic numbers, as above, those in the same vertical column have spectra of very similar (and often identical) character, and are usually similar in chemical properties. There is good reason to believe that for the atoms listed in each column the structure of the outer parts is the same and that the groups of two, six, and ten columns correspond to the "building on" of successive "shells of electrons" in "orbits" of different quantum numbers.

For the inert gases, for which the symbols are printed in heavy type, the atoms are believed to be of especially symmetrical structure, all the "shells" of electrons being complete, so that they have practically no tendency to combine with other atoms.

The rare earths form an additional group which should have a place farther to the left, as indicated. Th and U probably belong below Hf and W.

TABLE XXXVIII. IONIZATION POTENTIALS OF ASTROPHYSICAL IMPORTANCE

H	13.54	Na	5.13	Ca +	11.82	Ge	7.83
He	24.41	Na +	30:	Sc	6.57	As	11:
He +	54.18	Mg	7.61	Sc +	12.80	Rb	4.16
Li	5.37	Mg +	14.97	Sc + +	24.1	Sr	5.67
Li +	68:	Al	5.96	Ti	6.80	Sr +	10.98
Be	9.6:	Al +	18.18	Ti +	13.60	Y	6:
B	8.0:	Al + +	28.32	Ti ++	28.4	Mo	7.35
C	11.3	Si	8.11	Ti + + +	43.07	Ag	7.54
C +	24.3	Si +	16.27	V	6.76	Cd	8.95
C + +	46.4	Si + +	31.66 +	Cr	6.72	In	5.76
N	14.18	Si + + +	44.95	Mn	7.41	Sn	7.30
N +	29.6	P	12 ±	Fe	7.83	Ba	5.19
N + +	47.2	P + +	30.4	Fe +	16.5	Ba +	9.96
O	13.56	S	10.31	Co	7.81	La	6:
O +	35.00	A	15.69	Ni	7.64	Au	9.20
O + +	54.8	K	4.32	Cu	7.69	Hg	10.39
F	16.9	K +	22:	Zn	9.35	Tl	6.08
Ne	21.47	Ca	6.09	Ga	5.97	Pb	7.38

NOTE. These measure the least energy in *volts* (§ 637) which is capable of removing an electron from a neutral atom (or an additional electron from an ionized atom) (see Chapter XVII). The values given to two decimal places are derived from spectroscopic data, and should be highly accurate. The others have been measured or estimated in different ways, and are only approximately correct.

Table XXXIX. Physical Constants of Astrophysical Significance

Constant of gravitation (§ 308) $G = 6.673 \times 10^{-8}$ cm.^{3}/gram sec.^{2} (Woodward.)
Velocity of light $c = 2.998 \times 10^{10}$ cm./sec. (Michelson, 1926.)
Planck's constant $h = 6.56 \times 10^{-27}$ erg sec. (Birge, 1926.)
Charge of electron $e = 4.775 \times 10^{-10}$ electrostatic units. (Birge, 1926.)
Mass of electron $m = 9.05 \times 10^{-28}$ grams. (Birge, 1926.)
Rydberg's constant for hydrogen (§ 629) $= 109{,}677.6$ cm.$^{-1}$ (Birge, 1926.)
Mass of "proton," or hydrogen nucleus $= 1.663 \times 10^{-24}$ gram. (*I.C.T.*)
Mass of the unit of atomic weight $= 1.650 \times 10^{-24}$ gram. (*I.C.T.*)
Avogadro's number, or number of units of atomic weight per gram, $N_0 = 6.06 \times 10^{23}$. (*I.C.T.*)
Loschmidt's number, or number of molecules per cm.^{3} of a gas under standard conditions, $n_0 = 2.705 \times 10^{19}$/cm.^{3} (*I.C.T.*)
Value in dynes/cm.^{2} of a pressure of 1 atmosphere $= 1{,}013{,}250$. (*I.C.T.*)
The gas constant (§ 962) $R = 8.315 \times 10^7$ if pressure is measured in dynes, density in grams/cm.^{3} (*I.C.T.*)
The gas constant $R = 82.06$ if pressure is measured in atmospheres. (*I.C.T.*)
The molecular gas constant $k_0 = 1.37 \times 10^{-16}$ erg/degree. (*I.C.T.*) ($N_0 k_0 = R$. The theoretical mean translational energy of a monatomic molecule at absolute temperature T is $3/2\ k_0\ T$.)
Energy density of radiation in (evacuated) inclosure at absolute temperature T is aT^4 ergs, where $a = 7.63 \times 10^{-15}$ erg/cm.^{3} degree^4. Radiation pressure in this inclosure $= \frac{1}{3} aT^4$ dynes/cm.^{2}
Stefan's constant (§ 607) $\sigma = 5.72 \times 10^{-5}$ erg/cm.^{2} sec. degree$^4 = ac/4$.
Wien's displacement constant (§ 608) $= 0.289$ cm. degree.
Mechanical equivalent of heat: 1 calorie $= 4.18 \times 10^7$ ergs. (*I.C.T.*)
The equivalent in ergs of 1 "volt" (§ 637) $= 1.59 \times 10^{-12}$ ergs.

Note 1. Except where otherwise expressly stated, the foregoing values are given in the centimeter-gram-second system of units. The heat units refer to the centigrade degree. In some cases slightly different values of some of these constants have been employed in the text of this book. Certain values of this table represent later determinations, and have been revised during the reading of the proof.

Note 2. The references to Birge are to a brief paper by him in *Science*, Vol. LXIV, p. 180, August, 1926; the value of e there employed has been changed here to take account of Michelson's latest determination of c. The references to *I.C.T.* are to *International Critical Tables*, Vol. I (1926), p. 18.

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(References are to pages. Those to Volume I are in italics.)